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A note on the role of robustness analysis in decision-aiding processes[1]

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Abstract. Robustness analysis is an increasingly popular response to the difficulties in setting the parameter values of decision aiding models, for individual or group decisions. This note discusses three possible roles for robustness analysis in a decision aiding process. Considering the difficulties of eliciting values for preference-related parameters, we will argue the potential benefits of using robustness analysis throughout the whole decision process as a tool to guide that process, which is the idea behind tools such as VIP Analysis or IRIS and their extensions for group decision-making.

Introduction

The formal models used in decision aiding processes are subject to many sources of uncertainty, imprecision, and ignorance (Roy,1989). We will consider that a formal model is defined by a structure and a combination of parameter values. For instance, the structure of a linear program consists of a linear function to maximize (or minimize) and a set of linear constraints; its parameters are the cost coefficients, the technical coefficients and the right-hand-side values. A multicriteria decision-aiding model, can be structured as an ELECTRE model, a MAUT model, or another type of structure, and has parameters related to the performance of each alternative on each criterion, criteria weights, etc. For a given model structure, different combinations for the parameter values will lead to different model "versions" (to use the expression recently proposed by Roy (2003)).

It is well known that setting technical and economical parameter values is often problematical: instruments and statistics can be imprecise (e.g., tolerances for precision in measurement, confidence intervals in statistics), measurement can be arbitrary and subjective (e.g., measuring noise pollution or

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a firm's performance), some information (e.g. data from clinical trials) may be controversial or contradictory, let alone uncertainties about the future.

When considering multi-criteria decision aiding, as we wish to do, we also incorporate in the models parameters related to the preferences of the Decision Maker (DM). Eliciting parameter values about preferences is also problematical. In cognitive terms, the parameters are artifacts whose semantic may be difficult to understand for the DM, not to mention biases related to the way questions are posed. For him or her, value judgments are naturally easier to express through words than through numbers. Furthermore, preferences may evolve, as they are often unstable outcomes of unresolved internal conflicts in the DM's mind. Adding to these fundamental difficulties, other constraints of a more pragmatic nature may be present, e.g., the DM is reluctant to divulge precise parameter values about his/her preferences in public, or his/her time and patience is rather limited.

Moreover, we often need to address the concerns of a group of actors, rather than a single DM. The above mentioned difficulties of fixing preference-related parameter values are still present, if not reinforced by the diversity of judgments and subjective perceptions of what is at stake. In such cases, the existence of "hidden agendas" may hinder an open discussion about parameter values. Even in case of consensus, one must be aware of phenomena such as groupthink (Janis 1972).

Robustness Analysis (RA) is motivated by these difficulties in setting the parameter values of decision-aiding models, i.e., the difficulties of isolating a single model version (or problem version, in Roy's words)[2] to serve as a basis to recommend a decision. The general idea behind RA approaches is to accept multiple model versions and to try to progress in the decision aiding progress despite the diversity of versions, possibly to identify a solution that is seen as good (or at least acceptable) in any model version (or at least in most of the versions). As a matter of fact, we may find a reference to "robustness" in this sense in one of the pioneering books on Operational Research (OR), which described a famous OR problem referring to the need of solutions «which are robust in a quite large variety of circumstances» (Beer, 1966: p.44).

In this note we start by reviewing some of the multiple conceptions of RA. Then, we discuss three possible roles for RA in a decision aiding process, emphasising the role underlying the use of RA in VIP Analysis (Dias and Clímaco 2000) and IRIS (Dias and Mousseau 2003).

[2] We will use *model version* instead of *problem version*, to emphasize that it is the model that will vary, not the problem. This distinction is relevant only if we consider the word "problem" from a structuring perspective ("what is the problem?"). If we consider the word "problem" from an algorithmic perspective (the input, or the instance), then there is no distinction.

Concepts of Robustness Analysis

Sensitivity Analysis (SA) is a traditional answer to the difficulties in setting the model's parameter values. In model building it may indicate which parameters contribute more to the variance of the outputs. In optimization, it indicates how much the parameters may vary without changing some conclusion of interest. RA is often seen as a reverse perspective of SA, but that would depend on the notion of RA that is being considered. Indeed, we may find multiple perspectives about the concept (Roy, 2002). To Rosenhead (2001), RA is used to choose one action (a first choice in a succession of choices) that is flexible enough as to leave many good options regarding the choices to be made in the future. Kouvelis and Yu (1997) define robust solution to an optimization problem as the one which has the best performance under in its worst case (e.g., max-min rule), which could be replaced the criterion proposed by Aloulou et al. (2005). Mulvey et al. (1995) differentiate between the quality of solution robust, for a solution whose value is always near the optimal one for every acceptable model version, from the quality of model robust, for a solution which is always feasible or almost feasible for every version. Sevaux and Sörensen (2004) introduce a concept of solution robustness meaning the solution (a plan) does not change much in optimization programs that are to be repeated regularly. More generally, Hites et al. (2003) recommend a multicriteria evaluation of solution robustness.

The perspectives that are nearer to the reverse of SA are those appearing in (Roy 1998; Roy and Bouyssou, 1993), defining robust conclusion as an assertion that is valid for the set of results compatible with the different model versions envisaged, and Vincke's (1999) definition of robust solution as one that is always near (or that does not contradict) any other solution obtainable using an acceptable version (Vincke has also introduced the notion of robust method). Roy (1998) also introduced the notions of approximately robust conclusion (if it is true "almost everywhere") and pseudo-robust conclusion (if not perfectly formalized). It may also be of interest to distinguish absolute vs. relative conclusions, as well as to distinguish between conclusions about the intermediate results vs. the final result of a method (e.g., see Dias and Clímaco 2002).

Usually, the model versions are considered as equally acceptable, without attempting to define a "meta-model" that would attribute different degrees of probability (or possibility, or importance…) to different versions. Indeed, an attempt to model the model versions would bring new difficulties to surmount: a simple model may become too complex for being used by the DM in a transparent manner; the "meta-model" would itself need to be parameterized; and some "meta-models" are not adequate to some types of parameters, e.g., it would be unnatural to consider criteria weights as random variables.

Roles for Robustness Analysis

The role of RA in decision aiding does not seem to have been much discussed so far. In an attempt to classify most of the proposed RA approaches, we could separate them according to their placement (ex-ante vs. ex-post) with respect to using a method to obtain a solution.

One of the possibilities is to consider RA as an ex-ante concern, which amounts to imbed this concern in a model to be optimized. In these cases, usually optimization problems, a model is built and an algorithm is used to obtain a solution that is robust according to some pre-specified criterion. The obtained solution will be optimal with respect to that criterion (e.g., it minimizes the maximum cost or the maximum regret). It may even happen that the optimal solution for the RA-imbedded model might never have been optimal for any of the versions considered for the original model. Examples of these approaches are, e.g., (Kouvelis and Yu, 1997; Mulvey et al. 1995). An approach that seems particularly promising is to use several criteria rather than a single one to be optimized (as suggested by Hites et al., 2003), possibly using tools analogous to those employed in interactive multiobjective programming to explore the set of efficient solutions. Another type of approach considering RA ex-ante is to identify a subset of robust solutions (possibly empty) according to some robustness criterion (e.g., Aloulou et al., 2005), rather than a single optimally robust solution.

A second possibility is to consider RA as an ex-post concern, substituting or complementing SA, to assess how robust is a solution derived from a decision aiding process and to supply additional robust conclusions. Arguably, the first example of this type of approach is found in (Roy and Bouyssou, 1993). Such an approach may be useful to question the validity of the recommendation and how its evaluation might change from version to version, possibly identifying its limits and enriching the information that may be provided to the DM. For instance, rather than saying that x is the best alternative in a choice problem, one may inform the DM that all alternatives are outranked by either x or y, explaining what are the main differences between the versions that favour x and those favouring y, and adding that y is always a relatively good choice, while there are versions where x receives a poor evaluation.

Before discussing a third possibility, we may note that for the approaches we mentioned before the set of versions is considered to have been defined a priori. As Roy (2002) notes, this may cause a dilemma between the wish to take into account every conceivable version and the wish to obtain some useful conclusions. It is perhaps because of this potential dilemma that Roy (1998) had earlier proposed the notion of approximately robust conclusion: a formal assertion that is verified for all the versions, except a few ones, which are considered negligible.

When we consider preference-related parameters, a third possibility is based on the idea of using RA throughout the whole decision process as a tool to guide that process. At the outset of the process, the participants will be open to accept a wide set of model versions, which they will try to progressively reduce as the process advances. Instead of a process that starts with an elicitation of parameter values, followed by the computation of a solution and an ex-post RA, we will have a reiteration of elicitation and RA phases.

In elicitation phases, the DM will be questioned about parameter values, without requesting precise numbers (e.g., the answer can be an interval, or a comparison relation between two parameters). The elicitation may even proceed indirectly, as in aggregation-disaggregation approaches (Jacquet-Lagrèze and Siskos 2001), where the DM may indicate results that the model should reproduce. The DM's answers to the elicitation phases will be used to constrain the set of versions considered in RA phases. In the first elicitation phases the more difficult elicitation questions may be avoided, allowing the DM to learn about the methods used and his/her preferences before reaching those difficult questions. In the case of group decisions, this way of proceeding allows to postpone (or even avoid) those elicitation questions that are more prone to cause conflict among the members of the group.

In RA phases, the robust conclusions corresponding to the current set of versions are to be discussed. It will normally be visible that some conclusions are robust, whereas other results are very dependent on the version chosen. In group decisions, this will show where agreement exists and where disagreement is stronger. This may in turn motivate new elicitation questions or issues to be discussed, when returning to an elicitation phase.

The examples of VIP Analysis and IRIS

If the RA is adopted in the spirit of using it throughout the decision aiding process as a means to guide an informed discussion, then RA becomes interactive. This is best achieved when there is software to aid the DM (and possibly an analyst) during the successive iterations. We next provide two examples of such software.

VIP Analysis (for details see Dias and Clímaco, 1999)

This software is intended to support choice decisions using additive value functions, allowing to draw robust conclusions when using different versions for the scaling weights ($k_1,\ldots,k_n$). In elicitation phases, the DM may indicate any information that can be translated as a linear constraint, such as intervals for weights or weight ratios, parameter comparisons (e.g., $k_1 \geq k_2$), or holistic comparisons (e.g., a_1 is not worse than a_2). In RA phases, VIP Analysis uses linear programming to identify the minimum and maximum value that each alternative may achieve, as well as the minimum and maximum differences of value between each alternative and the other ones (Fig 1).

The outputs of RA indicate which alternatives are most affected by imprecision, indicating also the versions leading to the extreme results (hence inviting the DM to ponder whether such versions are acceptable or not). In a choice problematic, RA also highlights which alternatives may be discarded (dominated or quasi-dominated with respect to versions), allowing a progressive reduction of the number of alternatives.

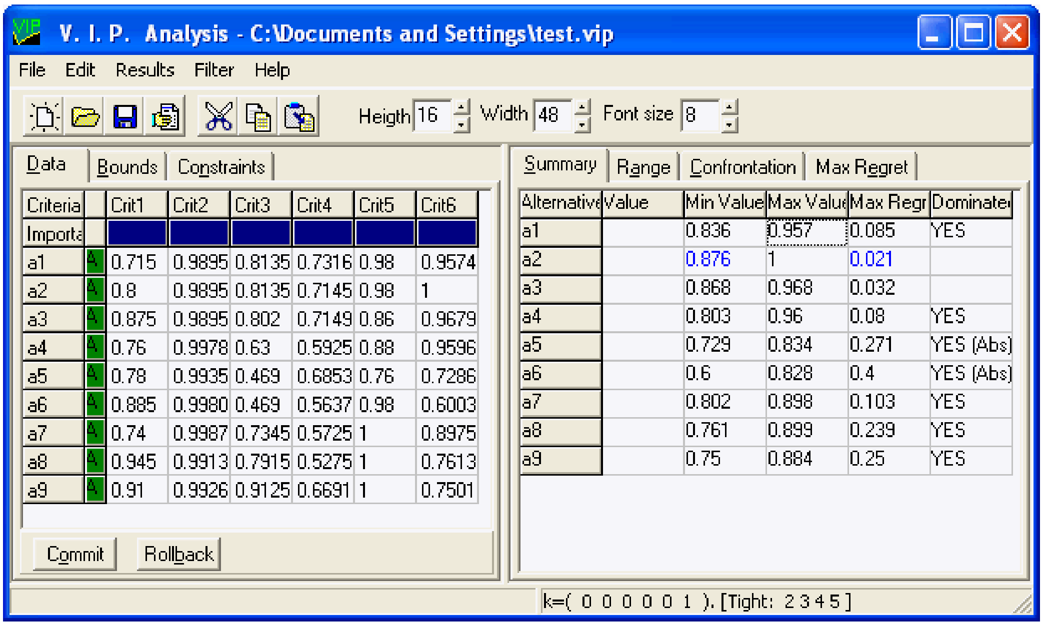

Figure 1. Given the set of acceptable versions for the scaling weights (not shown in the figure), the results on the right indicate the minimum and maximum value for each alternative, as well as the maximum difference (max. regret) by which each one can lose to another. Dominated alternatives are those that lose to another one (either a_2 or a_3) in all model versions. When selecting the cell containing the maximum value of a_1, the corresponding version (all scaling weights equal to zero except the last one) appears at the bottom of the screen.

In extending the VIP Analysis methodology to group decision aiding, Dias and Clímaco (2005) suggest how a decision panel forming a democratic decision unit may proceed to reach a final decision in a choice problem using RA. The idea is not to impose an aggregated model from the individual ones, but rather to reflect to each group member the consequences of his/her inputs, confronting them with analogous reflections of the group members' inputs. Each group member can study what is robust from his/her perspective and from a group perspective. Some conclusions will be robust only to a subgroup, which will be enlarged if a tolerance is considered to accept conclusions that are almost robust, bearing in mind a trade-off between this tolerance and the proportion of the group members that agree with the conclusion.

IRIS (for details see Dias and Mousseau, 2003)

This software is intended to support sorting decisions using ELECTRE TRI models, allowing different versions for the weights ($k_1,\ldots,k_n$) and cutting level (λ). It implements the idea of integrating RA with an aggregation/disaggregation approach (parameter inference) proposed by Dias et al. (2002). In elicitation phases, besides linear constraints on the weights, the DM may indicate sorting examples, which should be reproduced by IRIS. In RA phases, IRIS uses linear programming to show the range of categories where each alternative may be sorted, and to infer which of the versions would satisfy the constraints with maximum slack (Fig. 2). It also provides some guidance when the inputs happen to be inconsistent.

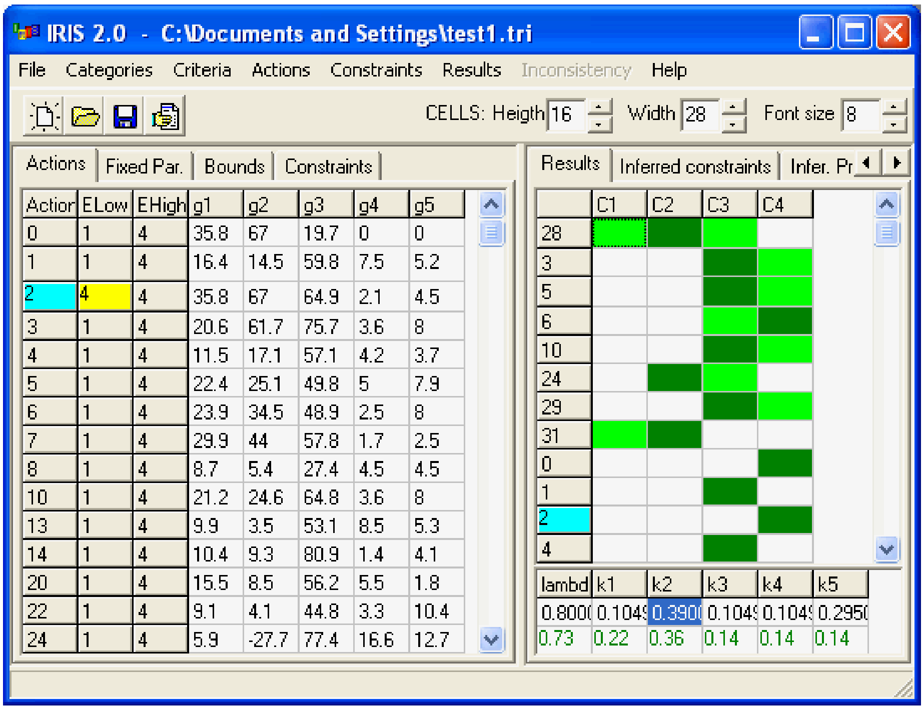

Figure 2. Given the set of acceptable versions for the criteria weights (not shown in the figure), the results on the right indicate the minimum and maximum category where each alternative may be sorted, as well as the sorting suggested by the inference program. When selecting the cell where a_{28} is assigned to *C1*, the corresponding version appears at the bottom right of the screen.

IRIS encourages the DM to interact with it through communicating sorting examples, aiming to reduce progressively the interval of categories where each alternative may be sorted. As in VIP Analysis, IRIS indicates the versions corresponding to extreme results (worst and best categories for each alternative), thus inviting the DM to ponder the acceptability of such versions.

An extension of IRIS to group decision settings has also been proposed (Damart et al., to appear). The idea is that the group may progress towards a common and unique ELECTRE TRI model while maintaining consistency (i.e., the ability to reproduce the examples with a suitable model version) along the whole process both at the individual and at the collective levels. The interaction is based on agreements concerning sorting examples. By using an individual instance of IRIS to privately build his/her model, each DM is helped to keep a consistent set of examples. Moreover, the DM can check the impact on his/her model that results from agreeing to a sorting example being discussed by the group. At the same time, an analyst uses an instance of IRIS to maintain a consistent set of examples agreed by the group, and may use the software to answer "what-if" questions or to verify whether a "package agreement" involving several sorting decisions simultaneously is consistent or not. By aggregating the robust intervals of possible categories for each alternative from all the group members, it is also possible to suggest the sorting possibilities that are more likely to generate agreement.

Concluding remarks

There are many distinctions that can be made regarding the approaches to deal with difficulties in setting the parameter values of decision aiding models. In this note we have focused the distinction of RA approaches with regard to their role in a decision aiding process that develops through time: ex-ante approaches imbed RA in a model to be optimized or to identify a subset of interesting solutions; ex-post approaches avoid *aggregating* what happens for the multiple model versions, aiming at *exploring* how robust is a result or to derive some robust conclusions about the results; a third option is to use RA as a tool to guide the process.

We have focused the role of RA as a tool to guide a decision-aiding process, prompting questions for the DM to analyze, indicating what are the results more affected by his/her answers, and showing what can be robustly concluded. As in ex-ante approaches, it intends to avoid the problem of eliciting a model version to work with. As in ex-post approaches, it follows a strategy of exploration of robust conclusions, rather than aggregation.

When the motivation for RA stems from the existence of multiple DMs, this type of approaches also seem promising as tools to guide a group decision process. In such processes, many versions may be needed to accommodate all the different views, and this set of versions can be discussed throughout an iterative process based on successive agreements. RA will show where disagreement is stronger, it will motivate the issues to be discussed, and will highlight robust conclusions (agreement). Besides (Dias and Clímaco 2005; Damart et al., to appear) we can also mention (Lamboray 2005) as an example of how RA can be used to support group decision processes.

The aim of using RA as a process-guiding tool will not be to select a version, but to highlight a set of robust conclusions that is found to be requisite (in the sense of (Phillips,1984)). Furthermore, the approach will not lead to a precise recipe or path to meet the objective of the decision aiding process. It is more like a general non-normative methodology of progressive construction (or perhaps delimitation) of a model, through the progressive reduction of the set of model versions considered, where the DM is informed by reiterated RA.

This type of approach seems particularly well-suited when the RA concerns parameters related with preferences, in that the set of versions can be reduced as a result of learning or increased effort from the DM. It may also be indicated for other parameters that can be known with higher precision but at an additional cost, e.g., data from surveys or experimental data.

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