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Opinion Makers Section

Harmonizing priority weights and indifference judgments in value function implementation

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Introduction

Value functions are one of the most popular ways of dealing with multiattribute decision making problems in real world problems, namely in situations like public tenders where the transparency of the evaluation methodology is an important aspect. The fact is that, many times, value functions used in practice are built in a naïve way, beginning with the definition of "weights" by the Decision Maker (DM), complemented by a more or less arbitrary specification of the individual value functions for each attribute. An additive model is generally assumed, sometimes without really checking the correspondent requirements.

The experience of the author in supporting this kind of processes, namely in the public transport and power systems sectors, showed that DM (and committees acting the DM, the most frequent case) adhere without difficulty to a formal process of building correctly a multiattribute value function and are able to express their preferences in terms of trade-offs or indifference judgments, after some discussion and having gained insight to the problem. However, there are frequently conflicts between the parameters of the multiattribute value function ("weights") determined by the formal process and the intuitive notion of relative importance of the criteria the DM keep in their minds, as a consequence of their personal qualitative analysis of the situation, or want to impose, due to political or social constraints in the decision framework. These conflicts must be taken seriously, since they may compromise the success of the entire decision-aid process.

This paper is devoted to the discussion of the means that can be used to dissolve these conflicts, in order to help building value functions that are technically correct and at the same time are compatible to the perception the DM has about the relative importance of the criteria.

In a first approach to the problem, it may seem that the inconsistency resides in the DM, since all the information comes from him. Of course this may happen: a DM that in his mind privileges environment protection versus cost may show the opposite spirit in specific situations, when faced with real costs and real environmental consequences. But, most of the times, the inconsistency is related with the definition of the range of the individual value function of each attribute, an aspect that generally is not the main concern in the decision-aid process.

In the next section we will review the main concepts associated to the use of value functions, including the usual procedure to determine the parameters, and discuss the origin of the conflict. Section 3 introduces the methodology we advocate to deal with the problem, for the most frequent case of linear additive functions, following our presentation in the 67th meeting of the European Working Group "Multiple Criteria Decision Aiding". A complete methodology for the linear case is presented in section 4. Illustrative examples are distributed along these sections. Conclusions and references conclude the paper.

The problem

Since the topic of value functions is well known - see, for instance, Keeney and Raiffa (1976), Keeney (1992), Clemen and Reilly (2001) or Belton and Stewart (2002) – we will just review the main definitions, in order to establish the nomenclature. However, in relation with the problem in hand, we must point out that methodologies like AHP (Saaty, 1980) and similar procedures do respect the perception of the DM of the relative importance of the attributes. In exchange, they do not use real individual value functions $v_i(x_i)$, since the score of an alternative in each attribute depends on all the remaining alternatives (leading sometimes to rank reversal when a new alternative is introduced).

Differently, the Macbeth approach (Bana e Costa and Vansnick, 1994) takes steps towards the agreement of the process of building the multiattribute value function with the DM's perception of the relative importance of the criteria, by using absolute reference levels in all the attributes. Recently, Bana e Costa et al (2008) show how to use Macbeth in situations close to the ones discussed in this paper, with similar objectives.

Finally, the methods of the "French School", like ELECTRE (Roy, 1991) and related methodologies, are based in a entirely different approach to decision-aid, where weights do not have the compensatory meaning they have in value function theory, so they are out of this discussion.

We will assume that the requirements for an additive function are fulfilled. In these circumstances, in a decision making situation with n attributes, the normalized value function will be:

$$v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} k_i . v_i(x_i)$$

where the individual (or partial) value functions $v_i(x_i)$ may be linear, polynomial, exponential, or any other specific shape that reflects the variation of the preference of the DM along the correspondent attribute. We will consider through the text that all the partial value functions are normalized, between $v_i(x_{i,worst})=0$ and $v_i(x_{i,best})=1$, so also $\sum_i k_i = 1$. Note that “worst” and “best” are used here to define the range limits, without any connection with possibly existing alternatives.

The typical procedure to build an additive value function for a specific problem has the following steps:

1) Define separately each of the individual value functions $v_i(x_i)$. This means establishing first the range of the function in attribute i, and then determining (or choosing) the appropriate shape for the function, after interaction with the DM;
2) Obtain additional information from the DM in order to determine the parameters k_i of the multiattribute value function. This may be done in a number of ways, the more frequent of which is based on judgments of indifference between (artificial) alternatives. Each indifference generates an equation on the parameters, so $n-1$ independent judgments are necessary (and sufficient) in the general case to calculate the totality of the parameters (since $\sum_i k_i = 1$).

We illustrate this process with a simple example that is also used to show the kind of conflict we are speaking about. Note that the method used to calculate the parameters is independent of the shape of the individual value functions.

<u>Example 1</u>

We want to build a methodology to evaluate candidates regarding cost, in millions of € (x_1) and duration, in weeks (x_2) of a specific task. Interaction with the DM led to the definition of the two individual value functions, in this case linear:

$$v_1(x_1) = \frac{x_1 - x_{1,worst}}{x_{1,best} - x_{1,worst}} = \frac{x_1 - 3.2}{2.6 - 3.2}$$
$$v_2(x_2) = \frac{x_2 - x_{2,worst}}{x_{2,best} - x_{2,worst}} = \frac{x_2 - 50}{30 - 50}$$

Now, in order to determine the parameters, two artificial alternatives A=(2.6, 50) and B=(3.2, 30) are presented to the DM. Since he prefers B, we change B until we reach an indifference, say A~(3.2, 35). This allows us to build the equation $v(A)=v(3.2, 35)$ and, since $k_1+k_2=1$,

$$k_1.1 + k_2.0 = k_1.0 + k_2.(0.75)$$
$$k_1 = 0.75(1 - k_1)$$
$$k_1 = 0.429,\ k_2 = 0.571$$

At this point, the DM rejected the value function, because in his mind he was convinced that duration was less important than cost, and the “weights” just determined display the opposite, since $k_1<k_2$.

The point here is: what to do? Of course we may try to convince the DM he is wrong and should accept our model anyway (generally a not very successful option) , but we also may review the process and check if we still have some degrees of freedom to accommodate the DM’s strong feeling that cost is more important.

Now, before going to our proposal to deal with this problem, it is worth remembering, for the sake of generality, that, with more than two attributes, the process is similar and subject to same comments. Finding indifference judgments departs generally from artificial alternatives where all the attributes except one are in their worst value. For instance:

$$A_1 = (x_{1,best} \quad x_{2,worst} \quad x_{3,worst} \quad \cdots \quad x_{n,worst})$$
$$B_1 = (x_{1,worst} \quad x_{2,best} \quad x_{3,worst} \quad \cdots \quad x_{n,worst})$$

Therefore, when comparing A_1 and B_1, the DM has just to consider the first two criteria, since A and B are equal in all the others. If $A_1 \succ B_1$, we immediately know that $k_1>k_2$, but would need again to improve the first attribute of B_1 until reaching an indifference.

It should also be noted that, in linear value functions, the judgment of indifference is equivalent to setting trade-offs between the attributes. For instance, in the case of Example 1, the trade-off is 0.6/15=0.04 M€/week. This judgment must of course be preserved by any methodology we choose. Of course, when the individual functions are not linear, trade-offs are only local and cannot be generalized to the whole range of the attributes, so the term “trade-offs between the attributes” makes no sense in that case.

A consistent approach for the linear case

In order to ease the explanation, we analyze, without loss of generality, the case of a two-attribute linear function. We start from the individual value functions, where R_i is the range of attribute i (note that it can be negative, in minimization attributes):

$$v_i(x_i) = \frac{x_i - x_{i,worst}}{x_{i,best} - x_{i,worst}} = \frac{x_i - x_{i,wst}}{R_i}$$

We also know that *A~B*, $A=(a_1, a_2)$ and $B=(b_1, b_2)$. Therefore, to calculate the parameters we must solve *v(A)=v(B)*:

$$k_1 \frac{a_1 - x_{1,wst}}{R_1} + k_2 \frac{a_2 - x_{2,wst}}{R_2} = k_1 \frac{b_1 - x_{1,wst}}{R_1} + k_2 \frac{b_2 - x_{2,wst}}{R_2}$$

or

$$\frac{k_1}{R_1} a_1 + \frac{k_2}{R_2} a_2 = \frac{k_1}{R_1} b_1 + \frac{k_2}{R_2} b_2$$

and

$$\frac{k_1}{k_2} = -\frac{R_1}{R_2} \cdot \frac{a_2 - b_2}{a_1 - b_1}$$

This turns manifest a well-known fact, sometimes forgotten: the relation between the parameters depends, not only on the trade-off assumed (directly or indirectly) by the DM, but also on the *ranges* of the individual value functions. For instance, if all preference information remains unchanged, but we modify the range of the first attribute to $R_1' = \alpha . R_1$, the result is:

$$\frac{k_1'}{k_2'} = \alpha \cdot \frac{k_1}{k_2}$$

The point here is: do you have really this degree of freedom regarding the range of values to consider? The answer is: most of the times, yes. All practitioners know that, in many situations, range limits are fuzzy and some discretionary judgment is used to set the values for the individual value function of the specific attribute. So, within certain limits, we can extend or restrict the range considered for the attribute without compromising the representation of the decision situation or include unrealistic values in the range.

A different discussion regards the *absolute* meaning of the value in a specific attribute. One could say that *v(cost)=0.5* should always mean that the corresponding alternative is halfway in the *internal value scale* of the DM. The fact, however, is that value functions (or utility functions) are valid up to a linear transformation of its scale, so we are just trying to build a mathematical model that respects the DM's relations between *value differences* between pairs of alternatives.

Coming again to the model (and taking α=2 to ease the explanation), the new multiattribute value function would be (note that $x_{1,wst}'$ may be different from the former worst value $x_{1,wst}$, since the range has changed):

$$v'(x_1, x_2) = k_1' \frac{x_1 - x_{1,wst}'}{R_1'} + k_2' \frac{x_2 - x_{2,wst}}{R_2}$$

$$v'(x_1, x_2) = \frac{k_1}{k_1 + 1} \cdot \frac{x_1 - x_{1,wst}'}{R_1} + \frac{k_2}{k_1 + 1} \cdot \frac{x_2 - x_{2,wst}}{R_2}$$

$$v'(x_1, x_2) = \frac{1}{k_1 + 1} \cdot \left(k_1 \frac{x_1 - x_{1,wst}}{R_1} + k_2 \frac{x_2 - x_{2,wst}}{R_2} \right) + \frac{k_1 (x_{1,wst} - x_{1,wst}')}{R_1 (k_1 + 1)}$$

and

$$v'(x_1, x_2) = M \cdot v(x_1, x_2) + N$$

so, for every alternatives X, Y:

$$\frac{v'(X) - v'(Y)}{v(X) - v(Y)} = constant$$

In summary, the modification of the range of an attribute leads to a change in the parameters, but the new value function is just a linear transformation of the preceding one, so it is equivalent to it, because it induces the same order in any set of alternatives and respects the ratio of difference of value between pairs of alternatives (this conclusion if valid for any value of α, as the reader may check easily).

This can be used to adapt the "weights" of the value function to the desires of the DM, without changing the order induced by the preference information he previously delivered. We now extend Example 1 to illustrate the idea.

Example 2

Following his disagreement with the weights, the DM indicated that k_1=0.6 and k_2=0.4 would be close to his perception of the relative importance of the two criteria. This allowed us to calculate the required change in R_1 through

$$\frac{R_1'}{R_2} = -\frac{k_1}{k_2} \cdot \frac{a_1 - b_1}{a_2 - b_2} = -\frac{0.6}{0.4} \cdot \frac{2.6 - 3.2}{50 - 35} = 0.06$$

leading to $R_1' = 0.06 R_2 = 1.2 = R_1 + 0.6$. Splitting the surplus in two equal parts (other hypotheses are possible), the modified $v_1(x_1)$ would be:

$$v_1'(x_1) = \frac{x_1 - 3.5}{2.3 - 3.5}$$

So, the old unacceptable (for the DM) multiattribute value function:

$$v(x_1, x_2) = 0.429 \; \frac{x_1 - 3.2}{2.6 - 3.2} + 0.571 \; \frac{x_2 - 50}{30 - 50}$$

is substituted by:

$$v'(x_1,x_2)=0.6\ \frac{x_1-3.5}{2.3-3.5}+0.4\ \frac{x_2-50}{30-50}$$

that fully agrees with the DM’s perception of the weights and also respects the preference information (indifference judgment) given by him. In this case, it is easy to see that:

$$v'(x_1,x_2)=0.7\,v(x_1,x_2)+0.15$$

A general procedure to reconcile preference information in the linear case

In order to tackle more general situations, we define the following starting point:

a) A multiattribute linear value function is defined, but the ranges of each attribute are not known in advance. However, an internal and an external range are defined for each attribute. The internal ranges should include all the possible values that may appear in the alternatives, while the external range limits the value functions to credible values of the attribute.
b) The DM defines a set of weights for the value function;
c) There are n-1 independent judgments of indifference (or trade-offs) supplied by the DM.

More formally, our data are the parameters k_1, k_2, …, k_n, judgments A_j~B_j, $j=2..n$, and internal and external ranges $\left[r_{i-}^{\text{int}}\quad r_{i+}^{\text{int}}\right]$ and $\left[r_{i-}^{\text{ext}}\quad r_{i+}^{\text{ext}}\right]$, respectively. The aim of the exercise is to determine the final ranges $\left[r_{i-}\quad r_{i+}\right]$ for each attribute that respect simultaneously the preferences of the DM and the internal and external ranges. The general form of the multiattribute value function is, assuming minimizing attributes (including maximizing attributes would be straightforward):

$$v(x_1,x_2,\ldots,x_n)=\sum_{i=1}^{n}k_i.\frac{r_{i+}-x_i}{r_{i+}-r_{i-}}$$

Without loss of generality, we will assume that the judgments of indifference involve only two criteria at a time (as mentioned in the previous section), but also that criterion 1 is part of all the indifferences, so

$$A_j\sim B_j\Rightarrow k_1\frac{a_1-b_1}{r_{1+}-r_{1-}}+k_j\cdot\frac{a_j-b_j}{r_{j+}-r_{j-}}=0$$

leading to the equation that relates the range limits (note that K_j is a constant):

$$r_{1+}-r_{1-}+\frac{k_1}{k_j}\cdot\frac{a_1-b_1}{a_j-b_j}\cdot\left(r_{j+}-r_{j-}\right)=r_{1+}-r_{1-}+K_j\cdot\left(r_{j+}-r_{j-}\right)=0$$

Besides these constraints ($j=2..n$), we must include ($i=1..n$):

$$r_{i+}^{\text{int}}\le r_{i+}\le r_{i+}^{\text{ext}}$$

$$r_{i-}^{\text{ext}}\le r_{i-}\le r_{i-}^{\text{int}}$$

Any solution $\left(r_{1-}\ r_{1+}\ r_{2-}\ r_{2+}\ \cdots\ r_{n-}\ r_{n+}\right)$ that respects the previous constraints would lead to the desired multiattribute value function. If the solution set is empty, expanding the external ranges would be needed in order to regain feasibility, unless the specifications of the DM are so conflicting that no solution exists really in plausible ranges of the attributes.

In order to solve the problem of finding a specific solution, the simplest way is to build a linear program with a convenient objective function, like for instance:

$$\min f=\sum_{i=1}^{n}\left(\frac{r_{i+}-r_{i-}}{r_{i+}^{\text{int}}-r_{i+}^{\text{ext}}}\right)$$

or

$$\min f'=y\text{ , with }y\ge\frac{r_{i-}^{\text{int}}-r_{i-}}{r_{i-}^{\text{int}}+r_{i+}^{\text{int}}}\ ,\ y\ge\frac{r_{i+}-r_{i+}^{\text{int}}}{r_{i-}^{\text{int}}+r_{i+}^{\text{int}}}$$

We now illustrate the procedure using again the situation of Examples 1 and 2.

<u>Example 3</u>

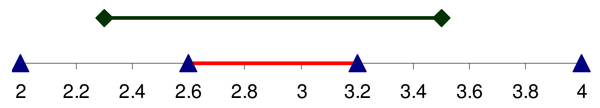
Besides the indifference stated in Example 1 and the weights indicated in Example 2, we obtained from the DM the following internal and external ranges:

$$2.0\le r_{1-}\le 2.6\qquad 3.2\le r_{1+}\le 4.0$$
$$20\le r_{2-}\le 30\qquad 50\le r_{2+}\le 60$$

We may now establish the set of constraints adding the equation:

$$r_{1+}-r_{1-}+\frac{0.6}{0.4}\cdot\frac{2.6-3.2}{50-35}\cdot\left(r_{2+}-r_{2-}\right)=0$$
or
$$r_{1+}-r_{1-}-0.06\cdot\left(r_{2+}-r_{2-}\right)=0$$

Using now the objective function f mentioned before, we find multiple solutions, as expected due to degrees of freedom of the problem. In fact, all the feasible solutions with R_1=1.2 and R_2=20, like (2.6 3.8 30 50) or (2.0 3.2 30 50), can be used, because they are equivalent in all senses, as discussed before. With *f'*, we would obtain the solution of Example 2, centered in the internal range. The next figure shows the ranges and depicts this solution.



Conclusions

The paper shows how to deal with the possible inconsistency between value functions built "by the book" and the perception of the DM regarding the relative importance of the criteria, in real decision-aid situations. The discussion turns clear that the answer resides in changing the ranges of the individual value functions, since expanding the range of an attribute leads to an increase of the correspondent weight, while respecting the remaining preference information (judgments of indifference) provided by the DM.

A complete procedure was presented to linear multiattribute functions, and can easily be extended to include exponential functions. The author believes that the material presented in the paper is useful for practitioners, since this kind of situation appears more often than not.

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