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A Hierarchical Multiobjective Routing Model for MPLS Networks with Two Service Classes – Analysis and Resolution Approach*

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Abstract

Modern multiservice network routing functionalities have to deal with multiple, heterogeneous and multifaceted QoS (Quality of Service) requirements. This led to routing models the aim of which is the calculation and selection of one (or more) sequences of network resources (designated as routes, which correspond to loopless paths in the network representation) satisfying certain QoS constraints and the optimisation of route related metrics. Therefore there are potential advantages in formulating important routing problems in these types of networks as multiple objective optimisation problems. These formulations enable the trade-offs among distinct performance metrics and other network cost function(s) to be pursued in a consistent manner. Note that the definition of the objective functions and constraints depends strongly on the nature of the considered routing principles, the type of network technological platform and the features of the offered traffic flows associated with different service types.

In the emergent MPLS (Multiprotocol Label Switching) technology for the Internet the implementation of connection-oriented services from origin to destination is possible. This is made feasible by using LSRs (Label Switching Routers) that forward the packets (grouped in Forward Equivalence Classes, FECs), through LSPs (Label Switched Paths) in the network using a specific packet label switching technique. This feature in association with other functional capabilities of MPLS enables the implementation of advanced QoS-based routing mechanisms, namely through the definition of "explicit routes" (determined at the originating node) for each traffic flow of a given FEC.

Having in mind these features and capabilities of MPLS routing a significant number of routing models have been proposed in the literature in recent years. These approaches often differ in key instances of the modelling framework. In particular such differences are concerned with: i) the scope of the routing optimisation (where we may distinguish network-wide optimisation models and flow-oriented models);

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ii) the nature of the optimisation model in terms of the objective function(s) and constraints (single/multiobjective, type of QoS-related or other constraints, etc); iii) the level of representation of the traffic flows (representation at the level of micro-flows of packet streams carried on a certain LSP or at the level of the "traffic trunks" – aggregation of traffic flows of the same class placed on the same LSP). Based on the analysis of the remarkable differences observed in the models proposed in this area, a discussion on key conceptual issues involved in the various modelling approaches and a proposal of a generic hierarchical multiobjective network-wide routing optimisation framework, was presented in [5].

This work presents, in detail, a model for multiobjective routing in MPLS networks formulated within the framework developed in [5], assuming that there are two classes of services (and different types of traffic flows in each class), namely QoS and Best Effort (BE) services. The flows of QoS type, when accepted by the network, have a guaranteed QoS level, related to the required bandwidth, while BE traffic flows, which are treated in the model as second priority flows, are carried by the network in order to obtain the best possible QoS level for the current network routing solution. Another feature of the routing model is the use of alternative routing: when a first choice route assigned to a given micro-flow, belonging to a certain traffic flow (corresponding to a "traffic trunk") is blocked a second choice route may be attempted. An important feature of this model is the use of hierarchical optimisation typically with two optimisation levels, including fairness objectives: the first priority objective functions refer to the network level objectives of QoS type flows, namely the total expected revenue and the maximal value of the mean blocking of all types of QoS flows; the second priority objective functions refer to performance metrics for the different types of QoS services and the total expected revenue associated with the BE traffic flows. Another important feature of the model is the use of an approximate stochastic representation of the traffic flows in the network, based on the use of the concept of effective bandwidth for macro-flows and on a generalised Erlang model for estimating the blocking probabilities in the arcs, as the one used in [25]. After describing in detail the routing model, including the underlying traffic model, we will present the theoretical foundations of a specialised heuristic strategy for finding "good" compromise solutions to the very complex bi-level routing optimisation problem. This theoretical foundation is based on a conjecture concerning the definition of marginal implied costs for QoS flows and BE flows, which is an extension and adaptation of earlier definitions of implied cost for mono-service networks with alternative routing in [16]. The structure of the heuristic procedure for resolving the problem is analogous to the one described in detail in [5] and [25] and is based on a constrained bi-objective shortest path model the objective functions of which are QoS or BE marginal path implied costs, depending on the class of the routed traffic, and path blocking probabilities. Also a description of the main features of a first version of this heuristic and some preliminary results for a test network will be revealed.

Keywords Routing models, Multiobjective optimisation, Telecommunication networks, MPLS-Internet.

1 Introduction and Motivation

Modern multiservice network routing functionalities have to deal with multiple, heterogeneous and multifaceted QoS (Quality of Service) requirements. This led to routing models the aim of which is the calculation and selection of one (or more) sequences of network resources (designated as routes, which correspond to loopless paths in the network representation) satisfying certain QoS constraints and the optimisation of route related metrics. Therefore there are potential advantages in formulating important routing problems in these types of networks as multiple objective optimisation problems. These formulations enable the trade-offs among distinct performance metrics and other network cost function(s) to be pursued in a consistent manner. Note that the definition of the objective functions and constraints depends strongly on the nature of the considered routing principles, the type of network technological platform and the features of the offered traffic flows associated with different service types.

In the emergent MPLS (Multiprotocol Label Switching) technology for the Internet the implementation of connection-oriented services from origin to destination is possible. This is made feasible by using LSRs (Label Switching Routers) that forward the packets (grouped in Forward Equivalence Classes, FECs), through LSPs (Label Switched Paths) in the network using a specific packet label switching technique. This feature in association with other functional capabilities of MPLS enables the implementation of advanced QoS-based routing mechanisms, namely through the definition of "explicit routes" (determined at the originating node) for each traffic flow of a given FEC.

Having in mind these features and capabilities of MPLS routing a significant number of routing models have been proposed in the literature in recent years. Note that routing in communication networks can be viewed from different perspectives and described at different levels.

Now let us introduce some concepts which will be used to describe routing approaches and their implementations. The term routing method will designate a given specification of a routing principle (involving broad essential features of the routing functionality, for example whether it is static/dynamic or single path/multipath). A key element of a routing method is the procedure used to perform the path calculation and path selection for any given node-to-node connection request at a given time. This procedure is usually designated as *routing algorithm* and its input information is associated with the current network representation (typically network topology, arc capacities, and possibly estimated offered demand and network status information) and the connection requirements.

The description of a routing method involves the definition of a routing model that includes the assumptions and logic-mathematical entities which are necessary for a full understanding and specification of a routing method. A central element is the routing calculation problem typically an optimisation problem (or routing optimisation problem) where the decision variables are the path(s) to be assigned to node-to-node connection requests or 'calls'. A routing model can be described in terms of various features. A key feature is the *routing optimisation framework* which has to do with the scope and nature of the formulation of the routing calculation problem; in this respect we may distinguish network-wide optimisation models and flow-oriented optimisation models. In the former the objective functions are formulated at network level and depend explicitly on all traffic flows in the network. Examples of these functions are average total traffic carried, total expected revenue, average packet delay or a function which seeks the optimisation of the utilisation of the arcs of the network in terms of their level of occupancy, as in [10] and [25]. In contrast, flow-oriented optimisation models consider the objective functions formulated at the level of each particular node-to-node connection or flow, for example number of arcs of the path, path cost (for a specific link usage path metric) or mean packet delay on the particular traffic stream. Examples of this type of models are the numerous QoS routing models which are based on single-objective constrained shortest path formulations (a review can be seen on [20] and an overview in [2]). Another feature refers to the *nature* of the chosen objective functions and constraints, namely whether the optimisation model is single or multiobjective, and the type of functions and constraints (technical, economic, social or other). The *representation* of node-to-node demand requests or traffic offered is also relevant in a telecommunication routing model. We can consider different types of traffic models in terms of the granularity of the representation (for example representation at connection request level or traffic flow level i.e. in terms of a sequence of connections throughout time) or the nature of this representation, namely whether it is deterministic or stochastic.

An overview of some applications of MCDA (Multicriteria Decision Analysis) tools in telecommunication strategic planning and negotiation is shown in [13]. A review on applications of MCDA in telecommunications network planning and design, including a section on routing models is presented in [2]. An overview of a significant number of contributions on multicriteria routing models in telecommunication networks followed by a description of a bi-level hierarchical multicriteria routing model of flow-oriented optimisation type, is put forward in [3].

As discussed in [5], a significant number of routing models for MPLS have been proposed in the literature in recent years which often differ in key instances of the modelling framework. In particular such differences are concerned with: i) the scope of the routing optimisation; ii) the nature of the optimisation model in terms of the objective functions and constraints (single/multiobjective, type of QoS-related or other constraints, etc); iii) the level of representation of the traffic flows (representation at the level of micro-flows of packet streams carried on a certain LSP or at the level of the "traffic trunks" – aggregation of traffic streams of the same class placed on the same LSP). Based on the analysis of the remarkable differences observed in the models proposed in this area, a discussion on key conceptual issues involved in the various modelling approaches and a proposal of a generic hierarchical multiobjective network-wide routing optimisation framework, was presented in [5].

The possibility of applying this modelling framework to a MPLS type network, already outlined in [5], namely by considering two classes of service, QoS traffic (first priority traffic) and Best Effort (BE) traffic (second priority traffic), was a major motivation for this work.

This work presents, in detail, a model for multiobjective routing in MPLS networks formulated within the framework developed in [5], assuming that there are two classes of services (and different types of traffic flows in each class), namely QoS and BE services. The flows of QoS type (first priority flows), when accepted by the network, have a guaranteed QoS level, related to the required bandwidth, while BE traffic flows, which are treated in the model as second priority flows, are carried by the network in order to obtain the best possible QoS level for the current network routing solution. Another feature of the routing model is the use of alternative routing: when a first choice route assigned to a given micro-flow, belonging to a certain traffic flow (corresponding to a "traffic trunk") is blocked a second choice route may be attempted. An important feature of this model is the use of hierarchical optimisation typically with two optimisation levels, including fairness objectives: the first priority objective functions refer to the network level objectives of QoS type flows, namely the total expected revenue and the maximal value of the mean blocking of all types of QoS flows; the second priority objective functions refer to performance metrics for the different types of QoS services and the total expected revenue associated with the BE traffic flows. Another important feature of the model is the use of an approximate stochastic representation of the traffic flows in the network, based on the use of the concept of effective bandwidth for macro-flows and on a generalised Erlang model for estimating the blocking probabilities in the arcs, as the one used in [31, 25]. After describing in detail in section 2 the routing model, including the underlying traffic model, we will present in section 3 the theoretical foundations of a specialised heuristic strategy for finding "good" compromise solutions to the very complex bi-level routing optimisation problem. This theoretical foundation is based on a conjecture concerning the definition of marginal implied costs for QoS flows and BE flows, which is an extension and adaptation of earlier definitions of implied cost for mono-service networks with alternative routing in [16]. The structure of the heuristic procedure for resolving the problem is analogous to the one described in detail in [5, 25]. The new version of the heuristic, presented here, is based on a constrained bi-objective shortest path model the objective functions of which are QoS or BE marginal path implied costs, depending on the class of the routed traffic, and path blocking probabilities. Also in section 4 a description of the main features of a first version of this heuristic will be outlined and some preliminary results for a test network will be revealed.

2 Description of the Routing Model

The present model can be considered as an application of the multiobjective modelling framework for MPLS networks described in [5], where the underlying concepts and methodological considerations are discussed in detail.

We will begin by reviewing the main features of the general model in [5]. This framework (or "meta-model") uses hierarchical optimisation with up to three optimisation levels: the first priority objective functions refer to the global network level; the second priority objective functions refer to performance metrics for the different types of services supported by the network; the third priority functions are concerned with performance metrics for the micro-flows of packet streams of the same FEC.

Another feature of the modelling framework is the consideration of a "dual" stochastic representation of traffic flows in the network: 'macro' level, simply designated as *traffic flow* level (corresponding to the MPLS concept of 'traffic trunk'), and 'micro' level, corresponding to micro-flows of packet streams.

It is a network-wide routing optimisation approach of new type, in the form of a hierarchical multiobjective optimisation model, which takes into account the nature and relations between the adopted objective functions related to the different types of traffic flows associated with different services. We would like to note that various multiobjective models previously proposed use objective functions chosen to reflect only indirectly network technical-economic objectives. A typical example is the minimisation of a utilisation cost for all arcs expressed, through empirical functions, in terms of the occupied bandwidth as in [11, 10, 9, 18]. In fact, the pursued objective is to optimise the total traffic carried in the network or the associated expected revenue. One can say that this type of approaches is just a rough approximation to the 'hidden' (or implicit) objective function the model seeks to reflect, especially taking into account the random nature of traffic patterns, even in stationary or quasi-stationary network working conditions. Instead, our model considers an explicit representation of the most relevant technical-economic objectives in a network-wide routing optimisation, such as the total expected revenue (expressed in terms of the traffic carried of all service types). This aspect of the modelling approach is in line with the school of thought adopted by [16, 17, 31], in the context of single-objective routing models.

We propose a hierarchy of objective functions by considering in a first approach two levels of optimisation with several objective functions in each level. The first level (first priority) includes objective functions formulated at network level for the QoS type traffic and considering the combined effect of all types of traffic flows in the network. The second level refers to average performance metrics of the QoS traffic flows associated with the different types of services supported by the network and the expected revenue of the BE traffic.

An important feature of the model is the explicit consideration, as objective functions, of 'fairness' objectives, at the two levels of optimisation. These are objectives of min-max type and seek to make the most of the proposed multiobjective formulation. In previous formulations of routing models for these networks, such type of aims related to fairness are usually not considered explicitly in any form or just represented through constraints on certain performance metrics. Another important feature of the model is the stochastic representation of the traffic flowing in the network as described in [5, 31].

We will consider two *classes* of services, namely QoS corresponding to services with certain guaranteed QoS levels, and BE, where the corresponding traffic flows are routed seeking to obtain the best possible quality of service but not at the cost of the QoS of the QoS traffic flows (first priority traffic flows). The *service types* in each class are grouped in the sets $\mathcal{S}_Q$ (for QoS service types) and $\mathcal{S}_B$ (for BE service types), and the traffic flows of each service type $s \in \mathcal{S}_Q$ or $s \in \mathcal{S}_B$ may differ in important attributes, namely the required bandwidth.

The consideration of two (or more) classes of traffic flows in a routing model is a complex issue and different approaches have been proposed in the literature. An example of flow-oriented models of this type is in [28], where an admission control technique is proposed based on the reservation in the links of a certain bandwidth BW_1 for the traffic flows of the QoS service class, while those with lower 'priority' (BE) will only be accepted if the available bandwidth is greater than BW_1. Other routing models have been proposed in this area based on the concept of residual virtual bandwidths associated with arc costs for the purpose of computing paths with minimal costs. Examples of these approaches are in [21, 19].

As for network-wide optimisation approaches, [32] describes a bi-objective routing model using lexicographic optimisation where a primary objective function is the weighted sum of the carried bandwidth associated with QoS traffic flows and a secondary objective function of the same type is defined for the BE traffic. A heuristic procedure based on a decomposition technique and multicommodity flow programming is developed for obtaining solutions to the problem.

In our model a *traffic flow* can be specified by $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}(f_s))$ for $s \in \mathcal{S} = \mathcal{S}_Q \cup \mathcal{S}_B$ and corresponds to a stochastic process, in general a marked point process, that describes the arrivals and basic requirements of μ-flows, originated at the MPLS ingress node v_i and destined to the MPLS egress node v_j, using the same LSP and characterised by the vectors of 'attributes' $\overline{\gamma}_s$ and $\overline{\eta}(f_s)$ for service type s. Vector $\overline{\gamma}_s$ describes the traffic engineering attributes of flows of service type s and vector $\overline{\eta}(f_s)$ enables the representation of the mechanism(s) of admission control to all links l_k in the network by calls of flow f_s. In the teletraffic modelling approach described in the Appendix, these attributes include the required effective bandwidth d_s and the mean duration $h(f_s)$ of each μ-flow in f_s. In our model a 'μ-flow' corresponds to a 'call', the term call being used in its broadest sense that is, as a connection request with certain features of the traffic flow. The use of the concept of effective bandwidth [15] in the present context (MPLS networks with explicit routes) was earlier proposed in [31] and used in [24, 25]. This enables an upper level network representation, that is in the traffic plane level, through an equivalent multirate loss traffic network.

Consider that we have an approximate teletraffic model that is capable of estimating the blocking probabilities $B(f_s)$ for all flows f_s of all service types, from which one can calculate the average blocking probability B_{ms}, for all traffic flows of type s, for a given routing choice for all traffic flows. Then the maximal average blocking probability among all QoS service types, $B_{Mm|Q}$, is

$$B_{Mm|Q} = \max_{s \in \mathcal{S}_Q} \{B_{ms}\} \tag{2.1}$$

The total expected network revenues, W_Q and W_B associated with QoS and BE traffic flows, can be calculated in terms of the expected revenues $w(f_s)$ associated with calls of flows $f_s, \forall s \in \mathcal{S}$ and of the values of carried traffic A_s^c for all service types,

$$W_Q = \sum_{s \in \mathcal{S}_Q} W_s = \sum_{s \in \mathcal{S}_Q} A_s^c w_s \tag{2.2}$$

$$W_B = \sum_{s \in \mathcal{S}_B} W_s = \sum_{s \in \mathcal{S}_B} A_s^c w_s \tag{2.3}$$

assuming that $w(f_s) = w_s, \forall f_s \in \mathcal{F}_s$, where $\mathcal{F}_s$ is the set of traffic flows of type s.

In the framework of the meta-model [5] we may formulate a two-level multiobjective routing optimisation problem by separating the total expected revenue in two parts: W_Q for the traffic flows of QoS type and W_B for the traffic flows of BE type as defined above, and by considering explicitly performance optimisation of QoS service types. While W_Q will be a first priority objective function, together with the maximal blocking probability for all QoS service types, $B_{Mm|Q}$, W_B will be a second level objective function. This seeks to guarantee that the routing of BE traffic, in a quasi-stationary situation, will not be made at the cost of the decrease in revenue or at the cost of an increase in the blocking probability of QoS traffic flows.

The second level of optimisation also concerns QoS service types and includes $2|\mathcal{S}_Q|$ objective functions to be minimised: the mean blocking probability for flows of type $s \in \mathcal{S}_Q$,

$$B_{ms|Q} = \frac{1}{A_s^o} \sum_{f_s \in \mathcal{F}_s} A(f_s) B(f_s) \tag{2.4}$$

where $A(f_s)$ is the mean traffic offered associated with f_s (in Erlang) and the maximal loss $B_{Ms|Q}$, defined over all flows of type $s \in \mathcal{S}_Q$,

$$B_{Ms|Q} = \max_{f_s \in \mathcal{F}_s} \{B(f_s)\} \tag{2.5}$$

$B_{Ms|Q}$ represents the fairness objective defined for each service type $s \in \mathcal{S}_Q$.

These considerations led to the following formulation of a two-level hierarchical optimisation problem for two service classes:

Problem P-M2-S2 (2.6)

$$\boxed{\begin{array}{ll}
\text{1st level} & \left\{\begin{array}{ll}\text{QoS - Network objectives:} & \min_{\overline{R}}\{-W_Q\} \\ & \min_{\overline{R}}\{B_{Mm|Q}\}\end{array}\right. \\
\text{2nd level} & \left\{\begin{array}{ll}\text{QoS - Service objectives:} & \min_{\overline{R}}\{B_{ms|Q}\} \\ & \min_{\overline{R}}\{B_{Ms|Q}\} \\ & \forall s \in \mathcal{S}_Q \\ \text{BE - Network objective:} & \min_{\overline{R}}\{-W_B\}\end{array}\right.
\end{array}}$$

subject to equations of the underlying traffic model.

It is important to note that while QoS and BE traffic flows are treated separately in terms of objective functions in order to take into account their different priority in the optimisation model, the interactions among all traffic flows are fully represented in the model. This is guaranteed by the traffic modelling approach underlying the optimisation model, since the traffic model enabling to estimate the blocking probabilities $B(f_s)$ integrates the contributions of all traffic flows which may use every link of the network. This is another major difference in comparison to other routing models proposed for networks with two service classes.

The definition and calculation of the parameters in the expressions are given in detail in Appendix A.

It should be noted that this model is a simplification of the general model for QoS and BE service classes outlined in [5, sec.3.3]. In fact, in this general model a third level of optimisation (third priority level) referring to objective functions (average packet delays) formulated at the level of μ-flows, is considered. These objectives are the average delay experienced by an arbitrary packet of any traffic flow, weighted by the corresponding effective bandwidths, and the maximum of the average packet delay experienced by all types of packet streams. These objectives are related to the bi-level stochastic representation of the traffic flowing in the network, an important feature enabled by the meta-model. That is, a 'macro' level representation of 'traffic flows' which correspond to a stochastic model for representing the MPLS traffic trunks associated with explicit routes, and a 'micro' level representation of streams of packet-flows, corresponding to end-to-end 'μ-flows' of a given traffic flow. This lower-level representation of traffic enables an estimation of average delay related performance parameters.

In the addressed routing optimisation model (2.6) only the macro level representation was considered having in mind to avoid the additional complexity which would result from the inclusion of a third optimisation level in the routing model, as well as the corresponding additional computational burden. Nevertheless the underlying traffic model can be extended in order to encompass a lower-level representation (μ-flow level) by using as first rough approximation the queueing model described in [5, Appendix C].

The traffic modelling approach is the one used in [5] and earlier in [31, 25] for tackling the calculation of blocking probabilities experienced by the traffic flows in network links. It is based on the concept of *effective bandwidth*, in association with the definition of MPLS explicit routes. The effective bandwidth can be considered as a stochastic measure of the utilisation of network resources enabling the representation (in an approximate manner) of the effects of the variability of the rates of different traffic sources, as well as the effects of statistical multiplexing of different traffic flows in a network. It is important to note that 'hiding' packet dynamics features is necessary, in network-wide routing optimisation models, for tractability reasons. In fact, the use of more exact traffic representations would lead to more complex traffic models with an extremely heavy computational burden. A review of the theoretical foundation and application of the effective bandwidth concept can be seen in [15]. This conceptual tool was used in routing optimisation models of multiservice networks of various types as in [31, 25]. Hence, the network may be represented in the traffic plane by a multiclass loss traffic network, equivalent to a multirate traffic circuit-switched network.

The basic calculation sub-model enables the blocking probabilities B_{ks}, for connection requests of service type s in link l_k, to be obtained in the form:

$$B_{ks} = \mathcal{L}_s \left(\overline{d_k}, \overline{\rho_k}, C_k \right) \tag{2.7}$$

$\mathcal{L}_s$ represents the basic function (implicit in the analytical model) that gives the marginal blocking probabilities, B_{ks}, in terms of $\overline{d_k} = (d_{k1}, \cdots, d_{k|\mathcal{S}|})$ (vector of equivalent effective bandwidths), $\overline{\rho_k} = (\rho_{k1}, \ \cdots, \rho_{k|\mathcal{S}|})$ (vector of reduced traffic loads ρ_{ks} offered by flows of type s to l_k) and the link capacity C_k.

This approximation was suggested in [31] in the context of off-line single-objective multiservice routing optimisation models and was also used in the multiobjective dynamic alternative routing model [25]. The use of very efficient and robust approximations is absolutely critical in a routing optimisation model of this type, for tractability reasons.

The basic parameters and equations used in the traffic model are reviewed in the Appendices A and B.

3 Foundations of the Resolution Approach

In the hierarchical multiobjective network-wide optimisation routing problem P-M2-S2 we will consider that the routing principle uses alternative routing i.e. the decision variables are the network routing plans $\overline{R} = \cup_{s=1}^{|\mathcal{S}|} R(s)$ for all the network services, where $R(s) = \cup_{f_s \in \mathcal{F}_s} R(f_s), s \in \mathcal{S}_Q \cup \mathcal{S}_B$ and $R(f_s) = (r^p(f_s)), p = 1, \cdots, M$ with $M = 2$. This means that for each flow f_s the first choice route $r^1(f_s)$ will be used unless it is blocked as a result of one of its links l_k not having the required available bandwidth d_s (or as prescribed by a general probabilistic availability function ψ_{ks}). If $r^1(f_s)$ is blocked then the second choice route $r^2(f_s)$ will be attempted by the connection request and the request will be blocked only if $r^2(f_s)$ is also blocked.

The hierarchical multiobjective alternative routing problem in question is highly ‘complex’ as a result of two major factors: the strong interdependencies among all objective functions (via the $\{B(f_s)\}$) and the interdependencies between the objective function parameters and the (discrete) decision variables $\overline{R}$ (network route plans). All these interdependencies are defined by the underlying traffic model.

Concerning overall complexity it can be said that the simplest, ‘degenerated’ single objective version of the problem, corresponding to the single objective function W_Q, one single service and no alternative routing ($M = 1$) is NP-complete in the strong sense, as shown in [8]. Note that our model is a bi-level, multiobjective formulation of this type of problem. This and the interdependencies among the objective functions, are a strong indication of extreme intractability of the problem.

Concerning the possible conflict between the objective functions in P-M2-S2, it can be said that in many situations, the maximisation of W_Q entails a deterioration on $B(f_s)$, $s \in \mathcal{S}_Q$, for “small” intensity traffic flows $A(f_s)$ which tends to increase $B_{Ms|Q}$ and, as a result, $B_{Mm|Q}$. In single-objective routing models this effect is usually tackled by imposing upper bounds on the values $B(f_s)$. These relations between objective functions of this type have been analysed in [27]. Note that this is a major factor to justify the interest and potential benefit in using multiobjective approaches when dealing with this type of routing problem.

The resolution (in a multicriteria analysis sense) of the routing model P-M2-S2 will be performed by a heuristic approach, a first version of which is presented in the next section. This heuristic is the extension and adaptation to this problem of the heuristic procedure described in [6] and [25].

The heuristic developed for problem P-M2-S2 is based on the calculation of solutions of a bi-objective shortest path problem. In this problem the path metrics to be minimised will be the marginal implied costs (as defined according to the following analysis) and blocking probabilities.

The implied cost c_{ku} resulting from the acceptance of a call of flow f_u in link l_k is a powerful mathematical concept in routing optimisation in circuit-switched networks and was originally proposed by Kelly [16] for single-rate traffic networks. It was extended to single route (i.e. without alternative routing) multirate traffic networks in [12] and [31]. It can be defined as the expected value of the loss of revenue in all network traffic flows which may use link l_k resulting from the acceptance of a call from f_u associated with the decrease on the capacity of this link. Therefore the implied cost measures the knock-on effects on all network routes (of all traffic flows) resulting from the acceptance of a call from f_u in a link l_k. The authors have adapted in [6] and [25] the definition of c_{ku} to multirate loss networks with alternative routing by extending the corresponding expression given for single-service networks in [16]. This extension implies that the c_{ku} can be calculated from the equation

$$c_{ku} = \sum_{s\in\mathcal{S}} \frac{1}{1-B_{ks}}\zeta_{kus}\left[\sum_{f_s:l_k\in r^1(f_s)} \lambda_{r^1(f_s)}\left(s_{r^1(f_s)}+c_{ks}\right)+\sum_{f_s:l_k\in r^2(f_s)} \lambda_{r^2(f_s)}\left(s_{r^2(f_s)}+c_{ks}\right)\right] \tag{3.1}$$

with

$$s_{r^2(f_s)} = w(f_s)-\sum_{l_j\in r^2(f_s)} c_{js} \tag{3.2}$$

$$s_{r^1(f_s)} = w(f_s)-\sum_{l_j\in r^1(f_s)} c_{js}-(1-L_{r^2(f_s)})s_{r^2(f_s)} \tag{3.3}$$

where $s_{r^p(f_s)}$ is the surplus value of a call on route $r^p(f_s)$, $\lambda_{r^p(f_s)}$ is the marginal traffic carried on $r^p(f_s)$, $L_{r^p(f_s)}$ is the blocking probability for calls of f_s on route $r^p(f_s)$ $(p=1;2)$, considering that $r^1(f_s)$ and $r^2(f_s)$ are disjoint paths, and

$$\zeta_{kus} = \mathcal{L}_s\left(\overline{d_k},\overline{\rho_k},C_k-d_{ku}\right)-\mathcal{L}_s\left(\overline{d_k},\overline{\rho_k},C_k\right) \tag{3.4}$$

is the increase in the congestion for type s calls on link l_k originated by a decrease in the arc capacity because of the acceptance of a type u call.

The calculation of the implied costs in this form is based on the following conjecture, which is an extension to multirate loss networks with alternative routing, of the results in [16, sec.7] and [30, sec.III].

Conjecture A. *In multirate networks with (one-stage) alternative routing the sensitivity of the revenue W_T with respect to the traffic $A(f_s)$ being offered to a pair of routes (r^1,r^2), when an approximation to the expected revenue is calculated from the solution of the fixed point equations in B_{ks}, can be written*

$$\frac{\partial W_T}{\partial A(f_s)} = (1-L_{r^1(f_s)})\left(w_{r^1(f_s)}-\sum_{l_k\in r^1(f_s)} c_{ks}\right)+L_{r^1(f_s)}(1-L_{r^2(f_s)})\left(w_{r^2(f_s)}-\sum_{l_k\in r^2(f_s)} c_{ks}\right) \tag{3.5}$$

where the c_{ks} are the implied costs and these satisfy the system of equations (3.1)-(3.3).

It will be further assumed that $w_{r^1(f_s)} = w_{r^2(f_s)} = w(f_s)$. The fixed point equations in this statement result from the traffic model and constitute a system of implicit non-linear equations enabling the calculation of the B_{ks} in terms of link capacities (expressed through matrix $\overline{C} = [C_k]$), the offered traffic matrix $\overline{A} = [A(f_s)]$, and the current network routing plan $\overline{R}$:

$$B_{ks} = \beta_{ks}(\overline{B}, \overline{C}, \overline{A}, \overline{R}) \tag{3.6}$$

with $k = 1, \cdots, |\mathcal{L}|; s = 1, \cdots, |\mathcal{S}|$ and $\overline{B} = [B_{ks}]$.

Note that the calculation of c_{ks} through (3.1)-(3.3) implies the solution of a system of equations of the form:

$$c_{ks} = \alpha_{ks}(\overline{c}, \overline{B}, \overline{C}, \overline{A}, \overline{R}) \tag{3.7}$$

with $\overline{c} = [c_{ks}]$. The numerical resolution of the systems (3.6) and (3.7) (in this order) is performed by fixed point iterators, for given $\overline{C}, \overline{A}$ and $\overline{R}$.

In [6] and [25], the authors formulated a bi-level *multiple objective dynamic alternative routing problem for multiservice networks* for a single service class with multiple service types:

$$\text{Problem } \mathcal{P}_G - S \tag{3.8}$$

$$\begin{array}{rl} \text{Network level:} & \min_{\overline{R}}\{-W_T\} \\ & \min_{\overline{R}}\{B_{Mm} = \max_{s\in\mathcal{S}}\{B_{ms}\}\} \\ \text{Service level:} & \min_{\overline{R}(s)}\{B_{ms}\}, s = 1, \cdots, |\mathcal{S}| \\ & \min_{\overline{R}(s)}\{B_{Ms} = \max_{f_s\in\mathcal{F}_s}\{B(f_s)\}\}, s = 1, \cdots, |\mathcal{S}| \end{array}$$

subject to the equations of the teletraffic model enabling to calculate $\{B(f_s)\}$ in terms of $\{A(f_s)\}$ and $\overline{R}$, where W_T is the total expected network revenue associated with the traffic carried by all service types, B_{ms} is the average blocking probability for all traffic flows of service type s and B_{Ms} is the maximal value of those blocking probabilities, and B_{Mm} is the maximal value among the B_{ms} and is, together with W_T the first priority objective function.

This routing model can be considered as an application of the meta-model P-M3-S2 [5] and as a particular case of the addressed model, P-M2-S2 by considering only one service class.

The resolution approach to $\mathcal{P}_G - S$ was based on the calculation of a solution to the bi-objective shortest path problem, formulated for every end-to-end flow f_s:

$$\text{Problem } \mathcal{P}^{(2)}$$

$$\min_{r(f_s)\in\mathcal{D}(f_s)} \left\{ m^n(r(f_s)) = \sum_{l_k\in r(f_s)} m^n_{ks} \right\}_{n=1;2} \tag{3.9}$$

where $m^1_{ks} = c_{ks}$ and $m^2_{ks} = -\log(1 - B_{ks})$, and $\mathcal{D}(f_s)$ is the set of feasible loopless paths for flow f_s, resulting from traffic engineering constraints. The logarithmic function is used to transform the blocking probability into an additive metric.

The use of this constrained bi-objective shortest path problem as a basis for the resolution approach to the network problem $\mathcal{P}_G - S$ relies on the fact that the metric blocking probability tends (at a network level) to minimise the maximal node-to-node blocking probabilities $B(f_s)$ while the metric implied cost tends to maximise the total average revenue W_T (see [7] and [27]). When one states that using the minimisation of path implied cost 'tends' to maximise W_T this would be in rigour only valid if the choice of such 'optimal' path for a given f_s, would not change in any form the network working condition, an assumption that is not true, having in mind the interdependencies among $\{c_{ks}\}$, $\{B_{ks}\}$ and $\overline{R}$ (see (3.6)-(3.7)). This is the ultimate source of difficulty in devising a heuristic based on this principle, as outlined in the next section.

Nevertheless it was possible to develop a heuristic approach based on this principle that gave very good results when compared with reference routing methods like RTNR (Real Time Network Routing) and DAR (Dynamic Alternate Routing) aimed at maximising the total expected revenue, as shown in [25].

In order to extend this resolution principle to the problem P-M2-S2 we need to extend the definition of implied costs to a network with two service classes. For this purpose we propose the following definition of *marginal implied costs* associated with QoS (BE) traffic by extending the original interpretation of implied costs by Kelly [16] to a multirate loss network with two service classes. Hence we will define the *marginal implied cost for QoS traffic*, c^Q_{ku}, associated with the acceptance of a connection (or "call") of traffic f_u of any service type $u \in \mathcal{S}$ on a link l_k as the expected value of the traffic loss induced on all QoS traffic flows resulting from the capacity decrease in link l_k. In an analogous form one can define the marginal implied cost c^B_{ku} for BE traffic associated with the acceptance of a connection of traffic flow f_u on link l_k.

We will assume, as a conjecture, that the marginal implied costs for QoS (BE) traffic can be estimated by solving a system of equations analogous to (3.1)-(3.3), by restraining the summation on the right hand side to the service types of QoS (BE) class

$$c^{Q(B)}_{ku} = \sum_{s \in \mathcal{S}_{Q(B)}} \frac{1}{1 - B_{ks}} \zeta_{kus} \quad \left[\sum_{f_s \in \mathcal{F}_s : l_k \in r^1(f_s)} \lambda_{r^1(f_s)} \left(s^{Q(B)}_{r^1(f_s)} + c^{Q(B)}_{ks} \right) + \right.$$
$$\left. + \sum_{f_s \in \mathcal{F}_s : l_k \in r^2(f_s)} \lambda_{r^2(f_s)} \left(s^{Q(B)}_{r^2(f_s)} + c^{Q(B)}_{ks} \right) \right] \tag{3.10}$$

with

$$s^{Q(B)}_{r^2(f_s)} = w^{Q(B)}(f_s) - \sum_{l_j \in r^2(f_s)} c^{Q(B)}_{js} \tag{3.11}$$

$$s^{Q(B)}_{r^1(f_s)} = w^{Q(B)}(f_s) - \sum_{l_j \in r^1(f_s)} c^{Q(B)}_{js} - (1 - L_{r^2(f_s)}) s^{Q(B)}_{r^2(f_s)} \tag{3.12}$$

$$\zeta_{kus} = \mathcal{L}_s \left(\overline{d_k}, \overline{\rho_k}, C_k - d_{ku} \right) - \mathcal{L}_s \left(\overline{d_k}, \overline{\rho_k}, C_k \right) \tag{3.13}$$

where

$$w^{Q(B)}(f_s) = \alpha^{Q(B)} w(f_s) \tag{3.14}$$

and the coefficients $\alpha^{Q(B)} \in]0.0; 1.0[$ satisfy the normalisation condition

$$\alpha^Q + \alpha^B = 1.0 \tag{3.15}$$

This condition and the calculation of the marginal costs through equations (3.10)-(3.13) are consistent with the definition of the sensitivity of the marginal revenues associated with QoS and BE traffic, through expressions analogous to (3.5):

Conjecture B.

$$\begin{aligned}\frac{\partial W_{Q(B)}}{\partial A(f_s)} &= (1-L_{r^1(f_s)})\left(w^{Q(B)}(f_s)-\sum_{l_k\in r^1(f_s)} c_{ks}^{Q(B)}\right)+\\ &+ L_{r^1(f_s)}(1-L_{r^2(f_s)})\left(w^{Q(B)}(f_s)-\sum_{l_k\in r^2(f_s)} c_{ks}^{Q(B)}\right)\end{aligned} \tag{3.16}$$

In fact, taking into account that $W_T = W_Q + W_B$, (3.16) and (3.14)-(3.15), together with the condition $c_{ks} = c_{ks}^Q + c_{ks}^B$, imply equation (3.5) (in conjecture A). The marginal expected revenues per call of f_s, $w^{Q(B)}(f_s)$ (such that $w^Q(f_s)+w^B(f_s) = w(f_s)$) in equation (3.16) may be interpreted as the part of the expected revenue $w(f_s)$ generated by a connection of f_s that is accepted by the network (for a given choice of the pair of routes $(r^1(f_s); r^2(f_s))$) that is assigned to the calculation of the sensitivity of the revenue either from the point of view of traffic losses induced in the QoS traffic flows or in the BE flows.

In the present model we will consider, as a first approach, $\alpha^Q = \alpha^B = 0.5$ so that no bias is induced in the calculation of the marginal costs through the choice of these factors.

The auxiliary bi-objective shortest path problem to be solved will have two possible configurations. The first one will use as link cost coefficients $m_{ks}^1 = c_{ks}^Q$ when one intends to obtain candidate solutions to improve the revenue of the QoS traffic, and the second one will use $m_{ks}^1 = c_{ks}^B$ when one seeks to improve the revenue associated with the BE traffic.

To solve this constrained shortest path problem we will use an adaptation of the previously developed algorithmic approach MMRA-S or Modified Multiobjective Routing Algorithm for multiservice networks (described in [6] and [25]). This adaptation of the algorithm, to be applied to the present model, will be outlined in the next section, together with the features of a first heuristic approach for solving the very complex network problem P-M2-S2.

4 A First Heuristic Approach

Next we describe the main features of a heuristic procedure for solving the model in the context of application to the MPLS network used in [32] for evaluating a reference routing model with two service classes, based on a multicommodity flow programming approach. The basic architecture of the heuristic is analogous to the MODR-S (Multiobjective Dynamic Routing for Multiservice networks) heuristic described in [6] and [25], with some relevant adaptations. These adaptations and changes in the heuristic have to do, on the one hand, with the different type of the considered network topology, as shown in figure 4.1, that unlike the one for which MODR-S was specified (a fully meshed network) has low connectivity. On the other hand, there are important differences in the objective functions as a result of the existence of two traffic classes, as previously analysed.

The 'core' of the heuristic is the generation of candidate solutions $(r^1(f_s), r^2(f_s))$ for each f_s, where $r^1(f_s)$ is defined according to the rules described hereafter and $r^2(f_s)$ is typically obtained through a constrained bi-objective shortest path algorithm, devised for problem $\mathcal{P}^{(2)}$, MMRA-S2. The main features of MMRA-S2 will be described later on.

Having in mind the network topology and the need to make a further distinction between real-time QoS services (video and voice services) and non-real time QoS services

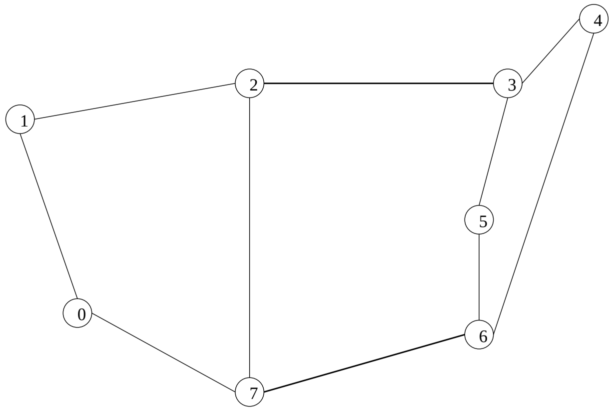

Figure 4.1: Test network M1, proposed in [32]

(such as 'premium data' service) special rules were defined for the selection of candidate first choice routes $r^1(f_s)$. An important parameter defined from these rules is the maximal number of arcs D_s per route for each service type s.

In general, for real-time QoS services, $r^1(f_s)$ is chosen as the direct arc whenever it exists or as one of the feasible paths with the least number of arcs. If there is more than one of these paths, the choice is made according to MMRA-S2, by using priority regions defined in the objective function space of $\mathcal{P}^{(2)}$. These criteria result from the more stringent constraints on delay and jitter of this type of services, and also having in mind to increase the connections reliability. For the remaining QoS services the initial choice of $r^1(f_s)$ is made by using the algorithm MMRA-S2 and the mentioned priority regions. A similar procedure is applied for obtaining $r^1(f_s)$ for BE traffic flows. Concerning the calculation of the second choice routes $r^2(f_s)$ for QoS or BE traffic, this is made according to the MMRA-S2 algorithm. These alternative routes may be eliminated through a mechanism designated as Alternative Path Removal (APR) proposed in [26] and [25], in order to prevent performance degradation in overload conditions. Therefore, $r_s = r^2(f_s)$ is eliminated whenever

$$m^1(r_s) > \alpha^{Q(B)} \cdot d_s \cdot z_{APR} \text{ and } m^2(r_s) > -\log(0.7) \cdot z_{APR} \tag{4.1}$$

where $z_{APR} \in [0.0; 1.0]$ is an empirical parameter which has to be adequately chosen through extensive experimentation with the model. The complete set of rules used in the given network to define candidate routes $(r^1(f_s), r^2(f_s))$ is described in Appendix C.

As for the 'core' algorithm MMRA-S2 it is basically an adaptation to the present model of the bi-objective constrained shortest path algorithm in [25] which is an extension of the algorithm in [7] to a multiservice environment. The main features of MMRA-S are now briefly reviewed. Note that in general there is no feasible solution which minimises both objective functions of $\mathcal{P}^{(2)}$ simultaneously. Since there is no guarantee of the feasibility of this ideal optimal solution, the resolution of this routing problem aims at finding a best compromise path from the set of non-dominated solutions, according to some relevant criteria previously defined. In this context, since path computation and selection have to be fully automated, such criteria are embedded in the working of the algorithm MMRA [7, 27] via preference regions in the objective function space.

The purpose of the MMRA version used in the present context is to calculate solutions to problem $\mathcal{P}^{(2)}$ and is a variant of the algorithm proposed in [1] for a bi-objective shortest path problem of the same type, adapted to the requirements and specifics of the dynamic routing model MODR-S [25]. The approach given in [1] (inspired by the one presented in [34] and [4] concerning a procedure to search interactively non-dominated paths) enables the calculation and selection of non-dominated paths in the framework of a routing control mechanism. The procedure satisfies this requirement by integrating the K-shortest paths algorithm [22] and a special concept designated as "soft constraints" (that is constraints not directly incorporated into the mathematical model). The main features of this approach are: i) the representation of QoS requirements through soft constraints corresponding to *requested* and *acceptable thresholds* for each QoS metric; ii) the consideration of this type of thresholds defines priority regions in the objective function space in which non-dominated solutions are searched for; iii) the non-dominated paths are computed by means of an extremely efficient K-shortest path algorithm proposed in [22], designated as MPS algorithm; iv) the adaptation of the preference thresholds to time-varying network working conditions, as required in dynamic routing applications.

Further details on MODR-S teletraffic model and MMRA, in a multiservice network, can be seen in the report [24].

This algorithm was adapted straightforwardly to the present routing model by incorporating in the definition of the feasible route set $\mathcal{D}(f_s)$ and in the route selection procedure, the rules described in detail in the Appendix C. Also the coefficients of the second objective function are the marginal implied costs, either c^Q_{ks} or c^B_{ks} depending on the expected revenue (associated with QoS or BE traffic) we are seeking to improve.

Note that successive application of MMRA-S2 to every traffic flow does not lead to an effective (even less robust) resolution approach to the network routing problem P-M2-S2. The essential reason for this is an instability phenomenon that arises in such path selection procedure, expressed by the fact that the route sets $\overline{R}$ tend to oscillate between certain solutions some of which may lead to poor global network performance under the prescribed criteria. This is associated with the complexity and interdependencies in the network problem P-M2-S2, namely the interdependencies between $\{c^{Q(B)}_{ks}\}$ and $\{B_{ks}\}$ and between these two sets of parameters and the current total route set $\overline{R}$.

A general idea of the heuristic is to seek, for each service, a routing solution $\overline{R}(s)$ which may lead to a better performance in terms of W_B, $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$, hence leaving network resources available for traffic flows of other services so that the solutions selected at each step may enable an improvement on the higher level objective functions W_Q and $B_{Mm|Q}$. Therefore the heuristic is constructed in order to seek, firstly for each QoS service and beginning by the higher bandwidth services (considering the numbering of s, $s = 1, \cdots, |\mathcal{S}_Q|$) and, secondly, for each BE service and beginning by the higher bandwidth services ($s = |\mathcal{S}_Q| + 1, \cdots, |\mathcal{S}|$), solutions which dominate the initial one, in terms of $B_{ms|Q}$ and $B_{Ms|Q}$ for QoS services and in terms of W_B for BE services, while not worsening any of the network metrics W_Q and $B_{Mm|Q}$ (taking into account the optimisation priorities in P-M2-S2).

The basis of the heuristic approach (similarly to MODR-S [25]) is to search for the subset of the path set

$$\overline{R}^a = \cup_{s=1}^{|\mathcal{S}|} \overline{R}^a(s) : \overline{R}^a(s) = \{(r^1(f_s), r^2(f_s)), f_s \in \mathcal{F}_s\} \tag{4.2}$$

the elements of which should be possibly changed in the next route improvement cycle. Detailed analysis and extensive experimentation with MODR-S led us to propose [27, 25] a specific criterion for choosing candidate paths for possible routing improvement by increasing order of a function $\xi(f_s)$ of the current $(r^1(f_s), r^2(f_s))$. The criterion depends explicitly on the first choice path $r^1(f_s)$ and on the alternative path $r^2(f_s)$. The adaptation of this criterion to the present model P-M2-S2 considers:

$$\xi(f_s) = F_C^{Q(B)}(f_s)F_L(f_s) \tag{4.3}$$

if the effect over QoS(BE) traffic is being considered, and where

$$\begin{aligned}
F_C^{Q(B)}(f_s) &= (n_2 - n_1)c_1^{\prime Q(B)} + c_{r^1(f_s)}^{Q(B)} - c_{r^2(f_s)}^{Q(B)} && (4.4)\\
c_{r(f_s)}^{Q(B)} &= \sum_{l_k \in r(f_s)} c_{ks}^{Q(B)} && (4.5)\\
c_1^{\prime Q(B)} &= \frac{1}{n_1} \sum_{l_k \in r^1(f_s)} c_{ks}^{Q(B)} = \frac{1}{n_1} c_{r^1(f_s)}^{Q(B)} && (4.6)\\
F_L(f_s) &= 1 - L_{r^1(f_s)}L_{r^2(f_s)} && (4.7)
\end{aligned}$$

The aim of the factor $F_C^{Q(B)}(f_s)$ is to favour (concerning the interest in changing the second choice route when seeking to improve W_Q or W_B) the flows for which the second choice route has a high implied cost and the first choice route a low implied cost. The factor $(n_2 - n_1)$ was introduced for normalising reasons having in mind that $r^1(f_s)$ has n_1 arcs and $r^2(f_s)$ n_2 arcs, in the considered network. The aim of the second factor $F_L(f_s)$ is to favour the choice of the flows with worse current blocking probability. In the cases where overload conditions led to the elimination of the alternative path (see explanation above), $F_C^{Q(B)}(f_s) = c_{r^1(f_s)}^{Q(B)}$ and $F_L(f_s) = 1 - L_{r^1(f_s)}$.

A second point to be tackled by the heuristic procedure is to specify how many of the routes with smaller values of $\xi(f_s)$ should possibly be changed by applying MMRA-S2 once again. For this purpose the effect of each candidate route, in terms of the relevant objective functions is anticipated by solving the corresponding analytical model.

4.1 Generic Descrition of the MOR-S2 Heuristic

An overview of the present heuristic (designated as MOR-S2 or Multiobjective Routing considering 2 classes of service) is presented in this sub-section. For a complete description of this heuristic, see Appendix D.

The heuristic is initialised with a set of paths $(r^1(f_s), r^2(f_s))$ that was defined without any previous "optimisation". The quality of the final solution obtained with the heuristic is dependent on the quality of the initial one. In order to have a "good" initial solution some alternative paths must be eliminated from the initial set of paths. According to the criterion of elimination of the alternative paths proposed in [6] and [25], all paths $r^2(f_s)$ satisfying

$$B(f_s) > \frac{\sum_{f_s \in \mathcal{F}_s} B(f_s)}{|\mathcal{F}_s|} \text{ or } B(f_s) > 10\% \tag{4.8}$$

should be eliminated. This procedure keeps only 'good' alternative paths in the initial solution, and it tends to improve both the service performances (especially for the services with higher bandwidth demands) and the network performance metrics. Note that if the final solution of the heuristic does not dominate the initial one (before the elimination of some alternative paths) in terms of the first level objective functions then it is this initial solution that should be adopted. However, this situation never occurred in all the tests performed.

The heuristic starts off with a ("`for`") cycle that covers all the services, beginning with the QoS services $s = 1, \cdots, |\mathcal{S}_Q|$ and ending with the BE services $s = |\mathcal{S}_Q| + 1, \cdots, |\mathcal{S}|$.

Note that QoS services are treated in the model as first priority traffic, and BE services as second priority traffic. Within each class of service, the algorithm begins with the types of services with higher bandwidth demands. The experience has shown that this ordering of the services generally leads to a better performance of the heuristic. Therefore, the heuristic is set up to find, for each service, and starting with the most demanding services solutions that dominate the previous one with respect to the first level objective functions W_Q and $B_{Mm|Q}$, if possible without worsening the partial criteria for each service, $B_{ms|Q}$ and $B_{Ms|Q}$ for QoS services and W_B for BE services.

The two main (“`while`”) cycles of the heuristic are improvement cycles of the objective functions. Two variables, $nPaths$ and $mPaths$, define the current number of paths which are candidates for possible improvements in these two cycles. The initial value of $mPaths$ is the total number of flows in the network and $nPaths$ controls the internal cycle where the heuristic seeks to obtain a new set of paths capable of improving the second level objective function(s) for the service in analysis, and the QoS network objectives; $mPaths$ defines the number of iterations of the external cycle in which $nPaths$ is re-initialised. One variable, ape, determines that the external cycle is executed three times, keeping the same value of $mPaths$ in the first two executions, and another variable, $nCycles$, determines that the internal cycle is executed twice, by re-initialising the value of $nPaths$. Extensive experimentation showed that more than two executions of this cycle would be redundant in most cases. Note that the solution found in the inner cycle depends on $nPaths$, which is the number of flows for which routes may change, and on the initial route set.

The $nPaths$ flows f_s for which the paths are liable to change are chosen by considering the value of the function $\xi(f_s)$, as was explained earlier. A specific service protection scheme to prevent excessive network blocking degradation in overload situations, the APR, is used, as described earlier. The parameter z_{APR} varies between 0.0 and 1.0, and its value is defined in the inner cycle of the heuristic.

This heuristic is formalised in Appendix D.

5 Application of the Model

In this section, computational results obtained with the MOR-S2 heuristic in a network case study analogous to the one in [32], are presented. The “quality” of these results concerning W_Q was compared to results obtained with another heuristic proposed in [32] for MPLS networks with two service classes that uses a lexicographic optimisation formulation based on a deterministic MCF (*Multicommodity Flow*) model, which gives an upper bound to our objective function W_Q in P-M2-S2. For this purpose the network case study for two service classes in MPLS addressed in [32], was considered.

5.1 Application of the Model to a Network Case Study

In [32], Mitra *et al.* propose a model for traffic routing and admission control in multiservice, multipriority networks supporting traffic with different QoS requirements. Having in mind a better understanding of the application case study we will begin by giving an overview of the relevant features of the model proposed in [32]. An important property of the proposed techniques is their scalability and quick response to the changing traffic and network conditions. Therefore, instead of using stochastic traffic models in the calculation of paths, deterministic models are used, in particular mathematical programming models based on MCFs. These models are only a rough approximation in this context and in fact, they tend to under-evaluate the blockings and the delays. As a result, an adaptation of the original model was introduced in [32] in order to obtain more 'correct' models, that is models which give a better approximation in a stochastic traffic environment. The authors of [32] propose a simple technique to adapt the MCF model to a stochastic environment: the requested values of the flows bandwidths in the MCF model are compensated with a factor $\alpha \geq 0.0$, so as to model the effect of the random fluctuations of the traffic that are typical of stochastic traffic models. The higher the variability of the point processes of the stochastic model, the higher is the need for compensation and therefore the higher should α be.

In this model [32], a deterministic traffic routing problem based on MCFs is defined, where traffic splitting is used. This means that the required bandwidth of each flow may be divided by multiple paths from source to destination, allowing for a better load balancing in the network.

The objective functions of this problem to be maximised are the revenues W_Q and W_B associated with QoS and BE flows. A bi-criteria lexicographic optimisation formulation is considered, concerning the revenues W_Q and W_B, so that the improvements in W_B are to be found under the constraint that the optimal value of W_Q is maintained. For solving this problem a two-stage heuristic procedure based on a MCF formulation, which enables to find the optimal value W_Q was developed. In the first stage of the heuristic, an admission control mechanism is applied. Initially only QoS traffic in the original network $\mathcal{N}$ is taken into account. In the application example described in [32], all the offered QoS traffic is carried, i.e. all the bandwidth demands are carried with QoS guarantees and an optimal value of W_Q is obtained under these assumptions. The capacities left in the network arcs are the residual capacities and a residual network $\mathcal{N}'$ is obtained. The BE traffic is offered to this residual network, and in the case study example, about 20.67% of the BE traffic bandwidth is not carried. At the end of this first stage, a routing plan for each service is obtained. In the second stage of the procedure the flow-based model is adapted to a stochastic environment with only a small amount of incremental effort. There are two basic ideas in this adaptation: the flow rates (input data for the flow-based problem) are inflated to compensate for the variabilities in traffic intensities in a stochastic environment as explained above, and the MCF-based result is mapped into the adapted model, keeping the relations between traffic intensities invariant.

In the deterministic flow-based model, a base matrix $T = [T_{ij}]$ with offered bandwidth values from node i to node j [Mbps] is given. A multiplier $m_s \in [0.0; 1.0]$ with $\sum_{s \in \mathcal{S}} m_s = 1.0$ is applied to these matrix values to obtain the offered bandwidth of each flow $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}(s, \mathcal{L}))$ to the network, $T(f_s) = m_s T_{ij}$. The transformation of this type of matrix into a matrix of traffic intensities, used in our stochastic traffic model is described in the Appendix E.1.

In the application example in [32], results for the QoS flows revenue W_Q are presented for three values of α: $\alpha = 0.0$ corresponds to the deterministic approach; $\alpha = 0.5$ is the compensation factor when calls arrive according to a Poisson process, service times follow an exponential distribution and the network is critically loaded; and $\alpha = 1.0$ for traffic flows with higher 'variability'. The results for the BE revenue W_B are not presented, but for $\alpha = 0.0$ the maximum value is 79.33% of the maximum possible value $W_B^{\max}$, because 20.67% of the traffic is not even admitted to the network due to the admission control scheme. In table 5.1 there are results for the revenues obtained from the information provided in [32]. More details on these calculations can be found in Appendix E.2.

Note that the revenue values W_Q in the model [32] should be viewed as upper bounds

Table 5.1: Revenue values according to the information in [32, Tables 3 and 6]

α	W_Q	W_B
0.0	65156.00	$\leq$ 17462.50
0.5	60829.72	
1.0	56338.65	

Table 5.2: Arc capacities on the test network M1

O-D Pair	C'_k [Mbps]	C_k [channels]	O-D Pair	C'_k [Mbps]	C_k [channels]
0-1	155	9688	4-3	155	9688
0-7	155	9688	4-6	155	9688
1-0	155	9688	5-3	155	9688
1-2	155	9688	5-6	155	9688
2-1	155	9688	6-4	155	9688
2-3	310	19375	6-5	155	9688
2-7	155	9688	6-7	310	19375
3-2	310	19375	7-0	155	9688
3-4	155	9688	7-2	155	9688
3-5	155	9688	7-6	310	19375

on the QoS revenue values of the problem P-M2-S2, because of the differences between the two optimisation problems and the important differences in the underlying routing control and traffic models, previously referred to. In fact an important feature of the resolution approach of the routing problem in [32] is the admission control of BE traffic flows at the first stage of resolution so the BE traffic that is actually offered to the network is the fraction of traffic that was not rejected by the admission control. Also note the absence of alternative routing as well as the use of traffic splitting. Therefore, for a specific traffic matrix, the model in [32] tends to obtain smaller values of blocking probability by comparison with a situation without admission control, and this tends to favour higher global revenues. Also the traffic representation, even in the approximated stochastic model [32, sec.5.4] is a bit rough and tends to under-evaluate the blocking probabilities and to over-estimate the revenues.

5.2 Some Experimental Results

The test network M1 proposed in [32] is displayed in figure 4.1. It has $N = 8$ nodes, with 10 pairs of nodes linked by a direct arc. The network has a total of $|\mathcal{L}| = 20$ unidirectional arcs, one in each direction for every pair of adjacent nodes. The bandwidth of each arc C'_k is shown in table 5.2, along with the number of channels C_k (with basic capacity $u_0 = 16$ kbps). When C_k is not an integer, the value was rounded to the nearest integer.

There are $|\mathcal{S}| = 4$ service types with the features described in table 5.3. The value of $d_s = \frac{d'_s}{u_0}$ [channels] $\forall s \in \mathcal{S}$ presented in the table (where d'_s is the required bandwidth in kbps) is calculated with $u_0 = 16$ kbps. Note that $w_s = d_s, \forall s \in \mathcal{S}$.

The traffic flow data information provided by [32] is a base matrix $T = [T_{ij}]$ with offered bandwidth values [Mbps] and a multiplier applied to these matrix values to obtain the offered bandwidth of each flow. Given this information and the variability compensation equations, the values of $A(f_s)$, the average number of offered μ-flows of f_s, during the

Table 5.3: Service features on the test network M1

Service	Class	d'_s [kbps]	d_s [channels]	w_s	h_s [s]	D_s [arcs]	m_s
1 - video	QoS	640	40	40	600	3	0.1
2 - Premium data	QoS	384	24	24	300	4	0.25
3 - voice	QoS	16	1	1	60	3	0.4
4 - data	BE	384	24	24	300	7	0.25

Table 5.4: Initial solution for MOR-S2 on the test network M1, $\forall s \in \mathcal{S}$

Node	0	1	2	3	4	5	6	7
0	–	0-1	0-1-2	0-1-2-3	0-7-6-4	0-7-6-5	0-7-6	0-7
1	1-0	–	1-2	1-2-3	1-2-3-4	1-2-3-5	1-0-7-6	1-0-7
2	2-1-0	2-1	–	2-3	2-3-4	2-3-5	2-7-6	2-7
3	3-2-1-0	3-2-1	3-2	–	3-4	3-5	3-4-6	3-2-7
4	4-6-7-0	4-3-2-1	4-3-2	4-3	–	4-3-5	4-6	4-6-7
5	5-6-7-0	5-3-2-1	5-3-2	5-3	5-3-4	–	5-6	5-6-7
6	6-7-0	6-7-0-1	6-7-2	6-4-3	6-4	6-5	–	6-7
7	7-0	7-0-1	7-2	7-2-3	7-6-4	7-6-5	7-6	–

average service time of a μ-flow can be calculated as shown in Appendix E.1.

In MOR-S2, an initial solution has to be chosen and applied as input data to the heuristic. We chose to consider an initial solution with only one path for each flow, i.e. without a second choice path. In [6], in the context of an application of MODR-S to a fully meshed network the initial solution without any alternative paths for all services proved not to be a 'good' initial solution. However, in the present case study our initial solution was determined without alternative routes and we concluded that this is more adequate and leave it up to the heuristic to find an adequate solution with second choice paths.

The initial solution is the same for all the services and the paths are symmetrical. The path chosen for every flow f_s is the shortest one (that is, the one with minimum number of arcs); if there is more than one shortest path, we choose the one with maximal bottleneck bandwidth (i.e. the minimal capacity of its arcs); if there is more than one shortest path with equal capacity, the choice is arbitrary. This initial solution is displayed in table 5.4. The objective function values obtained for this initial solution are given in table 5.5. The revenue values have 2 decimal places and the blocking probability values have 3 significant figures.

The MOR-S2 heuristic was used to solve the P-M2-S2 problem. The analytical results obtained with the experiments are in table 5.5. In this table the objective function values for both the initial and the final solution are displayed. The value of the QoS revenue in the final solution is also presented as a percentage of the optimal value obtained in [32].

The MOR-S2 heuristic manages to start off with an initial solution with poor values for the objective functions and still finish with a solution with significantly better values. The values for all the objective functions for all values of α are improved through the heuristic. The QoS revenue of the final solutions are slightly worse than those of the optimal solution in [32], as expected. However, these QoS revenues can be considered very good as they stand for approximately 99% of the optimal values in [32]. Note that no admission control is done in MOR-S2 and the tackled problem is hierarchical multiobjective, as opposed to a

Table 5.5: Objective function values for the initial and the final solution for MOR-S2 on the test network M1

Objective Functions	$\alpha = 0.0$		$\alpha = 0.5$		$\alpha = 1.0$	
	Initial	Final	Initial	Final	Initial	Final
W_Q	54803.69	64330.56 (98.73%)	51785.21	60097.78 (98.80%)	49010.41	55978.80 (99.36%)
$B_{Mm\|Q}$	0.413	0.135	0.413	0.0962	0.405	0.0582
$B_{m0\|Q}$	0.413	0.135	0.413	0.0962	0.405	0.0582
$B_{m1\|Q}$	0.314	0.0159	0.296	0.00811	0.275	0.00279
$B_{m2\|Q}$	0.0198	0.00489	0.0174	0.00263	0.0150	0.000436
$B_{M0\|Q}$	0.912	0.848	0.882	0.628	0.841	0.440
$B_{M1\|Q}$	0.766	0.0427	0.722	0.0305	0.667	0.0111
$B_{M2\|Q}$	0.0585	0.0456	0.0517	0.0241	0.0446	0.0143
W_B	15106.57	17391.44	13787.49	17031.62	12445.64	16509.86

two-level lexicographic formulation problem in [32]. Therefore, we can consider that MOR-S2 has managed to find an adequate "good" compromise routing solution to the routing problem P-M2-S2. In fact these experimental results for three traffic matrices showed that the expected QoS revenue obtained with our heuristic is never less than 98.7% of that upper bound while a substantial improvement on the other objective functions could be obtained with respect to the initial solution, using only shortest path first choice routing, typical of Internet routing conventional algorithms.

6 Conclusions and Further Work

In the emergent MPLS technology for the Internet the implementation of connection-oriented services from origin to destination is possible. This feature in association with other functional capabilities of MPLS enables the implementation of advanced QoS-based routing mechanisms, namely by establishing "explicit routes" (determined at the originating node) for each traffic flow.

Having in mind these features and capabilities of MPLS routing it is possible to explore the multicriteria nature of the routing environment and associated metrics (of technical and economic nature) and devise multicriteria routing models capable of explicitly incorporating various network revenues and performance metrics, including fairness QoS objectives at the level of the services. This enables the formulation of multiobjective network-wide optimisation routing models, namely hierarchical multicriteria models, for possible application at the top level of this type of networks.

In this work we described a bi-level multiobjective routing model in MPLS networks formulated within the general modelling framework developed by the authors in [5], assuming that there are two classes of services (and different types of traffic flows in each class), namely QoS and BE services. The flows of QoS type, when accepted by the network, have a guaranteed QoS level, related to the required bandwidth, while BE traffic flows, which are treated in the model as second priority flows, are carried by the network in order to obtain the best possible QoS level for the current network routing solution. The routing model also considers the possibility of using alternative routing when that is advantageous to the first priority objective functions. An important feature of this model is the use of hierarchical optimisation with two optimisation levels, including fairness objectives: the first priority objective functions refer to the network level objectives of QoS type flows, namely the total expected revenue and the maximal value of the mean blocking of all types of QoS flows; the second priority objective functions refer to performance metrics for the different types of QoS services and the total expected revenue associated with the BE traffic flows. Another feature of the model is the use of an approximate stochastic representation of the traffic flows in the network, based on the use of the concept of effective bandwidth for macro-flows and on a generalised Erlang model for estimating the blocking probabilities in the arcs, as the one used in [25]. Also note that while QoS and BE traffic flows are treated separately in terms of objective functions in order to take into account their different priority in the optimisation model, the interactions among all traffic flows are fully represented through the traffic model. This is another advantage in comparison to other routing models proposed for networks with two service classes.

After describing in detail the routing model, including the underlying traffic model, we presented the theoretical foundations of a specialised heuristic strategy for finding "good" compromise solutions to the very complex bi-level routing optimisation problem. This theoretical foundation was based on a conjecture concerning the definition of marginal implied costs for QoS flows and BE flows, which is an extension and adaptation of earlier definitions of implied costs for single-service networks with alternative routing in [16]. The structure of a first version of a heuristic procedure for resolving the problem, analogous to the one described in [6] and [25], was presented. This heuristic was based on a constrained bi-objective shortest path model the objective functions of which are QoS or BE marginal path implied costs, defined and calculated according to that conjecture, and path blocking probabilities. The model was applied to a test network previously used in a benchmarking study [32] that uses a lexicographic optimisation routing approach, including admission control for BE traffic, based on a deterministic traffic representation, where the objective functions are the expected revenues associated with QoS and BE traffic. This model is solved through a heuristic based on a deterministic MCF model, the results of which can be considered as upper bounds with respect the QoS traffic revenue. These preliminary results seem quite encouraging concerning the potential performance of a multicriteria routing model of this nature.

The major limitation of this type of model is its inherent great complexity and the associated computational burden, which constitute the reverse of its 'ambitious' features, namely, network-wide optimisation, multiobjective nature with a significant number of objective functions, use of alternative routing and a stochastic representation of traffic flows of multiple service types. This makes, at present, its potential practical application restrained to networks with a limited number of nodes, such as the core and intermediate (metro-core) level networks of low dimension. The model could also be used as the basis of a periodic type dynamic routing method, similarly to MODR-S [25].

Further work on this model will involve the search for improvements in the heuristic procedure, through sensitivity analysis of the present version, or the possible development of metaheuristics for this very complex network routing problem. Also the extension of the model to broader routing principles such as probabilistic load sharing or traffic splitting might be explored and tested. Finally a discrete event simulation platform will be developed, which will enable a more exact evaluation of the results of the heuristic in a stochastic dynamic environment closer to real networking working conditions.

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Appendices

A Notation and Calculation of Basic Parameters

- $\mathcal{F}_s$: set of traffic flows of the service type s ($s \in \mathcal{S}$);
- $R(f_s) = \{r^1(f_s), \cdots, r^M(f_s)\}$: routing plan for the flow f_s;
- $\overline{R} = \{R(f_s),\ \forall f_s \in \mathcal{F}_s, s \in \mathcal{S}\}$: global routing plan for the network;
- μ-flow: flow which is equivalent to a "call" in the MPLS routing model with explicit routes;
- $b(f_s)$: continuous r.v. representing an approximation to the number of packets in a μ-flow of f_s. The average value of $b(f_s)$ is $\overline{b}(f_s)$;
- $S(f_s)$: continuous r.v. representing an approximation to the size of packets in a μ-flow of f_s. The average value of $S(f_s)$ is $\overline{S}(f_s)$;
- $I(f_s)$: traffic intensity for flow f_s (average number of μ-flows arriving during a second);
- $h(f_s)$: average service time for a μ-flow;
- $A(f_s)$: average number of offered μ-flows of f_s, during the average service time of a μ-flow, $h(f_s)$,
$$A(f_s) = I(f_s) \cdot h(f_s) \text{ [Erl]} \tag{A.1}$$
- ψ_{ks}: admission control function to the link l_k for calls of the service s (its values are probabilities of admission to l_k);
- ρ_{ks} [Erl]: total traffic of type s flows offered to the link l_k;

- d_{ks}: effective bandwidth of the flows $f_s \in \mathcal{F}_s$, of the service type s, in the link l_k, in number of channels (i.e. it corresponds to a bandwidth $d'_{ks} = d_{ks}u_0$ [bit/s]);

- d_s: effective bandwidth of the flows of the service type $s \in \mathcal{S}$;

- C'_k [bit/s]: transmission rate or capacity of the link l_k;

- C_k: capacity of the link l_k [bit/s] in terms of basic units of transmission u_0 [bit/s], where u_0 =64 kbit/s or 16 kbit/s, for instance. The unit of C_k is "channels" or "circuits",
$$C_k = \frac{C'_k}{u_0} \tag{A.2}$$

- $\rho^k(f_s)$ [Erl]: packet potential traffic offered by the flow f_s to the link l_k;

- $\rho^{k*}(f_s)$ [Erl]: packet traffic that is actually offered by the flow f_s to the link l_k;

- ρ^{k*} [Erl]: total packet traffic offered by all the flows to the link l_k (reduced offered traffic);

- $I'(f_s)$: total intensity of packet traffic associated with f_s,
$$I'(f_s) = I(f_s) \cdot \overline{b}(f_s) \text{ [packet/s]} \tag{A.3}$$

- $h^k(f_s)$: average service time for a packet of the flow f_s in the link l_k,
$$h^k(f_s) = \frac{\overline{S}(f_s)}{C'_k} \tag{A.4}$$

- B_{ks}: blocking probability of a call of the service s on the link l_k;

- $L_{r^p(f_s)}$: blocking probability of a call of f_s on the route $r^p(f_s)$;

- $B(f_s)$: point-to-point blocking probability for the flow f_s, considering M disjoint alternative routes,
$$B(f_s) = \prod_{p=1}^{M} L_{r^p(f_s)} \tag{A.5}$$

- B_{ms}: mean blocking probability of the flows of type s, calculated as in (2.4);

- $B_{ms|Q}$: mean blocking probability of the QoS flows of type $s \in \mathcal{S}_Q$,
$$B_{ms|Q} = \frac{1}{A^o_s} \sum_{f_s \in \mathcal{F}_s} A(f_s)B(f_s),\ s \in \mathcal{S}_Q \tag{A.6}$$

- B_{Mm}: maximum of the average blocking probabilities experienced by all types of flows, calculated as in (2.1);

- $B_{Mm|Q}$: maximum of the average blocking probabilities experienced by all types of QoS flows,
$$B_{Mm|Q} = \max_{s \in \mathcal{S}_Q}\{B_{ms}\} \tag{A.7}$$

- B_{Ms}: maximum of the point-to-point blocking probabilities for all the flows of type s, calculated as in (2.5);
- $B_{Ms|Q}$: maximum of the point-to-point blocking probabilities for all the QoS flows of type s,
$$B_{Ms|Q} = \max_{f_s \in \mathcal{F}_s} \{B(f_s)\}, \; s \in \mathcal{S}_Q \tag{A.8}$$
- A_s^o: total traffic offered by the flows of the service type s,
$$A_s^o = \sum_{f_s \in \mathcal{F}_s} A(f_s) \text{ [Erl]} \tag{A.9}$$
- A_s^c: total traffic carried for all flows of the service type s,
$$A_s^c = \sum_{f_s \in \mathcal{F}_s} A(f_s)(1 - B(f_s)) = A_s^o(1 - B_{ms}) \text{ [Erl]} \tag{A.10}$$
- $\lambda_{r^p(f_s)}$ [Erl]: marginal carried traffic of calls of the flow f_s in the route $r^p(f_s)$;
- W_s: expected revenue associated with the carried traffic for all flows of the service type s, where $w(f_s) = w_s$, $\forall f_s \in \mathcal{F}_s$,
$$W_s = A_s^c w_s \tag{A.11}$$
- W_T: expected revenue (on average) of the network, associated with the total traffic carried for all types of service, calculated as in (2.2);
- W_Q: total expected revenue for the QoS traffic,
$$W_Q = \sum_{s \in \mathcal{S}_Q} W_s = \sum_{s \in \mathcal{S}_Q} A_s^c w_s \tag{A.12}$$
- W_B: total expected revenue for the BE traffic,
$$W_B = \sum_{s \in \mathcal{S}_B} W_s = \sum_{s \in \mathcal{S}_B} A_s^c w_s \tag{A.13}$$

B Model for Calculating the Blocking Probabilities

The calculation of the blocking probabilities B_{ks} is made according to the implicit function $\mathcal{L}_s$ in (2.7) as outlined in [23, 25] for a multirate loss traffic network. By considering a simplification where the links are modelled through a multidimensional Erlang system with multirate Poisson traffic inputs, the classical Kaufman (or Roberts) algorithm [14, 33] can be used to calculate the functions $\mathcal{L}_s$ for small values of C_k. For larger values of C_k, approximations based on the uniform asymptotic approximation (UAA) [29] were used, having in mind its efficiency.

Consider that the arrivals occur according to a Poisson process, the service time follows a negative exponential distribution, there is one alternative path ($M = 2$), the occupations of the links in each path are statistically independent and the two paths $r^1(f_s)$ and $r^2(f_s)$ are disjoint. Then for the computation of ρ_{ks}, we begin by calculating

$$\begin{aligned} L_{r^p(f_s)} &= 1 - \textstyle\prod_{l_j \in r^p(f_s)} \psi_{js}(1 - B_{js}) \\ &= 1 - \textstyle\prod_{l_j \in r^p(f_s)} (1 - B_{js}) \ \text{ with } p = 1; 2 \end{aligned} \tag{B.1}$$

$$\begin{aligned} \lambda_{r^1(f_s)} &= A(f_s) \textstyle\prod_{l_j \in r^1(f_s)} \psi_{js}(1 - B_{js}) \\ &= A(f_s) \textstyle\prod_{l_j \in r^1(f_s)} (1 - B_{js}) \ \text{[Erl]} \end{aligned} \tag{B.2}$$

$$\begin{aligned} \lambda_{r^2(f_s)} &= A(f_s) L_{r^1(f_s)} \textstyle\prod_{l_i \in r^2(f_s)} \psi_{is}(1 - B_{is}) \\ &= A(f_s) L_{r^1(f_s)} \textstyle\prod_{l_i \in r^2(f_s)} (1 - B_{is}) \ \text{[Erl]} \end{aligned} \tag{B.3}$$

where we assume, in the present model, that the access is complete, i.e. $\psi_{ks} = 1, \forall l_k \in \mathcal{L}, s \in \mathcal{S}$. Therefore,

$$\begin{aligned} \rho_{ks} &= \textstyle\sum_{f_s : l_k \in r^1(f_s)} A(f_s) \prod_{l_j \in r^1(f_s) \setminus \{l_k\}} (1 - B_{js}) \\ &+ \textstyle\sum_{f_s : l_k \in r^2(f_s)} A(f_s) L_{r^1(f_s)} \prod_{l_i \in r^2(f_s) \setminus \{l_k\}} (1 - B_{is}) \ \text{[Erl]} \end{aligned} \tag{B.4}$$

C Set of Rules for Defining Candidate Routes $(r^1(f_s), r^2(f_s))$

The choice of candidate routes $(r^1(f_s), r^2(f_s))$ for a flow f_s follows a set of rules according to the class of service and to the specific features of the service. A maximal number of arcs D_s for each service type s is imposed. For real-time QoS services (video and voice services), D_s is equal to the network diameter. For the non-real time QoS services (data services), the limit for the maximal number of arcs is the network diameter + 1. For the BE services, no limits are imposed on D_s, so $D_s = N - 1$, where N is the total number of nodes in the network.

The rules for the path calculation for each service type are:

1. First choice route $r^1(f_s)$ calculation
   - (a) Feasible paths have to respect the constraint on the maximal number of arcs.
   - (b) For QoS flows, the first choice path should always be the direct path whenever it exists. If no direct path exists:
     - i. For real-time QoS services, the paths with minimal number of arcs should be used. These services are very delay-sensitive and by minimising the number of arcs of the path, the delay experienced by the flows tends to be smaller. The reliability of the connection also tends to increase. Therefore, for these services, the ordering of the candidate paths should abide by the following rules:
       - A. ordering of the feasible paths by increasing order of the number of arcs;
       - B. ordering of the feasible paths with smaller number of arcs according to their priority region, determined by MMRA-S2;
       - C. in the “best” possible priority region, consider firstly the non-dominated solutions;
       - D. ordering of the non-dominated solutions in the “best” possible priority region and with smaller number of arcs by increasing order of the QoS implied cost of the path.

    ii. For non-real time QoS services, the ordering of the candidate paths should follow these rules:
        A. ordering of the feasible paths according to their priority region, determined by MMRA-S2;
        B. in the "best" possible priority region, consider firstly the non-dominated solutions;
        C. ordering of the non-dominated solutions in the "best" possible priority region by increasing order of the QoS implied cost of the path.
  (c) For BE flows, the rules are the same as in 1(b)ii, where the implied costs are BE costs. The direct path, whenever it exists, is treated exactly as any other path, i.e. no preference is given to the direct path over the other paths.

2. Second choice route $r^2(f_s)$ calculation: no distinction on the type of service is made.
    (a) The feasible paths are the ones that
        - respect the constraint on the maximal number of arcs;
        - do not respect (4.1) (service protection mechanism for alternative routing);
        - are arc-disjoint of the first choice paths $r^1(f_s)$.
    (b) The ordering of the candidate paths is similar to the one defined in 1(b)ii, i.e. it should follow the rules:
        i. ordering of the feasible paths according to their priority region, determined by MMRA-S2;
        ii. in the "best" possible priority region, consider firstly the non-dominated solutions;
        iii. ordering of the non-dominated solutions in the "best" possible priority region by increasing order of the QoS or BE implied cost of the path, according to the class of service.

D Formalisation of the MOR-S2 Heuristic

I. $\overline{R}_a \leftarrow \overline{R}_o$
II. Compute $\overline{B}$ and $W_Q, B_{Mm|Q}$ for $\overline{R}_a$
III. $W_Q^o \leftarrow W_Q, B_{Mm|Q}^o \leftarrow B_{Mm|Q}$
IV. Eliminate the paths $r^2(f_s)$ in $\overline{R}_a$ that verify (4.8)
V. $\overline{R}_e \leftarrow \overline{R}_a$
VI. Compute $\overline{B}$ and $W_Q, B_{Mm|Q}$ for $\overline{R}_a$
VII. $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
VIII. For $s = 1$ to $s = |\mathcal{S}|$

1. $\overline{R}_a(s) \leftarrow \overline{R}_e(s); \overline{R}_*(s) \leftarrow \overline{R}_e(s)$

2. Compute $\overline{B}$ and $B_{ms}, B_{Ms}, s \in \mathcal{S}_Q$ or $W_B, s \in \mathcal{S}_B$ for $\overline{R}_a$

3. $\min\{B_{ms}\}_{ini} \leftarrow B_{ms}, \min\{B_{Ms}\}_{ini} \leftarrow B_{Ms}, s \in \mathcal{S}_Q$ or $\max\{W_B\}_{ini} \leftarrow W_B, s \in \mathcal{S}_B$

4. $mPaths \leftarrow |\mathcal{F}_s|$ (= total number of flows $\leq N(N-1)$), $z_{APR} \leftarrow 1, ape \leftarrow 0$

```
5. While ($mPaths \geq |\mathcal{F}_s| - 1$) do
    (a) $nCycles \leftarrow 2$
    (b) $nPaths \leftarrow mPaths$
    (c) $\overline{R}_a(s) \leftarrow \overline{R}_e(s)$
    (d) Compute $\overline{B}$ and $\overline{c}^Q, B_{ms}, s \in \mathcal{S}_Q$ or $\overline{c}^B, W_B, s \in \mathcal{S}_B$ for $\overline{R}_a$
    (e) $\min\{B_{ms}\} \leftarrow B_{ms}, s \in \mathcal{S}_Q$ or $\max\{W_B\} \leftarrow W_B, s \in \mathcal{S}_B$
    (f) While ($nPaths > 0$) do
        i. Compute and order the values of the function $\xi(f_s)$ – see (4.3)
        ii. Find the $nPaths$ with lower value of $\xi(f_s)$
        iii. Compute with MMRA-S2 new candidate paths for the corresponding O-D pairs and define a new set of first and second choice paths for the service $s$, $\overline{R}_a(s)$, according to the rules established in Appendix C.
        iv. Compute $\overline{B}$ and $B_{ms}, B_{Ms}, s \in \mathcal{S}_Q$ or $W_B, s \in \mathcal{S}_B$ for $\overline{R}_a$
        v. If $s \in \mathcal{S}_Q$ then
            A. If ($B_{ms} < \min\{B_{ms}\}_{ini}$ and $B_{Ms} < \min\{B_{Ms}\}_{ini}$) then
                • Compute $W_Q, B_{Mm|Q}$
                • If $W_Q > \max\{W_Q\}$ and $B_{Mm|Q} < \min\{B_{Mm|Q}\}$ then
                    – $\min\{B_{ms}\}_{ini} \leftarrow B_{ms}, \min\{B_{Ms}\}_{ini} \leftarrow B_{Ms}$
                    – $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
                    – $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$
            B. If ($B_{ms} < \min\{B_{ms}\}$) then
                • $\min\{B_{ms}\} \leftarrow B_{ms}$
            C. Otherwise go to 5(f)vii
        vi. Otherwise ($s \in \mathcal{S}_B$)
            A. If ($W_B > \max\{W_B\}_{ini}$) then
                • Compute $W_Q, B_{Mm|Q}$
                • If $W_Q > \max\{W_Q\}$ and $B_{Mm|Q} < \min\{B_{Mm|Q}\}$ then
                    – $\max\{W_B\}_{ini} \leftarrow W_B$
                    – $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
                    – $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$
            B. If ($W_B > \max\{W_B\}$) then
                • $\max\{W_B\} \leftarrow W_B$
            C. Otherwise go to 5(f)vii
        vii. (Update $nPaths$)
            A. $nPaths \leftarrow nPaths - 1$
            B. If ($nPaths = 0$ and $nCycles = 2$) then
                • $nCycles \leftarrow nCycles - 1$
                • $nPaths \leftarrow |\mathcal{F}_s|$
            C. Compute $\overline{B}$ and $\overline{c}^Q, s \in \mathcal{S}_Q$ or $\overline{c}^B, s \in \mathcal{S}_B$ for $\overline{R}_a$
            D. If ($nPaths \leq 10$ and $ape \geq 1$) then
                • $z_{APR} \leftarrow nPaths \cdot 0.1$
```

Table E.1: Base matrix T with point-to-point bandwidth demands [Mbps], as in [32]

	0	1	2	3	4	5	6	7
0	–	15.7	18.1	2.7	23.5	13.1	5.3	7.9
1	18.3	–	70.1	8.4	90.4	39.2	15.6	23.5
2	20.8	72.7	–	10.4	96.4	46.8	18.1	26.8
3	2.7	8.4	7.9	–	10.6	5.3	2.7	2.7
4	28.7	96.3	101.6	13.2	–	62.3	23.5	39.2
5	13.1	36.4	41.6	5.3	54.9	–	10.4	15.6
6	5.2	13.1	15.6	2.7	20.8	10.4	–	5.3
7	7.9	20.8	26.0	2.7	31.2	15.6	5.3	–

```
                E. Otherwise $z_{APR} \leftarrow 1$
            End of the cycle While ($nPaths$)
        (g) $ape \leftarrow ape + 1$
        (h) If ($ape > 1$) then
                i. $mPaths \leftarrow mPaths - 1$
        End of the cycle While ($mPaths$)
    6. $\overline{R}_a(s) \leftarrow \overline{R}_*(s)$
End of the cycle For ($s$)
IX. If $W_Q^o > \max\{W_Q\}$ or $B_{Mm|Q}^o < \min\{B_{Mm|Q}\}$ then
    1. The best solution is $\overline{R}_o$
X. Otherwise, the best solution is $\overline{R}_*$
XI. Compute the objective function values for the best solution
```

E Traffic Matrices and Revenue Values for the Computational Experiences

In this section, details on the calculation of the input values used in the application model and the values used to compare the final results are given.

E.1 Calculation of Traffic Intensities

In [32], a base matrix $T = [T_{ij}]$ with point-to-point expected bandwidth demands (with values in Mbps) is given. A multiplier $m_s \in [0.0; 1.0]$ with $\sum_{s \in \mathcal{S}} m_s = 1.0$ is used to compute the offered bandwidth of each flow $T(f_s)$ of service s to the network. In table E.1, the values of T_{ij} which represent the total effective bandwidth offered from node v_i to v_j by all the calls of all the services are given. $T(f_s) = m_s T_{ij}$ [Mbps] is the average bandwidth offered by $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}(s, \mathcal{L}))$.

The adaptation of the MCF model to a stochastic model is based on a compensation mechanism that models the effect of random fluctuations of traffic that are typical of a stochastic traffic model. This compensation mechanism is proposed in [32, eq.(5.1)],

$$\frac{T(f_s)}{d'_s} = A(f_s) + \alpha\sqrt{A(f_s)} \tag{E.1}$$

with the compensation factor $\alpha \geq 0$. The greater the variability of the point processes of the stochastic model the higher the value of α. This expression establishes a relation between the bandwidth demand $T(f_s)$ for the MCF model and the parameters $A(f_s)$ and d'_s of the stochastic model. From [32, eq.(5.2)], the average number of μ-flows of $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}(s, \mathcal{L}))$ offered during the average duration of a μ-flow is

$$A(f_s) \approx \frac{T(f_s)}{d'_s} - \alpha\sqrt{\frac{T(f_s)}{d'_s}} = \frac{m_s T_{ij}}{d_s u_0} - \alpha\sqrt{\frac{m_s T_{ij}}{d_s u_0}} \text{ [Erl]} \tag{E.2}$$

if $\frac{T(f_s)}{d'_s} = \frac{m_s T_{ij}}{d_s u_0} > \alpha^2$ and both $T(f_s)$ and $A(f_s)$ are high. Otherwise,

$$A(f_s) \approx \frac{T(f_s)}{d'_s} = \frac{m_s T_{ij}}{d_s u_0} \text{ [Erl]} \tag{E.3}$$

where u_0 is a basic unit of transmission [bit/s].

The input data in terms of traffic intensity can also be computed. The traffic intensity for the flow $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}(s, \mathcal{L}))$ is

$$I(f_s) = \frac{A(f_s)}{h_s} \approx \frac{m_s T_{ij}}{(d_s u_0) h_s} - \frac{\alpha}{h_s}\sqrt{\frac{m_s T_{ij}}{d_s u_0}} \text{ [}\mu\text{-flows/s]} \tag{E.4}$$

if $\frac{m_s T_{ij}}{d_s u_0} > \alpha^2$. Otherwise,

$$I(f_s) = \frac{A(f_s)}{h_s} \approx \frac{m_s T_{ij}}{(d_s u_0) h_s} \text{ [}\mu\text{-flows/s]} \tag{E.5}$$

Considering the values in the matrix T and the other parameter values, the traffic intensity for each flow $f_s \in \mathcal{F}_s$, $s \in \mathcal{S}$ for each value of $\alpha = 0.0; 0.5; 1.0$ is presented in tables E.2-E.4.

The traffic offered to the network is not symmetrical, i.e. the traffic offered from node v_i to node v_j is not necessarily the same as the traffic offered from node v_j to node v_i. Note however that the arc capacities in the network are symmetrical.

E.2 Computation of the revenue values

The values of the QoS revenue W_Q and the maximal possible value of the BE revenue W_B (due to the admission control in phase 1, the deterministic phase, of the resolution in [32]) can be computed following the information provided in [32]. If all the offered traffic was actually carried, the maximal ideal values of the revenues would be

$$W_Q^{\text{ideal}} = \sum_{s \in \mathcal{S}_Q} \sum_{f_s \in \mathcal{F}_s} A(f_s) d_s \tag{E.6}$$

$$W_B^{\text{ideal}} = \sum_{s \in \mathcal{S}_B} \sum_{f_s \in \mathcal{F}_s} A(f_s) d_s \tag{E.7}$$

The values in [32, Tab.6] allow us to compute the revenue values W_Q obtained for each α as a percentage of the ideal value W_Q^{ideal}. As for W_B, a maximal limit is known only for $\alpha = 0.0$ – see [32, Tab.3]. In table E.5, the revenue values used for comparison purposes are presented.

Table E.2: Traffic intensity [μ-flows/s] offered to the test network M1, for $\alpha = 0.0$

s=1	0	1	2	3	4	5	6	7
0	–	0.004089	0.004714	0.000703	0.006120	0.003411	0.001380	0.002057
1	0.004766	–	0.018255	0.002188	0.023542	0.010208	0.004063	0.006120
2	0.005417	0.018932	–	0.002708	0.025104	0.012188	0.004714	0.006979
3	0.000703	0.002188	0.002057	–	0.002760	0.001380	0.000703	0.000703
4	0.007474	0.025078	0.026458	0.003438	–	0.016224	0.006120	0.010208
5	0.003411	0.009479	0.010833	0.001380	0.014297	–	0.002708	0.004063
6	0.001354	0.003411	0.004063	0.000703	0.005417	0.002708	–	0.001380
7	0.002057	0.005417	0.006771	0.000703	0.008125	0.004063	0.001380	–

s=2;4	0	1	2	3	4	5	6	7
0	–	0.034071	0.039280	0.005859	0.050998	0.028429	0.011502	0.017144
1	0.039714	–	0.152127	0.018229	0.196181	0.085069	0.033854	0.050998
2	0.045139	0.157769	–	0.022569	0.209201	0.101563	0.039280	0.058160
3	0.005859	0.018229	0.017144	–	0.023003	0.011502	0.005859	0.005859
4	0.062283	0.208984	0.220486	0.028646	–	0.135200	0.050998	0.085069
5	0.028429	0.078993	0.090278	0.011502	0.119141	–	0.022569	0.033854
6	0.011285	0.028429	0.033854	0.005859	0.045139	0.022569	–	0.011502
7	0.017144	0.045139	0.056424	0.005859	0.067708	0.033854	0.011502	–

s=3	0	1	2	3	4	5	6	7
0	–	6.541667	7.541667	1.125	9.791667	5.458333	2.208333	3.291667
1	7.625	–	29.208333	3.5	37.666667	16.333333	6.5	9.791667
2	8.666667	30.291667	–	4.333333	40.166667	19.5	7.541667	11.166667
3	1.125	3.5	3.291667	–	4.416667	2.208333	1.125	1.125
4	11.958333	40.125	42.333333	5.5	–	25.958333	9.791667	16.333333
5	5.458333	15.166667	17.333333	2.208333	22.875	–	4.333333	6.5
6	2.166667	5.458333	6.5	1.125	8.666667	4.333333	–	2.208333
7	3.291667	8.666667	10.833333	1.125	13	6.5	2.208333	–

Table E.3: Traffic intensity [μ-flows/s] offered to the test network M1, for $\alpha = 0.5$

s=1	0	1	2	3	4	5	6	7
0	–	0.002783	0.003312	0.000162	0.004523	0.002219	0.000622	0.001131
1	0.003356	–	0.015497	0.001233	0.020410	0.008146	0.002761	0.004523
2	0.003914	0.016124	–	0.001646	0.021870	0.009934	0.003312	0.005274
3	0.000162	0.001233	0.001131	–	0.001688	0.000622	0.000162	0.000162
4	0.005709	0.021846	0.023138	0.002241	–	0.013624	0.004523	0.008146
5	0.002219	0.007492	0.008709	0.000622	0.011856	–	0.001646	0.002761
6	0.000603	0.002219	0.002761	0.000162	0.003914	0.001646	–	0.000622
7	0.001131	0.003914	0.005091	0.000162	0.006285	0.002761	0.000622	–

s=2;4	0	1	2	3	4	5	6	7
0	–	0.028743	0.033558	0.003650	0.044479	0.023562	0.008406	0.013364
1	0.033961	–	0.140867	0.014332	0.183394	0.076650	0.028543	0.044479
2	0.039006	0.146303	–	0.018233	0.195998	0.092363	0.033558	0.051198
3	0.003650	0.014332	0.013364	–	0.018625	0.008406	0.003650	0.003650
4	0.055079	0.195788	0.206931	0.023760	–	0.124585	0.044479	0.076650
5	0.023562	0.070880	0.081604	0.008406	0.109176	–	0.018233	0.028543
6	0.008218	0.023562	0.028543	0.003650	0.039006	0.018233	–	0.008406
7	0.013364	0.039006	0.049567	0.003650	0.060197	0.028543	0.008406	–

s=3	0	1	2	3	4	5	6	7
0	–	6.376570	7.364400	1.056535	9.589680	5.307525	2.112409	3.174554
1	7.446756	–	28.859476	3.379239	37.270505	16.072459	6.335430	9.589680
2	8.476637	29.936400	–	4.198962	39.757569	19.214956	7.364400	10.950964
3	1.056535	3.379239	3.174554	–	4.281010	2.112409	1.056535	1.056535
4	11.735115	39.716114	41.913347	5.348617	–	25.629457	9.589680	16.072459
5	5.307525	14.915282	17.064591	2.112409	22.566273	–	4.198962	6.335430
6	2.071652	5.307525	6.335430	1.056535	8.476637	4.198962	–	2.112409
7	3.174554	8.476637	10.620874	1.056535	12.767263	6.335430	2.112409	–

Table E.4: Traffic intensity [μ-flows/s] offered to the test network M1, for $\alpha = 1.0$

s=1	0	1	2	3	4	5	6	7
0	–	0.001478	0.001911	0.000703	0.002926	0.001027	0.001380	0.000206
1	0.001947	–	0.012739	0.000278	0.017278	0.006084	0.001460	0.002926
2	0.002412	0.013315	–	0.000584	0.018636	0.007681	0.001911	0.003569
3	0.000703	0.000278	0.000206	–	0.000615	0.001380	0.000703	0.000703
4	0.003945	0.018613	0.019818	0.001044	–	0.011024	0.002926	0.006084
5	0.001027	0.005504	0.006584	0.001380	0.009415	–	0.000584	0.001460
6	0.001354	0.001027	0.001460	0.000703	0.002412	0.000584	–	0.001380
7	0.000206	0.002412	0.003412	0.000703	0.004445	0.001460	0.001380	–

s=2;4	0	1	2	3	4	5	6	7
0	–	0.023414	0.027837	0.001440	0.037960	0.018694	0.005310	0.009585
1	0.028208	–	0.129608	0.010434	0.170608	0.068230	0.023231	0.037960
2	0.032873	0.134837	–	0.013896	0.182794	0.083163	0.027837	0.044236
3	0.001440	0.010434	0.009585	–	0.014247	0.005310	0.001440	0.001440
4	0.047874	0.182591	0.193376	0.018874	–	0.113971	0.037960	0.068230
5	0.018694	0.062766	0.072931	0.005310	0.099212	–	0.013896	0.023231
6	0.005152	0.018694	0.023231	0.001440	0.032873	0.013896	–	0.005310
7	0.009585	0.032873	0.042709	0.001440	0.052685	0.023231	0.005310	–

s=3	0	1	2	3	4	5	6	7
0	–	6.211473	7.187133	0.988069	9.387693	5.156717	2.016486	3.057442
1	7.268513	–	28.510619	3.258477	36.874342	15.811584	6.170860	9.387693
2	8.286608	29.581131	–	4.064591	39.348471	18.929912	7.187133	10.735261
3	0.988069	3.258477	3.057442	–	4.145353	2.016486	0.988069	0.988069
4	11.511897	39.307229	41.493360	5.197235	–	25.300580	9.387693	15.811584
5	5.156717	14.663897	16.795849	2.016486	22.257546	–	4.064591	6.170860
6	1.976637	5.156717	6.170860	0.988069	8.286608	4.064591	–	2.016486
7	3.057442	8.286608	10.408415	0.988069	12.534525	6.170860	2.016486	–

Table E.5: Revenue values used for comparison purposes

α	W_Q^{ideal}	$W_Q^{\max}$	W_B^{ideal}	$W_B^{\max}$
0.0	66037.50	$\frac{1042.2}{1056.3}66037.50 = 65156.00$	22012.50	$\leq \frac{279.4}{352.2}22012.50 = 17462.50$
0.5	61004.23	$\frac{976.0}{978.8}61004.23 = 60829.72$	19586.47	
1.0	56344.90	$\frac{901.2}{901.3}56344.90 = 56338.65$	17160.44	