A Bicriteria Approach To Hub Location Problems

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Hub Location Problems (1)

- Some traffic must be transported from a origin point \((i)\) to a destination point \((j)\)
- The direct link between each origin-destination pair is impossible to establish or too expensive
- Hubs will act as consolidation, switching and distribution centres for traffic
- They enable economies of scale, resulting in lower transportation costs
- Hubs are fully interconnected
- Non-hub nodes can be linked directly to one hub
We have studied the Capacitated Single Allocation Hub Location Problem (2).
Notations (1)

- **Input Data**
  - $W_{ij}$: flow from location $i$ to location $j$
  - $O_i = \sum_j W_{ij}$: total flow from $i$
  - $D_i = \sum_j W_{ji}$: total flow to $i$
  - $d_{ij}$: distance from $i$ to $j$
  - $F_k$: fixed cost of establishing a hub at node $k$
  - $\Gamma_k$: capacity of hub $k$
  - $T_k$: time hub $k$ takes to process one unit of flow
  - $P_k$: fixed time to initiate the service at hub $k$
Notations (2)

- Constants
  - $\alpha$: coefficient of the transfer cost between hubs ($0 < \alpha < 1$)
  - $\chi$: coefficient of the collection cost
  - $\delta$: coefficient of the distribution cost
  - $n$: number of nodes

- Variables
  - $Y_{km}^i$: total amount of flow from location $i$ that is routed via hubs $k$ and $m$
  - $Z_{ik}$: 1 if node $i$ is allocated to a hub in node $k$
  - $Z_{kk}$: 1 if node $k$ is selected as a hub
Mathematical Formulation

\[
\begin{align*}
\min & \quad \sum_{i} \sum_{k} d_{ik} Z_{ik} (xO_i + SDS_i) + \sum_{i} \sum_{k} \sum_{m} \alpha d_{km} Y_{km}^i + \sum_{k} F_k Z_{kk} \\
\text{s.t.:} & \\
\sum_{k} Z_{ik} &= 1, \quad \forall i \\
Z_{ik} &\leq Z_{kk}, \quad \forall i, k \\
\sum_{i} O_i Z_{ik} &\leq \Gamma_k Z_{kk}, \quad \forall k \\
\sum_{m} Y_{km}^i - \sum_{m} Y_{mk}^i &= O_i Z_{ik} - \sum_{j} W_{ij} Z_{jk}, \quad \forall i, k \\
Z_{ik} &\in \{0, 1\}, \quad \forall i, k \\
Y_{km}^i &\geq 0, \quad \forall i, k, m
\end{align*}
\]

Formulation proposed by A. Ernst and M. Krishnamoorthy
Two main reasons motivated the use of a bicriteria model:

- The increase of information provided by a multicriteria model
- Specific structure of the Capacitated Single Allocation Hub Location Problem
The Bicriteria Model

\[
\begin{align*}
\min \sum_{i} \sum_{k} d_{ik}Z_{ik} (xO_i + \delta D_i) + \sum_{i} \sum_{k} \sum_{m} \alpha d_{km} Y_{km}^i + \sum_{k} F_k Z_{kk} \\
\min \sum_{i} \sum_{k} O_i T_k Z_{ik} + \sum_{k} P_k Z_{kk}
\end{align*}
\]

s.t.: 

\[
\sum_{k} Z_{ik} = 1, \quad \forall i
\]

\[
Z_{ik} \leq Z_{kk}, \quad \forall i, k
\]

\[
\sum_{m} Y_{km}^i - \sum_{m} Y_{mk}^i = O_i Z_{ik} - \sum_{j} W_{ij} Z_{jk}, \quad \forall i, k
\]

\[
Z_{ik} \in \{0,1\}, \quad \forall i, k
\]

\[
Y_{km}^i \geq 0, \quad \forall i, k, m
\]
Calculation Of The Non-Dominated Solution Set

- Interactive procedure based upon the progressive and selective knowledge of the non-dominated solution set (n.d.s.s.)
- Two main phases:
  - Dialogue phase
  - Calculation phase
- These two phases go on, alternately and interactively, only ending when the DM considers having sufficient knowledge of the n.d.s.s.
General diagram of the interactive procedure

1. **Start**
   - Calculation of the lexicographic minimum and ideal point

2. **Decision phase**
   - The ideal point is feasible?
     - Yes: STOP There is no conflict between the criteria
     - No: The DM considers to have "sufficient" knowledge about the set of non-dominated solutions?
       - Yes: STOP
       - No: The DM indicates a sub-region to carry on the search for non-dominated solutions by:
         - indicating upper bounds for the value of the objective functions
         - choosing a pair of n.d.s. candidate to be adjacent (through their indexes)

3. Determination of non-dominated solutions on the region of interest
Dialogue Phase

- The decision maker is asked to give indications about the sub-region to carry on the search for new non-dominated solutions.

- These information can be transmitted by:
  - Indicating upper bounds on both values of the objective functions.
  - Indicating two non-dominated solutions candidate to be adjacent.
Calculation Phase (1)

- Optimization of a single criterion problem representing the weighted sum of both objective functions, imposing limits on their values accordingly to the preferences expressed by the DM during the dialogue phase.

- For a minimum bicriteria problem, this corresponds to solve the next single-criterion problem:
Calculation Phase (2)

\[
\min Z = \lambda Z_1(x) + (1 - \lambda) Z_2(x)
\]

s. t.:

\[
Z_1(x) \leq Z_1^{(s)} - \varepsilon
\]
\[
Z_2(x) \leq Z_2^{(r)} - \varepsilon
\]
\[
\chi \in S
\]
\[
0 < \lambda < 1
\]

\(\varepsilon > 0\) is sufficiently small to allow the calculation of a new non-dominated solution distinct from \(Z^{(s)}\) and \(Z^{(r)}\).
Input Data

- AP data set (Australian Post)
  - 200 nodes representing postcode districts
  - Nodes coordinates
  - Flow volumes between nodes
  - Fixed costs of establishing hubs (2 types: T and L)
  - Flow capacities (2 types: T and L)

- We have studied problems with 10, 20, 25 and 40 nodes, in a total of 16 problems ($\chi = 3, \alpha = 0.75, \delta = 2$)
Non-Dominated Solutions Set Analysis

Problems grouped in three different sets:

- **Group I**: optimal solution to single-criterion model corresponds to one of the non-dominated solutions of the bicriteria model.
- **Group II**: optimal solution to single-criterion model corresponds to the lexicographic minimum of first objective function.
- **Group III**: optimal solution to single-criterion model corresponds to a dominated solution of the bicriteria model.
Group I

- Optimal solution of the original model is one of the n.d.s. of the bicriteria model
- Problems 10LT, 20LT, 25LL, 25TL and 40LL
- The bicriteria model identifies alternative solutions that allow to improve the value of one of the two criteria:
  - lower total cost values
  - lower service times
Problem 10LT
Group II

- Optimal solution of the single-criterion model is the lexicographic minimum of the cost function for the bicriteria model.
- Problems 10LL, 10TL, 10TT, 20LL and 20TL
- Capacity constraints of the original model are not restrictive.
- The entire set of n.d.s. gives solutions with lower service times and greater values on the costs.
- N.d.s. point out to the DM some alternatives characterized by an improvement of service quality.
Problem 10TL
Group III

- Optimal solution of the single-criterion model is a dominated solution of the bicriteria model
- Problems 20TT, 25LT, 25TT, 40LT, 40TL and 40TT
- Those solutions that dominate the original solution have excessive flow concentration on hubs
- It is interesting to compare the improvement in both criteria with the excessive flow incurred

<table>
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<tr>
<th>Problem</th>
<th>Number of n.d.s.</th>
<th>N.d.s. that improve $f_1$ and $f_2$</th>
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<td>1</td>
</tr>
<tr>
<td>40TT</td>
<td>53</td>
<td>6</td>
</tr>
</tbody>
</table>
Problem 20TT
Final Remarks

- The bicriteria model points out to the DM new solutions that, although exceeding hub capacities, allow to improve total cost values, service times and, in some cases, both criteria.
- The final choice of the solution to be implemented belongs to the DM.
- From the 16 problems we have studied:
  - 6 with alternatives that dominate the original solution
  - 5 to which capacity constraints are not restrictive
  - 5 with alternative solutions to improve cost values or service times