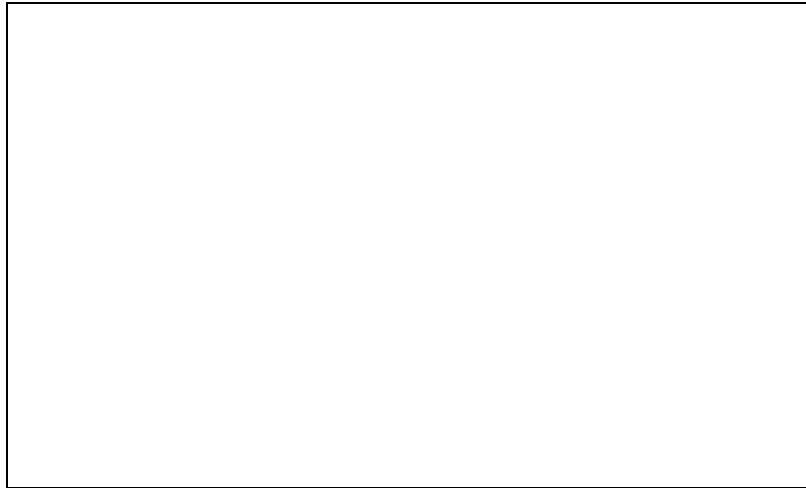


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# Some Thoughts for Discussion on a Mathematical Representation of Differences among a Decision Group

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## Abstract

This paper presents a mathematical representation of a decision group process, emphasizing the differences among the preferences of all group members. An underlying assumption is that group members can be concerned with different sets of criteria and preference parameters. Moreover, if some group members consider the same criterion, it is possible to have different evaluations of the alternatives according to that criterion. Even the information processing method to achieve the results can differ among the elements of the group. The mathematical representation allows to explicitly reflect the differences among the elements of the groups through differences on the preference parameters, considering another criterion, differences on the evaluation of the alternatives or through differences on the results of the information processing methods. We build a matrix with the differences stemming from the pair wise comparisons of each member's individual decision making process and then we point out some techniques to explore this matrix. In this setting it is not proposed to create a single group result; rather it is intended to provide tools in order that each decision maker can explore the differences between his/her opinions and the group. Some shortcomings of this mathematical representation are also presented.

## 1. Introduction

Most decision problems involve several decision makers (DMs). In addition, decision problems frequently have to deal with several different, often conflicting objectives (which can usually be associated to particular opinions of each particular DM, but some may be shared by several of them, eventually at different levels and with different intensities). However, it is usually required the cooperative work of several DMs to achieve a final decision. Recognizing that, for some decades, Group Decision Support Systems (GDSS) have been developed, to help in this task. See for instance Iz and Gardiner (1992), Turoff et al. (1993) for reviews, Lewandovski (1989), Lewis and Butler (1993), Brans et al. (1998), Macharis et al. (1998), Costa et al. (1998), Xanthopoulos et al. (2000) for particular examples and Teghem et al. (1986), Urli and Nadeau (1990) for systems that can be used in a group setting since they combine uncertainty with interactive procedures.

In this work we assume a particular model of group decision process (explained in deeper detail in section 2). We consider that each participant acts as an individual problem solver and concentrates his or her attention on the specific aspect he or she feels relevant. An underlying assumption is that group members can be concerned with different sets of criteria, with different parameters and with different alternative evaluation. Moreover, they can create and test their own alternatives (which may be hypothetical) just to learn, to better understand or to structure their knowledge. Thus, we must consider both individual and group decision processes and how they interact. Research in group decision making (Nadler, 1979) has shown that collaborative decision making tasks encompass three distinct types of information processing, corresponding to different reference levels: individual – each DM concentrates on their own thoughts and preferences; interpersonal – each group member compares, contrasts and integrates the opinions of others; and collective – the group tries to arrive to a joint decision.

There are other models of group decision processes. Dias and Clímaco (2000) present a model focusing on the moment where the individual perspectives are aggregated. Either the preference parameters of the group elements are aggregated before using the approach or method that

yields the joint group result, or the method is repeatedly used over the preference parameters of each group member yielding individual results that are then aggregated to achieve the joint result. Xanthopoulos et al. (2000) consider another group decision process consisting of three main stages. In the first stage there is an aggregation process of the group members opinions in order to have a unique interval of variation for each parameter. The second stage is an exploration and learning phase, ending with the indication of the preferred results for each group member. Finally the last stage is an aggregation one where a joint result is achieved.

In this paper we do not focus on the aggregation of the preferences of the group members, but in the representation of differences among them. It is not our intention to meld the opinions of each DM in a single point of view, rather its intended to let each DM to check the differences between his/her opinions and the ones of their colleagues, and see how their differences influence the outcome. The approach followed choses to present the differences so that the group can study them in order to enhance feedback. Feedback in our context means that a group member can change his or her preferences, so that they more closely reflect the other group members' preferences (as perceived by that member).

It is considered (Vetschera, 1991) that one criterion representing the 'group's evaluation' can cause less cognitive strain then dealing with all the criteria at once. This criterion can be interpreted by each group member as an aggregation of the other members' special knowledge that warrants distinction between the alternatives. In this paper we present a generalization of this idea. We present a mathematical representation of the decision process of each group member, allowing the explicit representation of the differences among the preferences of all group members. We then build a matrix with the differences stemming from the pair wise comparisons of each member's individual decision making process and point out some techniques to explore this matrix. The pair wise comparisons correspond to the interpersonal level of information processing whereas the matrix exploration phase corresponds to the collective level.

The needed changes on the preferences of the DM, in order to integrate the other group members' preferences, can be represented through several distinct processes. Each group member can

- change the structure of his/her evaluation system (e.g. changing the attention paid to the criteria); and/or
- alter his/her own set of criteria, adding or deleting some criteria (it is also possible to create a new criterion reflecting the other's opinion); and/or
- adjust his/her assumptions and predictions (more or less subjective) according to some extra information provided by the other group's elements; and/or
- use another method or process of aggregating the criteria that he or she considers relevant.

The mathematical representation proposed in this paper is very general. It doesn't need any assumption on the approach or method followed to yield the results neither on the problematics (selection, classification or ranking) in which the decision situation falls. It is a framework that can be useful in many situations, even if not fully implemented. It provides some tools that allows for a better understanding on the differences among a group's opinions, and possibly some grounds to overcome those differences.

To show an example of usage of the general mathematical representation we present a modification of a standard multiple criteria decision method, ELECTRE TRI (Yu, 1992), that can be used for classification of alternatives, and we describe how it can be used in a group setting. This setting follows our framework and so it does not intend to create a single group result; rather it proposes to provide tools in order that each DM can explore the differences

between his/her individual classification and the group. This example was previously presented in Melo and Costa (2000).

This task of comparing differences between the DMs is fraught with difficulties. There is doubt over what differences can really be compared, particularly if the DMs use different decision methodologies. Is not know whether all the group members will (or even can) provide their actual preferences, so the differences can be computed. Finally, and more important, the fact remains that knowing the differences between the participants in a group decision is a far cry from knowing how to overcome them.

However, we decided to proceed in this study, because there is some hope that it can be use regardless of the previous points. It's a fact that some differences found on decision processes may not reflect the actual preferences of the DMs, but be merely artefacts of the decision process followed. Also, in some circumstances, namely when the decision process is iterative, knowing who shares the same ideas may be useful in forming alliances.

The following section presents with some detail the model of group decision process that is assumed. Then, section 3 shows the mathematical representation of the differences among a decision group. Section 4 starts by a very brief review of the ELECTRE TRI method followed by the needed mathematical changes required in order to use ELECTRE TRI under our approach. The penultimate section highlights some results of the application of the framework to a known problem. Finally we conclude with some strengths and shortcomings of the approach presented in this paper.

## **2. A particular model of group decision process**

Grounded in theories of interpersonal information processing from Brehmer (1976) and of collective decision making from Ono and Davis (1988), Sengupta and Te'eni (1996, pp 120) consider that a group decision situation incorporates three iterative levels of information processing:

- At the individual level, group members process information individually, concentrating on their own decision processes.
- At the interpersonal level, they learn about the thoughts and opinions of other members, and incorporate them in their own decision processes to arrive at an individual decision.
- At the collective level, the group exchanges and processes information as a collective entity in order to arrive at a joint decision.

Even though these levels are intertwined in any collaborative situation, it is important to draw a conceptual distinction among them because each level supposes distinct modes of interaction and requires different levels of support.

At the individual level, it is assumed the decision makers concentrate on their own thoughts. This can be justified from an organizational point of view: organizations often delegate decisions because individual group members have specialized knowledge of different particular areas that otherwise cannot be brought together in a decision process. In contrast, information processing at the interpersonal level entails assimilating the views of others. This calls for interactive facilities that enable a decision maker to compare, to contrast and to integrate the views of others.

From the need for multiple modes of interaction stems the problem of supporting easy transitions from one level to the other. When individuals revise their respective strategies to take into account the thoughts of others, they can lose control over the revised versions of their strategies. There is a dual need to support: discrimination and integration (Sengupta and Te'eni,

1996). Individuals need to discriminate their own views from those of others, and integrate ideas from multiple views in order to enrich their perspectives.

At the collective level there is the need to support activities such as communication, creation of common views (e.g. by voting), extraction of common factors, other aggregation, etc. This level can be viewed as an extension of the interpersonal level to all members, with an increase of different perspectives (some of which may be antagonistic) requiring processes to achieve a common decision, and allowing some members to explicitly disagree with it.

According to this model, we now intend to discover the differences among the positions of the group elements regarding a decision situation.

### 3. Mathematical representation of differences among a decision group

In this section we present a mathematical representation of the decision process of each group member, allowing the explicit representation of the differences among the preferences of all group members. This mathematical representation is kept very general by design.

#### 3.1. Individual level of information processing

We have a set  $I = \{i, i = 1, \dots, m\}$  of elements of a decision group and a set  $A = \{a_j, j = 1, \dots, n\}$  of alternatives. For each element of the decision group we consider a set of criteria  $F_i = \{f_k^i, k = 1, \dots, p\}$ , where  $f_k^i(a_j)$  is the evaluation of the alternative  $j$ , according to the criterion  $k$  of the group element  $i$ .

We represent the process, method or technique used by the element  $i$  in order to achieve the result,  $R^i$ , by  $G_i(F_i, A, \alpha_i)$ , where  $\alpha_i$  is the vector of preference and technical parameters used by the element  $i$ , e.g.:

$$G_i(F_i, A, \alpha_i) \rightarrow R^i$$

#### 3.2. Interpersonal level of information processing

At this level, we consider a pair wise comparison between the group elements  $i$  and  $i'$ , with  $i, i' \in I$  and  $i \neq i'$ . We represent the differences between the two elements through differences on the parameters' vector, considering another criterion, differences on the evaluation of the alternatives or through differences on the results of the information processing methods.

*Differences on the parameters' vector*

We represent the differences between the two elements of the group,  $i$  and  $i'$ , through  $\delta_{ii'}$ . That is the lowest variation of the absolute value of the parameters of the element  $i$ , in order that the result of the information processing method of the element  $i$  turns to be equal to the result of the information processing method of the element  $i'$ . The following problem must be solved:

$$\min \left\{ \delta_{ii'} \mid R^{ii} = R^{i'}, G_i(F_i, A, \alpha_i + \delta_{ii'}) \rightarrow R^{ii} \text{ and } G_{i'}(F_{i'}, A, \alpha_{i'}) \rightarrow R^{i'} \right\}$$

*To add another criterion – Differences criterion*

The differences between the two elements of the group can be reflected on a new criterion,  $\phi^{ii}$ , added to the criterion set of the element  $i$ . This can be viewed as the aggregated the opinion of the other(s). The introduction of a new criterion can imply some changes on the parameters' vector of the element  $i$ , because it is possible that the information processing method of element  $i$  needs to have some parameters associated to each criterion. Moreover, these new parameters will possible change some necessary balance among the other criterion parameters, and so we

must consider a change in the parameters' vector in order to accommodate these changes. We state the following problem:

$$\begin{aligned} & \min |\delta_{ii'}| \\ & \min \varphi^{ii'}(a_j), j = 1, \dots, n. \\ & \text{s.t.: } R^{ii'} = R^{i'} \\ & \text{where } G_i(F_i + \{\varphi^{ii'}\}, A, \alpha_i + \delta_{ii'}) \rightarrow R^{ii'} \quad \text{and} \quad G_{i'}(F_{i'}, A, \alpha_{i'}) \rightarrow R^{i'} \end{aligned}$$

We search for the lowest possible evaluation value of the alternatives, according to the new criterion, and the lowest possible absolute value for the needed changes in the parameters' vector that equalize the results of the two group elements.

#### *Differences on the evaluation of the alternatives*

On another level, it is possible that group elements disagree on the actual evaluation of the alternatives. The differences between the two group elements will now be represented by the lowest needed amount of changes on the evaluation of the alternatives by the element  $i$ , in order to have the same result of the element  $i'$ . That is to deal with the following problem:

$$\begin{aligned} & \min \gamma_k^{ii'}(a_j), j = 1, \dots, n; k = 1, \dots, p. \\ & \text{s.t.: } R^{ii'} = R^{i'} \\ & \text{where } G_i(F_i, A, \alpha_i) \rightarrow R^{ii'} \quad \text{and} \quad G_{i'}(F_{i'}, A, \alpha_{i'}) \rightarrow R^{i'} \\ & F_i = \{f_k^i(a_j) + \gamma_k^{ii'}(a_j), k = 1, \dots, p; a_j \in A\} \end{aligned}$$

This can be combined with the approaches previously described to support also changes on the parameters and inclusion of new criteria.

#### *Differences on the information processing methods*

If the differences between group members are such that they don't share the information processing method to arrive at the decision, the changes will be more extreme, and the problem is less defined. We will establish a new information processing method associated with the group element  $i$ , such that its result will be equal to the result of the element  $i'$ , maintaining the parameters' vector, the set of criteria and the evaluation of the alternatives of the  $i^{\text{th}}$  group element. This new method,  $H_i$ , should be established as close as possible to the original method,  $G_i$ . Thus we need some way or technique to 'measure' how far is  $H_i$ , from  $G_i$ . This technique will depend on the decision problematic that is being considered by the decision group. There is more than just one way of computing this measure and its quality will depend on the methods actually used. The following example illustrates the approach.

The addressed problematic assumed by the algorithm is the selection of the preferred alternative among a set of alternatives. A simple idea is to count how many times does the result of method  $H_i$  differs from the method  $G_i$ , considering small sets of alternatives built by successively eliminating from the set  $A$  the result of  $G_i$  for the remaining alternatives. This counting is weighted by a function,  $\eta$ , of the number of alternatives remaining in  $A$ . The result  $R^i$  of the  $i^{\text{th}}$  group element assumes now the particular form  $a^{i*}$ , that is the preferred alternative of the participant  $i$ .

We state the following problem:

$$\begin{aligned} & \min \theta_{ii'} \\ & \text{s.t.: } H_i(F_i, A, \alpha_i) \rightarrow a^{i'*} \\ & \text{Where } G_{i'}(F_{i'}, A, \alpha_{i'}) \rightarrow a^{i'*} \quad \text{and} \quad \theta_{ii'} \text{ is defined by the following algorithm:} \\ & \text{Begin} \\ & B \leftarrow A \setminus \{a^{i'*}\}; \theta_{ii'} = 0 \\ & \text{While } B \neq \emptyset \text{ do} \end{aligned}$$

```

    If  $[H_i(F_i, B, \alpha_i) \rightarrow a_h \neq a_g \leftarrow G_i(F_i, B, \alpha_i)]$  then  $\theta_{i_i'} \leftarrow \theta_{i_i'} + \eta(\#B)$ ;
    B  $\leftarrow B \setminus \{a_g\}$ 
End while
End.

```

Other measures can be proposed, that may be better suited to particular decision problematics or group decision situations.

### 3.3. Collective level of information processing

This level will be represented by a matrix,  $\Delta_{m,m}$ , built with the final differences stemming from the pair wise comparisons made at the interpersonal information processing level. We characterize these differences as final, since we are expecting that the interpersonal level will be fulfilled in several interactions with some changes on the individual opinions. These changes are due to the possible integration of the view of others elements of the group in the preferences of a single element, such that her/his preferences more closely reflect the group preferences. However, it is possible that different decision situations may require the analysis of partial or otherwise not final information.

The matrix  $\Delta_{m,m}$  can be built with the differences among the group is defined by one of the prior presented ways, or by a set of them. That is, we can choose to represent the differences only by meanings of the parameters' vector or, for instance, by meanings of both the parameters and the evaluation of alternatives. We present the definition of this matrix as general as follows:

$$\Delta_{i_i'} = [\delta_{i_i'} | \varphi^{i_i'}(a_j), j=1, \dots, n. | \gamma_k^{i_i'}(a_j), k=1, \dots, p; j=1, \dots, n. | \theta_{i_i'}]; \quad i \neq i'; i, i' \in I$$

$$\Delta_{i_i'} = 0; \quad i = i'; i, i' \in I$$

We can devise several ways of exploring the information, contained in the matrix  $\Delta$ , about the differences among the elements of a decision group. This exploration should be pursued not only according to the objectives of the decision group but also taking in to account the structure of the decision process followed by the group. Moreover, the importance of the knowledge about the differences among a group will be different from one decision process to another. For instance if consensus is to be reached, this knowledge will be very important, in order to work out the differences among all the group elements. If a simple application of the relative majority rule is the group process to achieve a result, this knowledge will mostly be important if negotiations to form coalitions are allowed.

There are several well-known and simple techniques to explore pair wise comparisons (usually among alternatives – not directly among differences between opinions of decision makers, as it is required in the present case), that can be used to explore this matrix and draw some conclusions, e.g., minimum regret, MinMax, Condorcet or Borda techniques, etc. These techniques can however be adapted to the present case and, most important, their results are usually easy to interpret and so possibly very useful to gather some feedback to the elements of the group.

There also are other, more complex and powerful, techniques, used to explore outranking relations among a set of alternatives, that can be adapted to this case. Most of these techniques have the advantage of being able to deal with scenarios of incomplete information. It is not reasonable to expect to have all the members of a decision group disclosing all the information about their preferences and the process they followed to forming those preferences. Thus, the adaptation of those methods to explore the matrix  $\Delta$  seems a promising way of draw some useful conclusions to the elements of a decision group.

Another approach can be to use techniques from the location/allocation field in order to build clusters or sets of DMs the preferences of which are ‘more close’ to each other enabling to detect possible grounds of discussion or even some coalitions.

## 4. Group Cooperation with Electre TRI – exploring the differences

In this section we will start by presenting a simplified view of the ELECTRE TRI method, followed by a description of a modified version that can be used for group decision support, in an example of usage of the general mathematical representation presented in the previous section.

### 4.1. The ELECTRE TRI method – a simplified view

In the ELECTRE TRI method (for a detailed description, see Yu, 1992), a DM uses information that compares a set of actions  $A=\{a_1, a_2, \dots, a_m\}$  (which may reflect a set of alternatives), according to a set of criteria, and classifies it according to its relationship to a set of reference actions  $B=\{b_1, b_2, \dots, b_k\}$ , as a member of a particular class. Each action  $a_i$  is judged according to each criterion  $j \in F=\{1, 2, \dots, n\}$ , resulting in an evaluation  $g_j(a_i)$ .

As a member of the outranking family of methods, it intends to enrich the standard (Pareto) *dominance* relationship between actions (an action  $a_i$  dominates  $a_k$  iff  $g_j(a_i) \geq g_j(a_k)$  for all  $j \in F$ , with a strict inequality in at least one criterion) with an *outranking* relationship ( $a_i$  outranks  $a_k$  iff, for the DM,  $a_i$  is at least as good as  $a_k$ ).

ELECTRE (ELemination Et Choix Traduisant la REalité) methods are based in building and exploring the outranking relationship (as described in Roy, 1993). To arrive at this relationship, they use an outranking hypothesis over each pair of actions to be compared. To test this hypothesis, it's used the notion of *concordance* and *discordance*. As a result, if an action  $a_i$  is at least as good as  $a_k$  according to the majority of the criteria (concordance test) and in no criterion is  $a_i$  a lot worse than  $a_k$  (discordance test), than it can be said that  $a_i$  outranks  $a_k$ . To take into account the differences of importance between criteria, an importance coefficient  $k_j$  is associated to each criterion (different DMs may attach different importance coefficients to the same criterion).

ELECTRE TRI uses pseudo-criteria that allow for *indifference* between two actions when they have values that are near each other, and distinguish between *strict preference* and *weak preference*, the later meaning hesitation between indifference and strict preference. Thresholds are used to discriminate between indifference and weak preference ( $q_j$ ) and between weak and strict preference ( $p_j$ ) for each criterion. Using these thresholds and the evaluation of the actions, a concordance index  $c_j(a, b)$  regarding the hypothesis that action  $a$  is at least as good as (reference) action  $b$  according to criterion  $j$  can be computed. Using a veto threshold, a discordance index  $D_j(a, b)$  can also be computed.

A global concordance index  $C(a, b)$  can be computed as a simple additive weighting of the individual concordance indexes weighted by their importance coefficients, as:

$$C(a, b) = \frac{\sum_{j=1}^n k_j \cdot c_j(a, b)}{\sum_{j=1}^n k_j}$$

If all the discordance indexes are insufficient to override the concordance index, a credibility rating for the outranking relationship  $\sigma_s(a,b)$  can be assumed to be equal to  $C(a,b)$ . However, if any discordant criterion vetoes the hypothesis, that is if  $\bar{F} = \{j \in F : D_j(a,b) > C(a,b)\} \neq \emptyset$ , then:

$$\sigma_s = C(a,b) \prod_{j \in \bar{F}} \frac{1 - D_j(a,b)}{1 - C(a,b)}$$

This credibility ranking is then combined with a cutting threshold  $\lambda$  ( $1/2 < \lambda < 1$ ) to check the outranking relationship, that is (read  $a S b$  as  $a$  outranks  $b$ ):  $a S b$  iff  $\sigma_s > \lambda$ .

With the outranking relationships, relations of *preference* ( $a P b = a S b$  and not  $b S a$ ), *indifference* ( $a I b = a S b$  and  $b S a$ ) and *incomparability* ( $a R b =$  not  $a S b$  and not  $b S a$ ) can then be postulated between actions and their references. With these relationships, it is possible, using an assignment process to place each action in a class. One such assignment process, called pessimistic assignment, places an action to the highest class in which the action outranks the lower reference action of this class.

#### 4.2. Group cooperation – interpersonal level - using ELECTRE TRI

ELECTRE TRI, as just described, is a single user analysis tool. However, in many decision problems, we need to take in consideration the judgments of several agents. It is not our intention to meld the opinions of each DM in a single point of view, rather we intend to let each DM to check the differences between his/her opinions and the ones of their colleagues, and see how their differences influence the outcome. As this is an assignment problem, the proposal is to introduce a new criterion that reflects the  $i$ ' group element evaluation and to see which changes are needed on the importance coefficients of one DM to match the same classification of each one of the others. The idea is to perform pair wise comparisons between the DMs.

An easy way to get the same assignments as the other group element is to force the individual to have the same outranking relationships as the other. That is, to force the  $i^{\text{th}}$  individual outranking relationship ( $a S_i b$ ) to match the  $i^{\text{th}}$  ( $a S_i b$ ). In the following it is assumed that the group share the evaluations on the individual actions on each criterion  $g_j(a_i)$ , differing only on the importance coefficients they assign to each criterion. The goal then is to describe which are the smaller changes that can be made on the individual importance coefficients to arrive at the same result as the others. We assume that the change can be made on the individual criterion importance level (changing each criterion importance coefficient individually) and introducing a new criterion that reflects the  $i$ ' group element evaluation. This approach was inspired by the one followed in (Vetschera 1991).

To achieve the same outranking relationships, it is needed to replicate at the individual  $i$  the inequalities defining the  $i$ ' relationships, namely that  $\sigma_{S_i}(a,b) > \lambda$ , are replicated. If it is assumed that no veto is present,  $\sigma_{S_i} = C_g(a,b)$ , and hence a simple additive function can be used to compute it. Normalizing, it can be shown that:

$$\sigma_s(a,b) = \sum_{j=1}^n w_j c_j(a,b) \text{ where } w_j = \frac{k_j}{\sum_{i=1}^n k_i}$$

The problem is then to alter  $w_j$  in a way that the original outranking relations can be maintained. To achieve the desired inequalities, we can make:

$$\sigma'(a,b) = \sum_{j=1}^n (w'_j c_j(a,b)) + w_{i'} c_{i'}(a,b)$$

where  $w'_j = w_j(1 - w_{i'}) + \delta_j^+ - \delta_j^-$ , that is the original weighting  $w_j$  of each concordance index by criteria  $c_j(a,b)$  is replaced by one that takes into account the  $i'$  element's opinion ( $c_{i'}(a,b)$ ), and slack variables  $\delta_j^+$  and  $\delta_j^-$ . The original evaluation by the individual  $i$ ,  $c_j(a,b)$  and  $w_j$  will be constant during this analysis. Notice however, that in order to make this simple substitution it was needed to suppose the absence of significant discordance (veto) in both the individual  $i$  and  $i'$  evaluation of each criterion.

The original  $i'$  relationships are then satisfied if  $\sigma'(a,b) > \lambda$  whenever  $\sigma_{s_{i'}}(a,b) > \lambda$ . To do this we resort to two kinds of change, one modifying the original attribute weights ( $\delta_j^+$  and  $\delta_j^-$ ) relating to each particular criterion, and the other on the weight given to the actual value of the  $i'$  opinion ( $w_{i'}$ ). To achieve the intended result, one change can be substituted by the other. If  $L$  is a weight used to represent the different rates of substitution between the changes, we can estimate the total change to be applied by  $\sum_{j=1}^n \delta_j^+ + Lw_{i'}$ .

Using this model, we can now solve a new problem:

$$\begin{aligned} & \text{Min } \sum_{j=1}^n \delta_j^+ + Lw_{i'} \\ & \text{s.t.} \\ & \sum_{j=1}^n (w'_j c_j(a,b)) + w_{i'} c_{i'}(a,b) > \lambda \quad \forall a \in A, \forall b \in B : c_{i'}(a,b) > \lambda \\ & \sum_{j=1}^n (w'_j c_j(a,b)) + w_{i'} c_{i'}(a,b) \leq \lambda \quad \forall a \in A, \forall b \in B : c_{i'}(a,b) \leq \lambda \\ & \sum_{j=1}^n \delta_j^+ = \sum_{j=1}^n \delta_j^- \\ & w'_j \geq 0 \quad \forall j = 1, \dots, n \end{aligned}$$

As can be seen this is a linear programming model, which can easily be solved by, e.g. the Simplex method. With the results, the DM can see how far must the coefficients be changed to achieve the DM  $i'$  classification.

## 5. Application example

To present an application of the model we decided to implement the previously shown changes to the ELECTRE TRI method, and use it as a starting point to explore differences among DMs, and how these differences affect the final results. To do so, the method was applied to a known model (Dimitras et. al. 1995), an example of a real live bank application, that intends to place 40 actions (each corresponding to an enterprise) to 3 risk categories, according to its performance in 7 criteria. The criteria used were:

- g1 – [quantitative] earnings before tax and interest / total assets (maximize)
- g2 – [quantitative] liquid results / owned capital (maximize)
- g3 – [quantitative] total liabilities / total assets (minimize)
- g4 – [quantitative] interest / sales (minimize)
- g5 – [quantitative] general and administrative expenses / sales (minimize)
- g6 – [qualitative] managerial experience (maximize)
- g7 – [qualitative] market position (maximize)

The list of the reference actions used to define each category, as well as the actions evaluated, can be found in Dimitras et. al. 1995 paper.

To apply the previously described method, we generated 3 DM profiles, differing between themselves only according to the importance given to the criteria (see Table 1 and figure 1). DM1 gives particular importance to the quantitative criteria (mainly to g2), DM2 thinks g2 by itself is almost enough to describe the risk class of a prospect, and DM3 is mostly drawn to the qualitative criteria, and only after those to the three first quantitative criteria. These profiles don't represent any real DMs, and were chosen by the authors with some liberty (although they try to reflect the common "a priori" positions of DMs in face of this problem).

Profile	g <sub>1</sub>	g <sub>2</sub>	g <sub>3</sub>	g <sub>4</sub>	g <sub>5</sub>	g <sub>6</sub>	g <sub>7</sub>
DM1	15,0%	30,0%	15,0%	15,0%	15,0%	5,0%	5,0%
DM2	8,3%	50,0%	8,3%	8,3%	8,3%	8,3%	8,3%
DM3	14,3%	14,3%	14,3%	0,0%	0,0%	28,6%	28,6%

Table 1 – DM importance coefficients

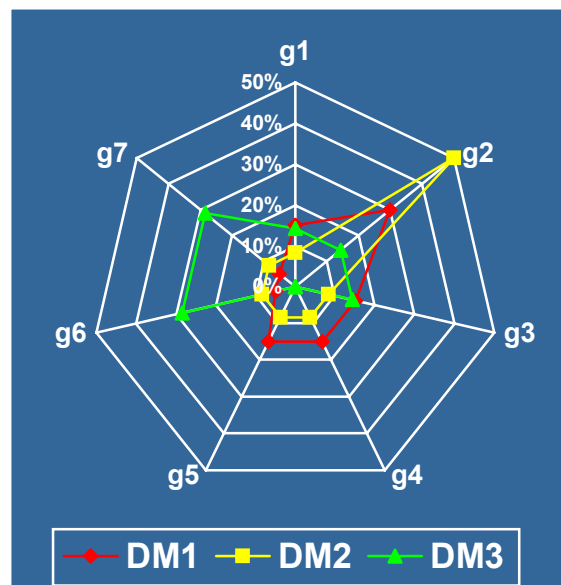


Figure 1 – DM importance coefficients

Applying the ELECTRE TRI method using each coefficient group (and a cutting threshold of  $\lambda=0.6$ ) to the previously referred 40 action, we get a set of three different classifications. Table 2 presents the classification of the 40 actions, according to the preferences of each DM. From this table is patent the relative similarity of DM1 and DM2 positions (only 5 actions are classified differently between them) and the differences between them and DM3 (21 and 19 actions classified differently). Notice also that some actions placed in classes completely different – like a38 and a39, considered by DM1 and DM2 as in the second best class, whereas DM3 places them in the worst class).

Classes	DM1	DM2	DM3
0	a31	a31	a35; a38; a39
1			
2	a24; a28; a35; a37	a24; a35	a10; a14; a21; a24; a25; a26; a27; a28; a31; a32; a33; a34; a36; a37
3	a1; a2; a4; a5; a8; a10; a11; a12; a13; a14; a15; a16; a17; a18; a19; a20; a21; a22; a23; a25; a26; a27; a30; a32; a33; a34; a36; a38; a39	a1; a2; a4; a5; a8; a10; a12; a13; a14; a15; a16; a17; a18; a19; a20; a21; a22; 23; a25; a26; a27; a28; a29; a30; a33; a34; a36; a37; a38; a39	a1; a3; a4; a5; a6; a8; a9; a11; a12; a13; a15; a16; a17; a18; a19; a20; a22; a23; a29; a30
4	a0; a3; a6; a7; a9; a29	a0; a3; a6; a7; a9; a11; a32	a0; a2; a7

Table 2 – Original classification of the actions

Using the changed method described (with a substitution weight  $L=0.5$ ), each pair of DMs choices were compared, and the minimum resulting changes to be applied are present in table 3. In this table, we can see the changes to apply to each importance coefficient ( $\delta g_i$ ) as well as the value to give to the other DM aggregated opinion ( $w_i$ ) for each pair of DMs.

Original	Intended	$\delta g_1$	$\delta g_2$	$\delta g_3$	$\delta g_4$	$\delta g_5$	$\delta g_6$	$\delta g_7$	$w_i$
DM1	DM2	-0,71%	5,00%	-4,29%	0,00%	0,00%	0,00%	0,00%	0,00%
	DM3	0,00%	-10,00%	5,00%	-15,00%	-15,00%	15,00%	20,00%	0,00%
DM2	DM1	15,00%	-18,33%	0,00%	3,33%	0,00%	0,00%	0,00%	0,00%
	DM3	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	89,09%
DM3	DM1	0,00%	11,43%	0,00%	20,00%	0,00%	-14,29%	-17,14%	0,00%
	DM2	0,00%	3,81%	-2,86%	0,00%	0,00%	-0,95%	0,00%	80,00%

Table 3 – Variations between the importance coefficients for group members.

Even the most cursory analysis of the data of table 3 can state its asymmetry (as expected, a group member may need to change its position more radically to achieve the results of the other than vice-versa). Its immediately visible that some DMs are relatively close to each other, so that the same results (for the total of 40 actions) can be achieved by only changing the importance coefficients of each criteria. However, for DM2 and DM3 their relative positions differ strongly, so that the achievement of the others position can only be achieved by giving a (very) strong weight to the criteria representing the others aggregated opinion (nearly 90% and 80% on each case). However it is patent from the analysis of results that mingle changes on the individual preferences and an aggregate value given to the “other” opinion, that the actual preferences may change a lot more than the DMs may suppose (table 4)

DM3->DM2	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$w_i$
Apparent changes	0,0%	3,8%	-2,9%	0,0%	0,0%	-1,0%	0,0%	80,0%
Final importance	2,9%	6,7%	0,0%	0,0%	0,0%	4,8%	5,7%	80,0%
Original weights	14,3%	14,3%	14,3%	0,0%	0,0%	28,6%	28,6%	
Changed weights	14,3%	33,3%	0,0%	0,0%	0,0%	23,8%	28,6%	

Table 4 – Changes in presence of an aggregated value

However, while there is no guarantee that a final solution without a significant value placed on the aggregate opinion of the other, it is usually possible to trade-off changes on the aggregate value by changes on the individual criteria preferences. This can be shown in table 5, were a higher weight given to the trade-off lever ( $L=0.6$ ) allowed results with all the changes on each criteria explicit.

		$\delta g_1$	$\delta g_2$	$\delta g_3$	$\delta g_4$	$\delta g_5$	$\delta g_6$	$\delta g_7$	$w_i$
DM2->DM3	Changes	0,0%	-30,0%	11,7%	-8,3%	-8,3%	11,7%	23,3%	0,0%
	Original weights	8,3%	50,0%	8,3%	8,3%	8,3%	8,3%	8,3%	0,0%
	Changed weights	8,3%	20,0%	20,0%	0,0%	0,0%	20,0%	31,7%	0,0%
DM3->DM2	Changes	-14,3%	45,7%	-14,3%	0,0%	0,0%	-17,1%	0,0%	0,0%
	Original weights	14,3%	14,3%	14,3%	0,0%	0,0%	28,6%	28,6%	0,0%
	Changed weights	0,0%	60,0%	0,0%	0,0%	0,0%	11,4%	28,6%	0,0%

Tabela 5 – Changes on the DM2-DM3 pair, with  $L=0,6$

As stated, in this case some measure of trust on the aggregate positions of the other can be exchanged by the explicit analysis of the changes in individual preferences. It falls now to the

DMs the choice to accept or reject the individual changes. From group matrix analysis, the DMs could either accept some minor changes (as the ones needed to achieve agreement between DM1 and DM2) and eventually iterate the analysis.

In figure 2 we can see graphically the results of the changes required to achieve the other group member's result. In the first graph we can see that when the original preferences are relatively close, like happened between DM1 and DM2, the changes needed still allow for a preference structure that share some characteristics of the original (while some may need to change more than others). However, when the individual preferences are markedly different, like between DM2 and DM3, the suggested way to achieve each others results is to alter their individual preference structures so they resemble the other. This may not be acceptable to the DMs.

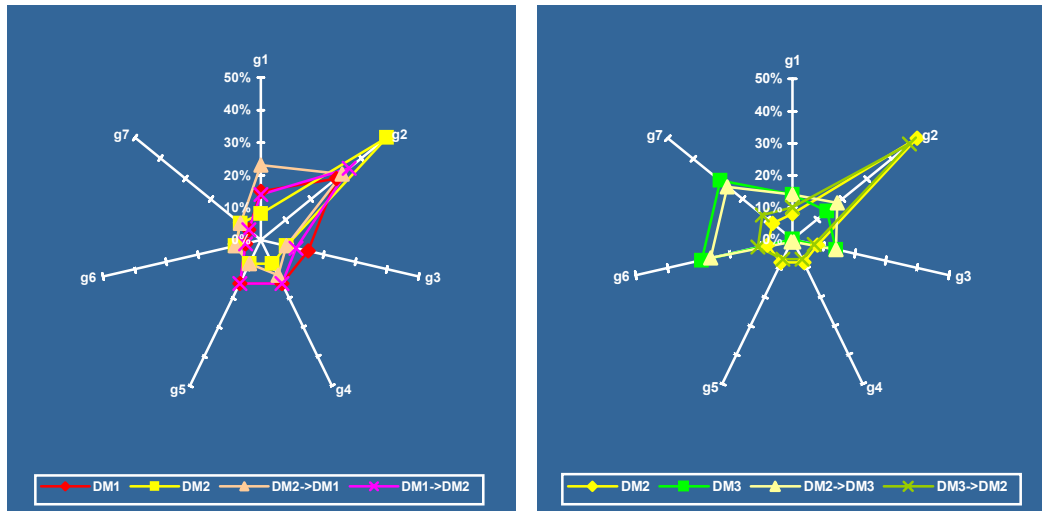


Figure 2 – Changes needed on the preferences

## 5. Conclusions

In this article we presented a mathematical representation of the decision process of each group member, allowing the explicit representation of the differences among the preferences of each pair of group members. It was described how to build a matrix with the differences stemming from the pair wise comparisons, and we pointed out some techniques to explore this matrix.

The chosen mathematical representation is purposely very general. It doesn't rely on any assumption regarding the approach or method followed, neither on the decision problematics (selection, classification or ranking) of the problem to solve. We thus think it to be a framework possibly useful on many situations, proving some tools for better understanding of the differences among group's members opinions, and hopefully some ground to overcome those differences.

As an practical application example, an analytical model following the presented general approach when using the ELECTRE TRI method was also described. However, it is an adaptation that doesn't take into account some important characteristics of the method, like discordance or veto.

Some shortcomings (and assumptions) of the model used are patent:

- it assumes a “perfect market” regarding complete information and access to the preferences of each group members, without hidden agendas;
- the pair wise comparison matrix is multidimensional, implying the need of additional aggregation techniques or work with a highly complex structure.

Decision groups of non-trivial dimension may make this approach difficult or even impossible;

- the mathematical representation doesn't address the decision situation context, in any operational way;
- finally, the human behaviour during the negotiation process falls outside the domain of the approach.

Regarding explicitly the ELECTRE TRI model presented, it was shown that the possibility of giving an importance to the aggregated preference of the others can be used to reflect some degree of trust on another group element's opinion, but risks obscuring the actual effects of the changes on the final preference structure required from that DM. It was however shown that sometimes that "blind" trust can be exchanged by individual changes on the individual preferences, allowing for a more precise (if not easier) choice regarding the required changes on each DMs preference structure.

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## 6. References

- Brans J-P, Macharis C., Mareschal B. And Mariame M., "A Two-Stage PROMETHEE-GAIA Based Procedure for Group Decision Support", *Working Paper STOOTW/287*, Vrije Univeriteit Brussel, Brussels, Belgium (1998).
- Brehmer, B., "Social Judgement Theory and the Analysis of Interpersonal Conflict", *Psychological Bulletin*, 83 (1976), pp 985-1003.
- Costa J., Melo P., Godinho P. and Dias L., "The Design of a GDSS for Project Analysis and Evaluation: the AGAP System", in *Proceedings of the ITI'98, 20th International Conference on Information Technology and Interfaces*, Kalpic D. and Dobric V. (eds.), Pula, Croatia (1998), pp 297-302.
- Costa, J., P. Melo, P. Godinho and L. Dias, "Evaluating Investment Projects with a Group Decision Support System" in N. Callaos, N. nada, A. Cherif, and M. Aveledo (eds.), *Proceedings of SCI'99 and ISAS'99 Conference, Volume I – Information Systems*, Orlando, Florida, International Institute of Informatics and Systemics (1999), pp 125-132.
- Dias, L. and J. Clímaco, "ELECTRE TRI for Groups with Imprecise Information on Parameter Values", *Group Decision and Negotiation*, 9 (2000), pp 355-377.
- Iz P. and Gardiner, L., "A Survey of Integrated Group Decision Support Systems Involving Multiple Criteria", in *Multiple Criteria Decision Making*, Tzeng G., Wang H., Wen U. and Yu P. (eds.), Springer-Verlag, New York (1992), pp 75-85.
- Lewandowski, A., "SCDAS – Decision Support System for Group Decision Making: Decision Theoretic Framework", *Decision Support Systems*, 5 (1989), pp 403-423.
- Lewis, H. and T. Butler, "An Interactive Framework for Multi-person, Multiobjective Decisions", *Decision Science*, 24 (1993), pp 1-22.
- Macharis C., Brans J-P and Mareschal B., "The GDSS PROMETHEE Procedure", *Journal of Decision Systems*, 7 (1998), pp. 283-307.

- Melo P. and Costa J., "Some Design Options for Persistent Distributed Information within Group Decision Support Systems", in Callaos N., Nada N., Cherif A. and Aveledo M. (eds.), *Proceedings of SCI'99 and ISAS'99 Conference, Volume I – Information Systems*, Orlando, Florida, International Institute of Informatics and Systemics (1999), pp. 178-184.
- Melo, P. and J. Costa, "Collaboration in a Distributed Environment", in *La Empresa del Siglo XXI: Finanzas, Tecnologías y Sistemas de Información, Vol II – Actas del I Encuentro Iberoamericano de Finanzas y Sistemas de Información*, (2000), pp 371-380.
- Nadler, D., "The Effects of Feedback on Task Group Behaviour: A Review of the Experimental Research". *Organizational Behaviour and Human Performance*, 23 (1979), pp 307-338.
- Ono, K. and J. Davis, "Individual Judgement and Group Interaction: a Variable Perspective Approach". *Organizational Behaviour and Human Decision Processes*, 41 (1988), pp 211-232.
- Roy B., *Méthodologie Multicritère d'Aide à la Decision*, Economica, Paris (1993).
- Sengupta, K. and D. Te'eni, "Incorporating Multiple Levels of Information Processing in CSCW: An Integrated Design Approach", in *Cooperative Work and GroupWare Systems*, D. Shapiro, M. Tauber and R. Traunmuller (Editors), Elsevier Science, North-Holland (1996), pp 119-134.
- Teghem, J., D. Dufrane, M. Thauvoys and P. Kunsch, "STRANGE: An Interactive Method for Multi-objective Linear Programming Under Uncertainty", *European Journal of Operational Research*, 26 (1986), pp 65-82.
- Turoff M., Hiltz S., Bahgat A. and Rana A., "Distributed Group Support Systems", *MIS Quarterly*, Vol. 17, N° 4 (1993), pp. 399-417.
- Urli, B. and R. Nadeau, "Stochastic MOLP with Incomplete Information: An Interactive Approach with Recourse", *Journal of the Operational Research Society*, 41 (1990), pp 1143-1152.
- Vetschera R., "Integrating Databases and Preference Evaluations in Group Decision Support: A Feedback-Oriented Approach", *Decision Support Systems*, Vol. 7 (1991), pp 67-77.
- Xanthopoulos, Z., E. Melachrinoudis and M. Solomon, "Interactive Multiobjective Group Decision Making with Interval Parameters", *Management Science*, 46 (2000), pp. 1585-1601.
- Yu W., "ELECTRE TRI – Aspects Methodologiques et Guide d'Utilization", *Document du LAMSADE* 74 (1992), Université Paris-Dauphine.