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**A fuzzy multiple objective decision support model for
energy-economy planning**

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Abstract

In this paper an interactive approach to deal with fuzzy multiple objective linear programming problems is presented, which is based on the analysis of the decomposition of the parametric (weight) diagram into indifference regions corresponding to basic efficient solutions. This approach is illustrated to tackle uncertainty and imprecision associated with the coefficients of an input-output energy-economy planning model, aimed at providing decision support to decision makers in the study of the interactions between the energy system and the economy on a national level.

Keywords: Fuzzy multiple objective linear programming; Decision support systems; Interactive methods; Energy system; Input-output analysis.

1. Introduction

The energy sector is of outstanding importance to the analysis of an economy on a national level, because of direct and indirect consequences on several well-being indicators ranging from economical aspects to social and environmental ones. For some industrialized countries, such as Portugal, energy dependence is a crucial issue because of the high level of imports of primary energy, namely fossil fuels. In these circumstances, well-founded information concerning economic development constrained by limited energy resources must be provided to decision makers (DMs). A decision support model addressing the energy sector in the broader context of the economic system has been developed enabling to study their interactions.

The interactions among different sectors of an economy can be dealt with input-output analysis. In the framework of input-output analysis an economic system is disaggregated into a number of interdependent sectors. Each sector in the static input-output table produces a

particular output, with fixed input and output structure, and no substitution between the outputs of the different sectors [Leontieff, 1951]. By providing a systemic view of macro-economic aggregates and economic flows in a given economic system, input-output analysis is an useful tool to assist in the formulation of economic planning models.

Economy-energy planning problems are characterized by the need of explicitly considering multiple, conflicting, and incommensurate axes of evaluation. Mathematical models for decision support must address, in an explicit manner, aspects of distinct nature such as social, economical, environmental, and technical ones rather than attempting to encompass them in an one-dimensional economic indicator [Zeleny, 1982; Steuer, 1986]. Moreover, the multiple objective approach intrinsically possesses a value-added role in the modeling process and in model analysis, supporting reflection and creativity in face of a larger universe of potential solutions rather than a single “optimal” solution.

A multiple objective linear programming (MOLP) model based on input-output analysis has been developed devoted to study the relationships between the economy and the energy sector on a national level. The model allows for the computation of the amount of energy required for the provision of goods and services in an economy, both for intermediate consumption (that is, for sectors producing other goods or services) and directly in final demand. Moreover, by associating the consumption of fossil fuels and the corresponding carbon content with the activity level of each sector it is possible determine the resulting amount of emissions of atmospheric pollutants (such as carbon dioxide).

In a model possessing a great diversity and complexity of input information, which is used to derive the coefficients to the MOLP model, several sources of uncertainty are at stake. These are taken into account herein by considering some model coefficients as triangular fuzzy numbers. Interactive techniques based on the analysis of the decomposition of the parametric (weight) diagram into indifference regions corresponding to (basic) efficient solutions have been developed and computationally implemented as the core of a decision support system (DSS) to deal with uncertainty in MOLP models.

Section 2 presents some key concepts of MOLP that are important to introduce the proposed visual interactive approach to deal with fuzzy MOLP problems. This approach is described in detail in section 3. In section 4 a multiple objective input-output model for energy planning is briefly presented, which is aimed at studying the energy sector in the context of the economy on a national level. Some illustrative results are reported in section 5. Finally, in section 6 some conclusions are drawn.

2. Multiple objective linear programming

Let us consider the following MOLP problem with p linear objective functions and m linear constraints:

$$\begin{aligned} & \text{“max” } \mathbf{z} = \mathbf{C} \mathbf{x} & (1) \\ \text{s. t.} & \\ & \mathbf{x} \in X = \{ \mathbf{x} \in \mathfrak{R}^n : \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0} \} \end{aligned}$$

where \mathbf{A} is a $m \times n$ matrix, \mathbf{b} is the m right-hand side (RHS) column vector and \mathbf{C} is a $p \times n$ matrix of objective functions coefficients. “max” denotes the operation of computing efficient solutions.

A feasible solution to (1) is called efficient if and only if there is no feasible solution for which no improvement in any objective functions value is possible without sacrificing on at least one of the other objective functions. A relaxed notion is also generally used: a feasible solution is called weakly efficient if and only if there is no other feasible solution that strictly improves all objective function values. For definitions and mathematical details see, for instance, Steuer [1986].

Let $C_{r\cdot}$ ($r=1, \dots, p$) be the r^{th} row of \mathbf{C} . When solving problem (1) by the weighted-sum approach, each objective $C_{r\cdot} \mathbf{x}$ is associated with a positive weight λ_r (an $\lambda_r=0$ could lead to a weakly efficient solution). Without loss of generality, each weighting vector can be normalized so that its elements sum to one

$$\Lambda = \{ \boldsymbol{\lambda}: \boldsymbol{\lambda}=(\lambda_1, \lambda_2, \dots, \lambda_p) \in \mathfrak{R}^p, \sum_{r=1}^p \lambda_r = 1, \lambda_r > 0, r=1, \dots, p \} \quad (2)$$

Therefore, basic efficient solutions can be obtained by optimizing a scalarizing function consisting of a weighted sum of the objective functions:

$$\begin{aligned} & \max \sum_{r=1}^p \lambda_r (C_{r\cdot} \mathbf{x}) & (3) \\ \text{s. t.} & \\ & \mathbf{x} \in X, \boldsymbol{\lambda} \in \Lambda. \end{aligned}$$

Let K be the index set of the $(n-m)$ non-basic variables associated with an optimal solution to (3), which is an efficient basic solution to (1), computed by using a given weighting vector. \mathbf{B} and \mathbf{N} are the submatrices of \mathbf{A} corresponding to the basic and non-basic variables,

respectively, and $C_B(x_B)$ and $C_N(x_N)$ are the submatrices (subvectors) of $C(x)$ corresponding to the basic and non-basic variables, respectively. An indifference region for the weights (set of weighting vectors that leads to the same basic efficient solution) is defined in Λ and can be achieved by the intersection of the $n-m$ hyper-halfspaces resulting from the reduced cost matrix of a multiobjective simplex tableau [Steuer, 1986] associated with a basic efficient solution ($W=C_B B^{-1}N-C_N$), that is:

$$\bigcap_{k \in K} \left\{ \sum_{r=1}^p \lambda_r w_{rk} \geq 0 \right\}, \lambda \in \Lambda \quad (4)$$

w_{rk} is the (r, k) element of the reduced cost matrix with respect to objective function $r=1, \dots, p$, and the non-basic variable $k \in K$. The DM may be indifferent to all combinations of weighting vectors within it because they lead to the same basic efficient solution. These indifference regions are defined in a geometrical $(p-1)$ -dimensional simplex in a p -dimensional Euclidean space.

For three objective functions the use of visual interactive graphical tools are particularly suited for the exchange of information with the DM. The decomposition of Λ into indifference regions lends itself well to a progressive and selective learning of the efficient solution set in MOLP [Clímaco and Antunes, 1987; 1989].

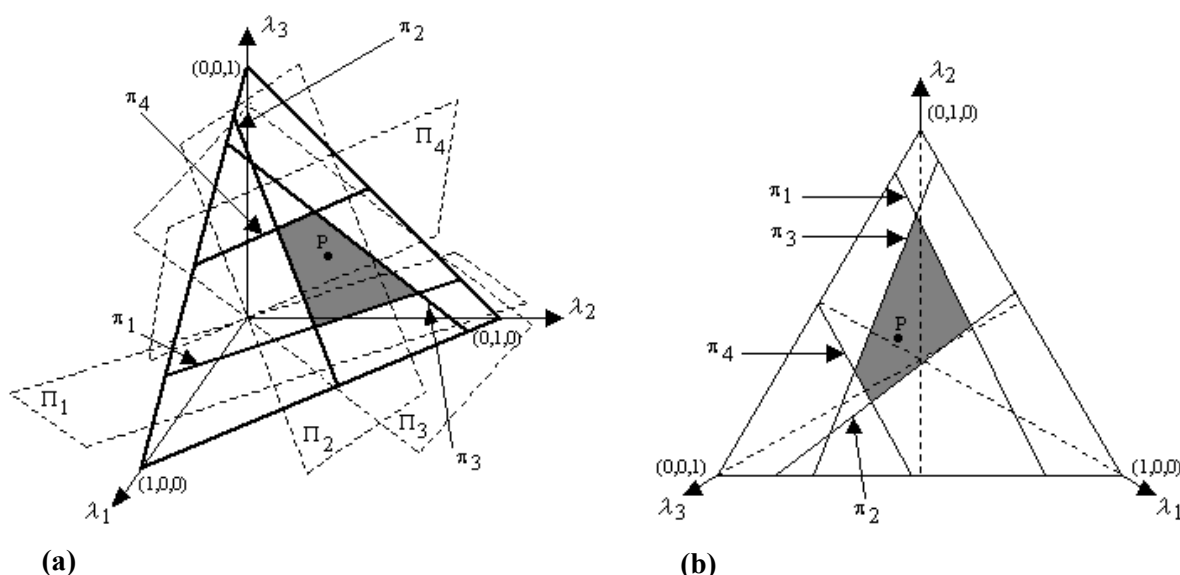


Figure 1 – Decomposition of Λ into indifference regions

Figure 1 shows a 3-dimensional weight space where the hatched polygon is the indifference region associated with the basic efficient solution computed by optimizing the weighted-sum

LP considering the weighting vector $\lambda=P$. Each of the $n-m$ ($n-m=4$ in this example) halfspaces defined in (4) corresponds to a non-basic variable. Π_k is the plane obtained from the k halfspace in (4) replacing the inequality ' \geq ' by '=' and it is defined by $\sum_{r=1}^3 \lambda_r w_{rk} = 0$. π_k denotes the intersection of Π_k with Λ .

In the operational framework of the proposed interactive fuzzy MOLP approach, the parametric diagram, a geometrical 2-dimensional simplex in a 3-dimensional Euclidean space, is used to display relevant information in the same graph to the DM. This enables the DM to visualize dynamically and interactively the behavior of efficient solutions according to changes in the initial model coefficients and DM's preferences.

3. Fuzzy analysis in MOLP

A great diversity of possible modifications to the classical (crisp) LP problem (1) have been proposed in a fuzzy environment and different ways to deal with the corresponding types of fuzziness in LP models are reported in the literature.

The coefficients of the vector \mathbf{b} or the matrices C or A can have a fuzzy character [Tanaka and Asai, 1984; Carlsson and Korhonen, 1986; Sakawa and Yano, 1990] either because they are fuzzy in nature or their perception is fuzzy.

The mathematical relations involved may also be fuzzy (fuzzy objectives and/or constraints) [Zimmermann, 1978, 1983; Chanas, 1983; Werners, 1987a, 1987b]. The DM may not be interested in optimizing some of the objective functions; rather he/she might want to "improve" as much as possible their values in order to reach some "aspiration levels" which may not be crisply defined. The constraints may also be fuzzy, that is the '=' sign might not be met in the strictly mathematical sense but the DM may accept small violations on it.

Moreover, the solution of a fuzzy linear programming problem may be crisp [Zimmermann, 1978, 1983; Tanaka and Asai, 1984; Werners, 1987a, 1987b; Sakawa and Yano, 1990] or fuzzy [Chanas, 1983; Carlsson and Korhonen, 1986]. In the latter case a solution set (of all fuzzy efficient solutions) is presented to the DM and he/she must choose the one that is more in accordance with his/her preferences.

In this study the objective function coefficients and the constraints' RHS as well as the coefficients in the technological matrix and in the objective functions associated with a new

decision variable are considered fuzzy coefficients and are characterized by triangular membership functions defined by (figure 2):

$$\mu_{\tilde{c}}(x) = \begin{cases} 0 & , \text{if } x \leq c_L \\ \frac{(x - c_L)}{(c_M - c_L)} & , \text{if } x \in]c_L, c_M[\\ 1 & , \text{if } x = c_M \\ \frac{(c_R - x)}{(c_R - c_M)} & , \text{if } x \in]c_M, c_R[\\ 0 & , \text{if } x \geq c_R \end{cases}$$

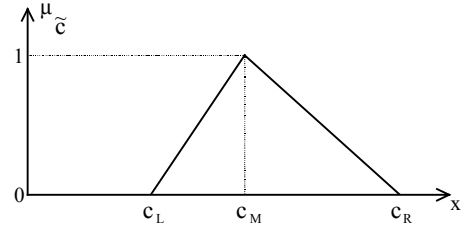


Figure 2 – A triangular membership function

A triangular fuzzy number can be denoted as $\tilde{c} = (c_L, c_M, c_R)$ where c_M is the central value (maximum grade of membership), $c_M - c_L$ is the left spread and $c_R - c_M$ is the right spread.

Interactive techniques to deal with fuzzy MOLP models have been developed and implemented as the core of a DSS. The DM can visualize dynamically the changes in the indifference regions corresponding to the initial (crisp) basic efficient solutions and compute new basic efficient solution(s) in an interactive manner by varying continuously the grades of membership as well as by changing the values of c_L and c_R (and also the value of c_M when introducing a new variable) for each fuzzy coefficient. The new basic efficient solutions are computed by using the Simplex or Dual-Simplex method starting from the multiobjective Simplex tableaux corresponding to an efficient solution previously computed with maximum grade of membership and considering weighting vectors within regions of Λ not yet filled with indifference regions [Borges and Antunes, 2000].

The aim of the proposed interactive DSS is to help the DM to exploit the uncertainty associated with the initial problem, modeled by means of fuzzy numbers, to gather further knowledge on the problem as well as to reinforce or weaken his/her own convictions and preferences in order to make a better informed decision. During the interactive study the DM is always allowed to revise prior preference information and exploit new search directions.

3.1 Objective function matrix

The objective function coefficients are defined as triangular fuzzy numbers where the central values (c_M) are the crisp model parameters, and the grade of membership (γ) is the same for all objective functions.

For each different y only the objective function coefficients are changing. Therefore, the extreme points of the feasible region remain unchanged. However, the reduced cost matrix values, $W=C_B B^{-1}N-C_N$, vary and the efficient region can eventually change. The indifference regions corresponding to the basic efficient solutions to the initial problem are then changing continuously, in size and shape, with changes of the grade of membership.

New basic efficient solutions can be computed by using weighing vectors within regions of Λ not yet filled with indifference regions. The new basic efficient solutions are computed by using the Simplex method starting from the multiobjective Simplex tableaux corresponding to an efficient solution previously computed with maximum grade of membership (corresponding to the C_M value) and selected by the DM.

The indifference regions associated with the computed basic efficient solutions can even disappear meaning that the corresponding extreme point becomes dominated.

3.2 RHS

The constraints' RHS coefficients are defined as triangular fuzzy numbers, with the central value (c_M) corresponding to the crisp model parameters, and the grade of membership (t) is the same for all constraints.

The values of the \mathbf{b} vector are changing with t and so the feasible region changes. If some decision variable values regarding a basic efficient solution, $\mathbf{x}_B=B^{-1}\mathbf{b}$, become negative, then the corresponding solution becomes infeasible. Since the objective function coefficients remain unchanged, then for a given basis the reduced cost matrix $W=C_B B^{-1}N-C_N$ does not change and the optimality condition is never violated. Therefore, the indifference regions corresponding to the basic efficient solutions do not change continuously, as in the case of objective functions, but they change 'suddenly' as they appear or disappear, meaning that the corresponding efficient basis becomes feasible or infeasible, respectively.

New basic efficient solutions can be computed by using weighing vectors within regions of Λ not yet filled with indifference regions by using the Dual-Simplex method and starting from the multiobjective Simplex tableaux corresponding to an initial basic efficient solution. The starting efficient solution is such that the selected weighing vector belongs to the initial corresponding indifference region.

If the selected weighing vector is within regions of Λ not initially filled with indifference regions then the DM is asked to previously compute the corresponding initial basic efficient

solution (computed with maximum grade of membership value, that is considering the c_M values).

3.3 Introduction of new decision variables

The coefficient vectors in the objective functions and in the technological matrix of a new decision variable are defined as triangular fuzzy numbers, $\tilde{C}_{x_{new}}$ and $\tilde{A}_{x_{new}}$, respectively. Two distinct grades of membership are considered: one for all the objective functions and another one for all constraints.

When both grades of membership are changing the reduced cost matrix column regarding the new variable ($W_{x_{new}} = C_B B^{-1} \tilde{A}_{x_{new}} - \tilde{C}_{x_{new}}$), must satisfy

$$\{\boldsymbol{\lambda}^T (C_B B^{-1} \tilde{A}_{x_{new}} - \tilde{C}_{x_{new}}) \geq 0\}, \boldsymbol{\lambda} \in \Lambda \quad (5)$$

for the basic solution under analysis to remain efficient.

The introduction of a new decision variable into a MOLP leads to the creation of new extreme points with non-zero value in the new variable as well as new edges and faces [Antunes and Clímaco, 1992]. The new variable may be classified, with respect to a selected basic efficient solution, as:

- non-efficient variable, whenever (5) does not affect the initial (4) associated with the selected efficient solution ((5) is redundant with respect to the initial (4));
- efficient variable, whenever (5) does intersect the initial (4) associated with the selected efficient solution;
- "must be made basic" variable, because the selected efficient solution becomes dominated (the initial (4) associated with the selected efficient solution does not belong to the hyper-halfspace defined by (5)).

A non-basic variable is efficient with respect to a given efficient basis if and only if when introduced into the basis it leads to an adjacent efficient basis through an efficient edge. In this situation, in addition to "update" the selected basic efficient solution (that is, to compute the intersection of the initial (4) with (5) to determine the new indifference region), new basic efficient solutions with non-zero value in the new variable can be computed.

If the variable "must be made basic" new basic efficient solutions with non-zero value in the new variable can be computed.

The new basic efficient solutions are computed as described in **3.1**.

Whenever the introduction of new decision variables is considered, the triangular membership functions associated with the parameters (corresponding to the new variable) can be changed by modifying not only the corresponding c_L and c_R but also the c_M values.

4. An Input-Output MOLP Model for Energy-Economy Planning

An input-output table based on statistical data available from several Portuguese and international sources has been developed which considers 21 economic sectors. The energy sector components have been disaggregated in detail, allowing the distinction between primary and secondary energy sources, by means of 23 artificial sectors that are used for distributing the output of the oil refining sector and the by-products through the consuming sectors. Energy flows (in toes, tons of oil equivalent) and monetary flows (in monetary units) are considered. The anatomy of the input-output model is as follows: a (44x44) matrix with the inter- and intra-sector flows, 6 column vectors with the components of final demand (private consumption, collective consumption, gross fixed capital formation, positive and negative stock changes, and exports), 1 column vector for the competitive imports and 3 row vectors for the primary inputs (wages, net indirect taxes, and operating surplus).

The consumption of fossil fuels is associated with the level of activity of each sector, enabling to evaluate the embodied energy required to manufacture a good or service. The analysis is then extended to account for emissions of air pollutants resulting from the burning of fossil fuels by incorporating the requirements of primary energy for the economic activities. Total emissions from each sector and the whole economy can be computed by using coefficients that relate the amount of carbon dioxide produced per unit of fuel consumed (through its calorific value). The top-down methodology proposed by the Intergovernmental Panel for Climate Change [IPCC, 1996] has been used to model carbon dioxide (CO_2) emissions, which is based on the principles of combustion and composition of fuels.

The model considers three objective functions:

- energy imports (to be minimized, taking into account the energy dependence of the country);
- self-production of electricity (to be maximized, in order to encourage the use of alternative forms of energy, valuing the recycle of wastes and allowing both energy economies and the minimization of waste disposal);

- CO₂ emissions (to be minimized, due to the impact of energy resources on the environment, specifically air pollution).

Energy imports and self-production of electricity are expressed in physical units of energy (toes) and CO₂ emissions is in Gg.

Several categories of constraints are considered in the MOLP model:

- balance of payments (to guarantee a certain level of external equilibrium);
- public deficit (to take into account European Union requirements);
- upper and lower bounds on the production capacity of each activity sector;
- upper and lower bounds on imports and exports (to avoid an over-specialization since they are not linked to the model coefficients);
- storage capacity and security stocks for hydrocarbons (to guarantee that positive stock changes never exceed storage capacity and negative stock changes are never below security stocks);
- coherence constraints for goods and services (imposing that the use of a specific good or service, for intermediate consumption and final demand, cannot exceed the resources available, resulting from national production and competitive imports);
- defining constraints for gross added value (an indicator enabling to quantify the resources generated within the country) and gross domestic product (both according to expense and product definitions).

Please see [Oliveira and Antunes, 2000] and [Antunes et al., 2002] for further details on the input-output structure and the mathematical model.

5. Some illustrative results

After performing a progressive and selective learning of the efficient solution set in a crisp environment, the DM is given the possibility of interactively studying the effects of the fuzziness arising in the parameters of the objective functions, the constraints' RHS or in the objective functions and constraints of a new decision variable. Different membership functions (associated with the fuzzy parameters) can be considered by the DM and for each set of them the grades of membership can be continuously changed. The comparative graphical analysis of the decomposition of the parametric (weight) diagram into indifference regions corresponding to the initial (crisp) basic efficient solutions and the new ones computed in a fuzzy

environment, as well as the numerical values provided by the DSS, enable the DM to study the fuzzy efficient solution set.

Firstly, a search for basic efficient solutions has been progressively performed in order to have an overview of solutions with different characteristics for the energy-economy planning model. Surprisingly, Λ become completely filled with few indifference regions, as displayed in figure 3, meaning that, in crisp environment, all basic efficient solutions have been found. The objective function values (as well as the indifference region areas) of those solutions are shown in table 1.

In general, the aim is to compute “well-dispersed” solutions to gain some insights into the problem, which a further selective search could be based on. This usually happens for medium-sized MOLP problems in which hundreds of basic efficient solutions can be found. In particular, it is the case of models similar to the one herein presented but considering other objective functions [Oliveira and Antunes, 2000; Antunes et al., 2002].

Figure 3 and the further ones are actual copies of the screens presented to the user.

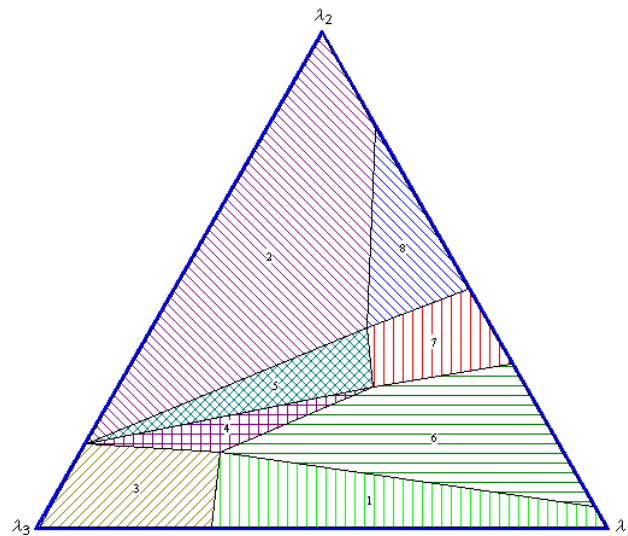


Figure 3 – Indifference regions corresponding to the initial basic efficient solutions

Table 1 – Initial basic efficient solution

Solution	Energy imports	Self-production of electricity	CO ₂ emissions	Area (%)
1	12 271 100	179 513	1 591 600	12.8149
2	87 769 700	22 622 700	30 606 300	35.5831
3	12 727 600	164 322	1 246 240	8.7248
4	13 222 500	1 334 770	2 776 380	3.6413
5	14 844 100	2 187 230	3 890 810	6.2979
6	12 395 100	986 354	2 799 170	19.4846
7	14 592 700	2 182 660	4 589 630	6.3779
8	85 810 100	22 499 000	34 996 400	7.0706

Let us take into account the effect of the uncertainty associated with the objective function coefficients modeled as triangular fuzzy numbers on the behavior of efficient solutions. Let us suppose that the DM considers solution 5 (previously computed with maximum grade of membership) as a good compromise solution and he/she is interested in studying its stability regarding changes of the grade of membership.

The DM can dynamically visualize the changes (in size and shape) of the indifference regions corresponding to the initial computed basic efficient solutions by changing the grade of membership γ (objective functions). New solutions corresponding to regions of Λ not yet filled that the DM is interested in studying can also be computed.

For instance, with $\gamma=0.0500_R$ efficient solutions 1, 6, 7 and 8 previously computed become dominated (their corresponding indifference regions disappear) and it is possible to calculate 6 new basic efficient solutions (L to Q) as displayed in figure 4(a) (table 2). Notice that the areas of the indifference regions corresponding to the efficient solutions computed in crisp environment have changed.

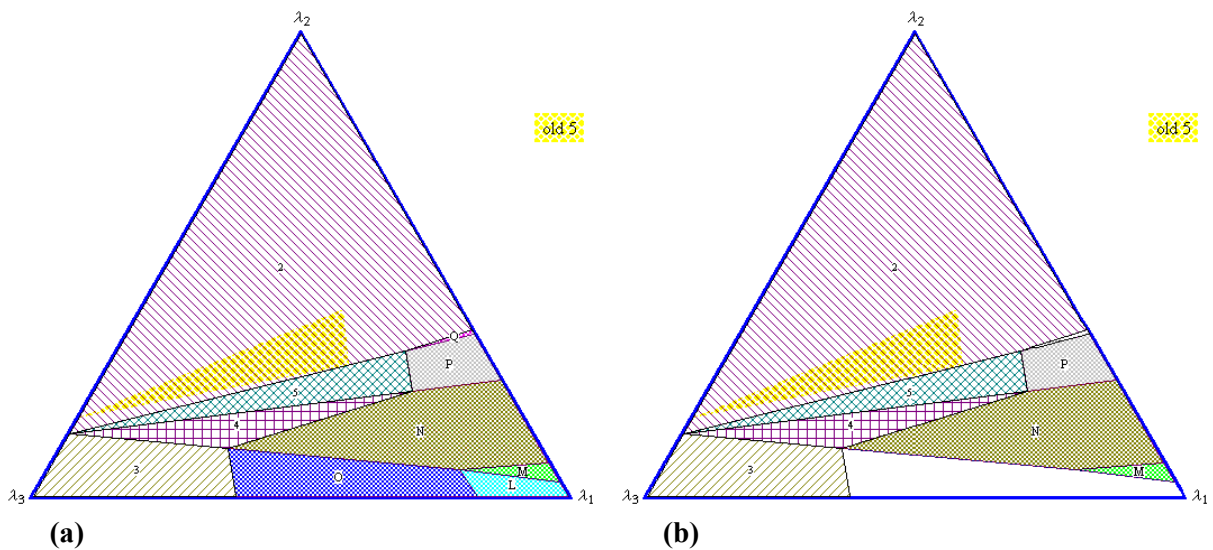


Figure 4 – Fuzzy analysis of the objective function coefficients using indifference regions ($\gamma=0.0500_R$)

If the DM is not interested in some solutions they can be disregarded further on. Once more the DM may conclude, namely by analyzing the objective function values of the relevant solutions, that certain regions of Λ are not worthwhile to search.

For example, in figure 4(b) solutions L, O and Q are eliminated because the DM may consider that the last one has a high value for energy imports and the other ones have small values with respect to self-production of electricity.

Table 2 – Fuzzy analysis of the objective function coefficients ($y=0.0500_R$)

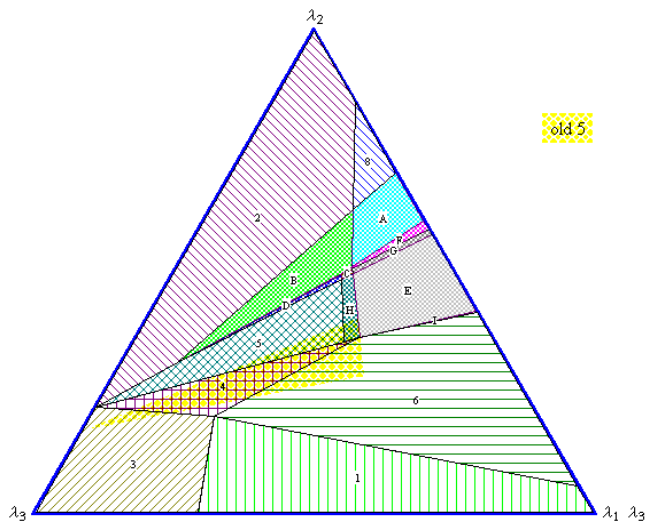
Solution ($y = 0.0500_R$)		Energy imports	Self-production of electricity	CO ₂ emissions	Area (%)	Number of iterations
2	Crisp	87 769 700	22 622 700	30 606 300	35.5831	-
	Fuzzy	59 014 100	25 846 400	27 751 500	56.0235	-
3	Crisp	12 727 600	164 322	1 246 240	8.7248	-
	Fuzzy	10 477 700	187 738	1 148 910	8.1414	-
4	Crisp	13 222 500	1 334 770	2 776 380	3.6413	-
	Fuzzy	10 668 100	1 524 980	2 496 820	4.8045	-
5	Crisp	14 844 100	2 187 230	3 890 810	6.2979	-
	Fuzzy	11 888 100	2 498 910	3 478 520	5.6871	-
L	Crisp	12 272 100	179 509	1 592 770	Non-efficient	
	Fuzzy	10 438 400	205 203	1 476 560	1.4773	11
M	Crisp	12 396 200	985 552	2 799 340	Non-efficient	
	Fuzzy	10 550 100	1 125 990	2 549 910	0.7168	8
N	Crisp	13 223 400	1 333 860	2 776 560	Non-efficient	
	Fuzzy	10 666 600	1 523 940	2 497 050	13.0926	8
O	Crisp	12 728 400	164 196	1 247 130	Non-efficient	
	Fuzzy	10 476 900	187 822	1 149 740	6.9128	7
P	Crisp	14 846 700	2 187 210	3 892 150	Non-efficient	
	Fuzzy	11 887 900	2 498 890	3 479 770	2.9264	2
Q	Crisp	87 785 700	22 620 800	30 805 200	Non-efficient	
	Fuzzy	59 009 500	25 844 300	27 750 600	0.1531	2

If the grade of membership y is changed from 0.0_L to 0.0_R the results presented in table 3 and in figure 5 ((a) to (r)) are obtained. Even though the core idea behind our approach is not to perform an exhaustive study of all basic efficient solutions it has been done for the sake of illustration and because it is not computationally heavy.

In Figs. 5(a) and 5(r) the decompositions of the weight diagram for $y=0.0000_L$ and $y=0.0000_R$ are displayed.

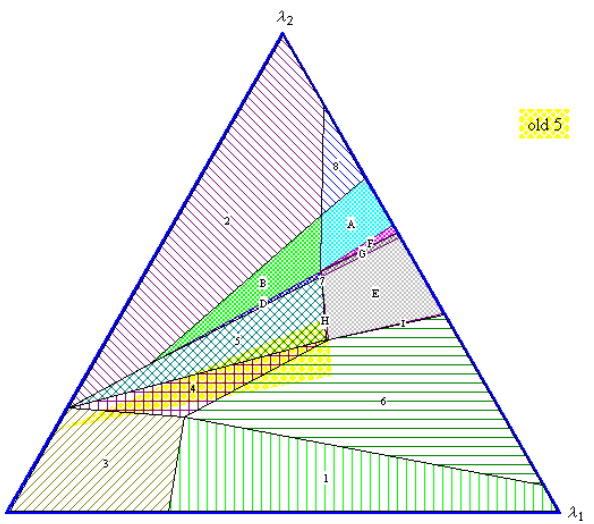
With $y=0.0000_L$, the efficient solution 7 previously computed becomes dominated and the new basic efficient solutions A to I can be reached.

For $y=0.0000_R$ the efficient solutions 1, 6, 7 and 8 previously computed become dominated and the new basic efficient solutions L to P can be reached. The solutions that can be reached with $y=0.0000_R$ and $y=0.0500_R$ are the same. Nevertheless, the areas of the indifference regions corresponding to the efficient solutions 3, 4, 5, L and M become smaller than the ones in figure 4(a); the areas corresponding to the efficient solutions 2, N, O, P and Q become greater than the ones in figure 4(a).



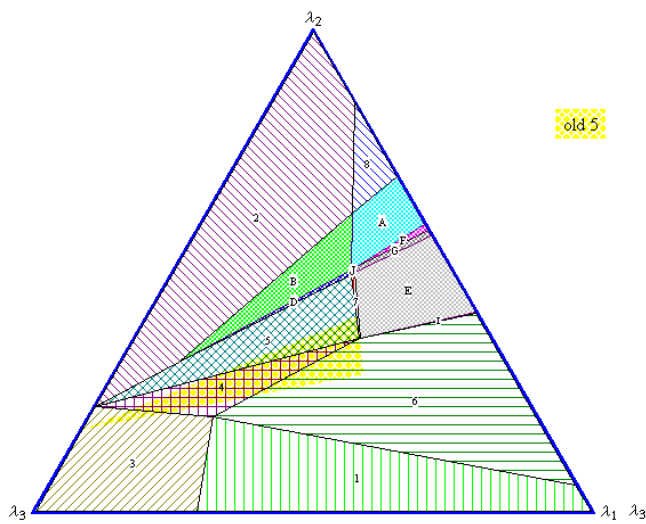
(a) ($y=0.0000L$)

old 5



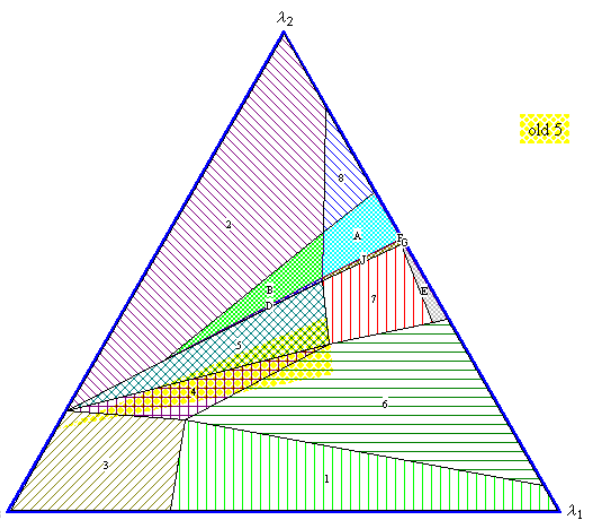
(b) ($y=0.0300L$)

old 5



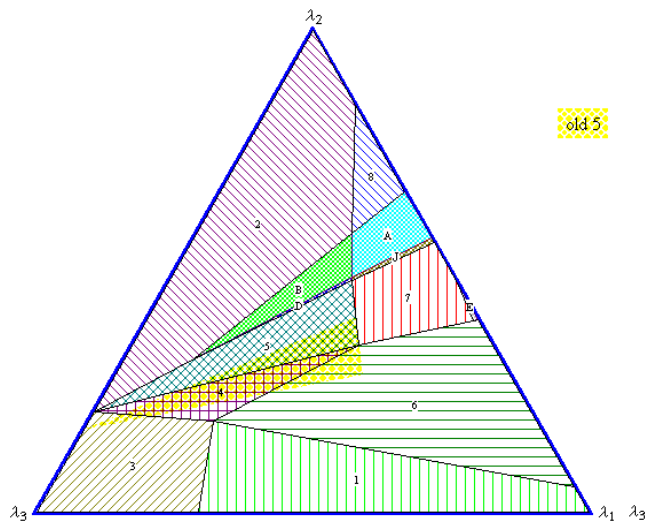
(c) ($y=0.0367L$)

old 5



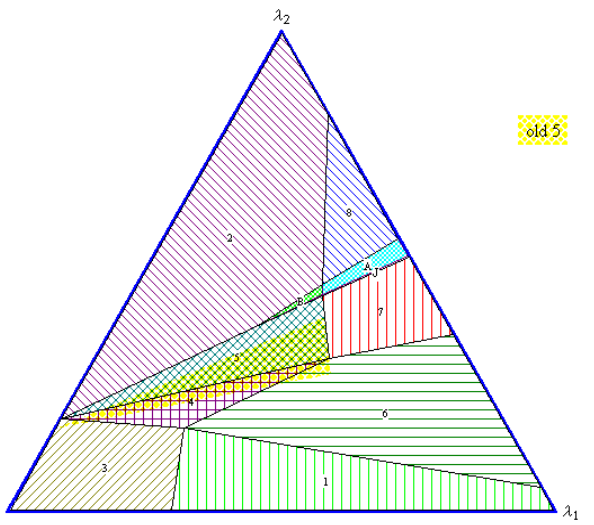
(d) ($y=0.1733L$)

old 5



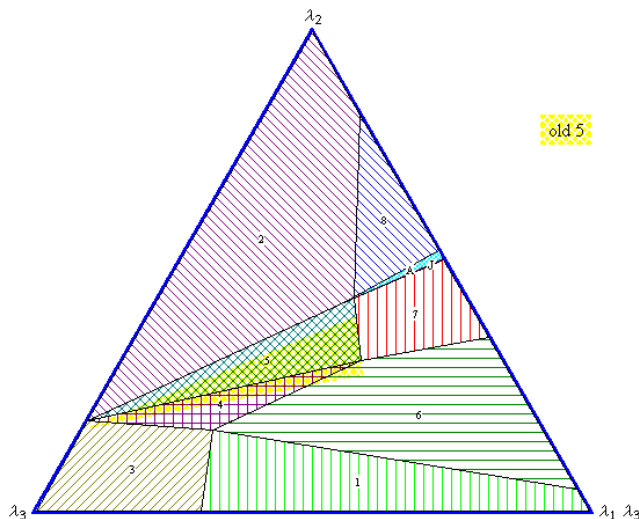
(e) ($y=0.1833L$)

old 5

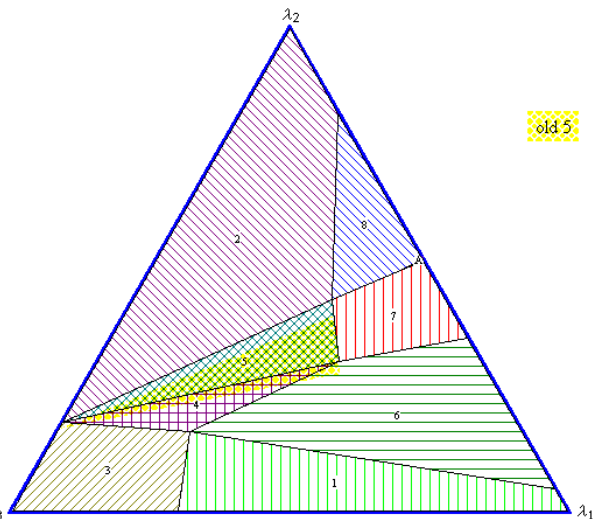


(f) ($y=0.5133L$)

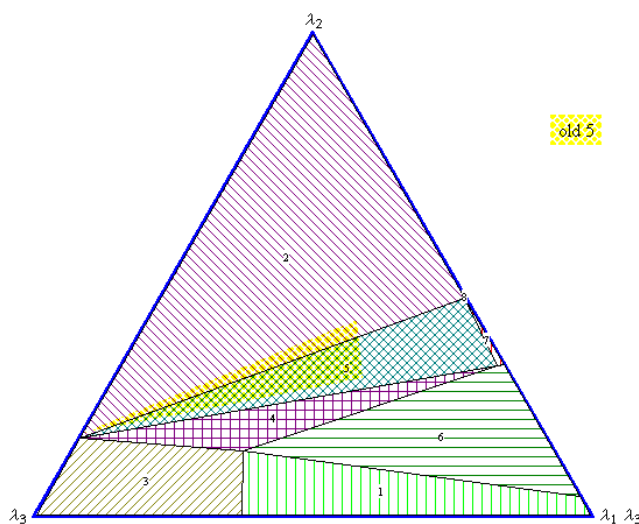
old 5



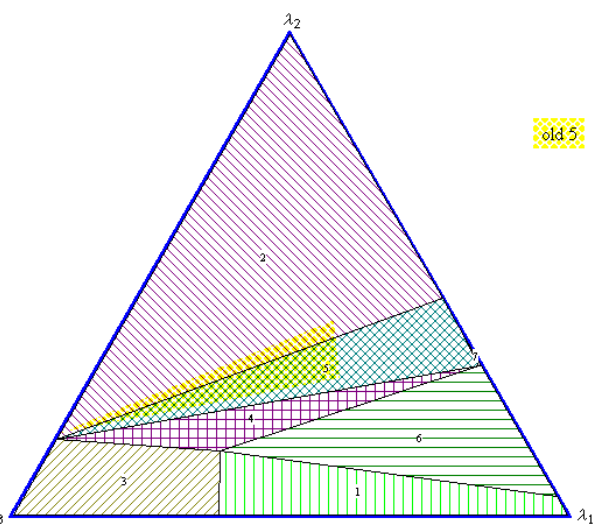
(g) ($y=0.5867_L$)



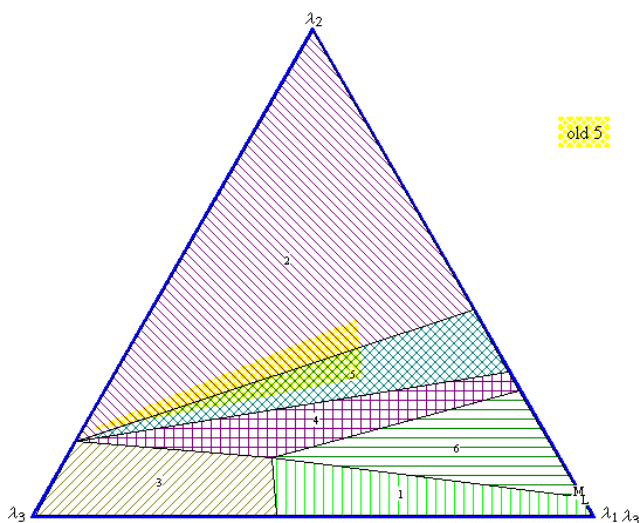
(h) ($y=0.6467_L$)



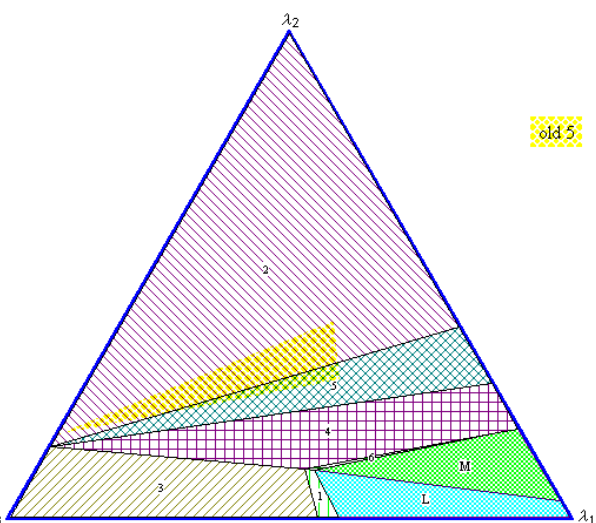
(i) ($y=0.7233_R$)



(j) ($y=0.7167_R$)



(k) ($y=0.5400_R$)



(l) ($y=0.3200_R$)

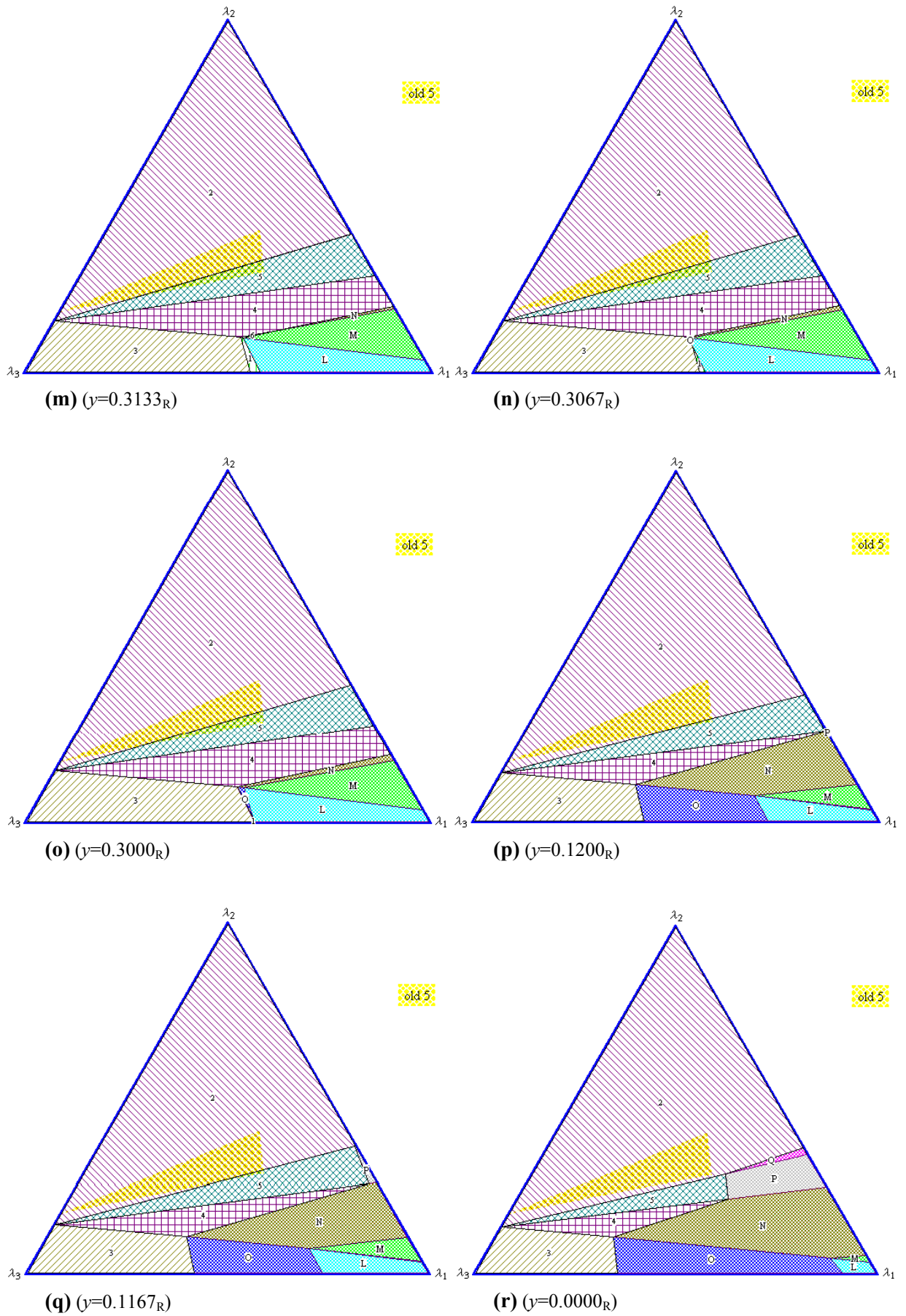


Figure 5 – Fuzzy analysis of the objective function coefficients using indifference regions

Table 3 – Fuzzy analysis of the objective function coefficients

$y \in$	Initial Efficient Solutions	New Efficient Solutions
(i) [0.0000 _L ; 0.0300 _L]	1, 2, 3, 4, 5, 6, 8	A, B, C, D, E, F, G, H, I
(ii) [0.0300 _L ; 0.0367 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, B, D, E, F, G, H, I
(iii) [0.0367 _L ; 0.1733 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, B, D, E, F, G, I, J
(iv) [0.1733 _L ; 0.1767 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, B, D, E, F, G, J
(v) [0.1767 _L ; 0.1867 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, B, D, E, J
(vi) [0.1867 _L ; 0.5133 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, B, D, J
(vii) [0.5133 _L ; 0.5867 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, B, J
(viii) [0.5867 _L ; 0.6100 _L]	1, 2, 3, 4, 5, 6, 7, 8	A, J
(ix) [0.6100 _L ; 0.6500 _L]	1, 2, 3, 4, 5, 6, 7, 8	A
(x) [0.6500 _L ; 0.7200 _R]	1, 2, 3, 4, 5, 6, 7, 8	
(xi) [0.7200 _R ; 0.7133 _R]	1, 2, 3, 4, 5, 6, 7	
(xii) [0.7133 _R ; 0.5400 _R]	1, 2, 3, 4, 5, 6	
(xiii) [0.5400 _R ; 0.3167 _R]	1, 2, 3, 4, 5, 6	L, M
(xiv) [0.3167 _R ; 0.3100 _R]	1, 2, 3, 4, 5, 6	L, M, N
(xv) [0.3100 _R ; 0.3067 _R]	1, 2, 3, 4, 5	L, M, N
(xvi) [0.3067 _R ; 0.2967 _R]	1, 2, 3, 4, 5	L, M, N, O
(xvii) [0.2967 _R ; 0.1200 _R]	2, 3, 4, 5	L, M, N, O
(xviii) [0.1200 _R ; 0.1133 _R]	2, 3, 4, 5	L, M, N, O, P
(xix) [0.1133 _R ; 0.0000 _R]	2, 3, 4, 5	L, M, N, O, P, Q

Having in mind the definition of α -level set [Zadeh, 1965; Zimmermann, 1987, 1992] it can also be envisaged a different way of analyzing the results presented in table 3. Often the DM may be interested in knowing the computed basic efficient solutions which are obtained if the grade of membership y is at least α . For instance, considering a grade of membership above 0.4 the solutions A, B, D, J, L, M and 1 to 8 can be reached (ranges (vi) to (xiii) on table 3). For a grade of membership above 0.65 only the crisp basic efficient solutions 1 to 8 can be obtained (ranges (x) to (xii) on table 3).

If the DM wants to proceed the fuzzy analysis of the objective function coefficients he/she could select another crisp basic efficient solutions (initially computed with maximum grade of membership) as the “starting” one and/or change some of the triangular membership functions associated with those parameters (by modifying the corresponding c_L and c_R values) and perform a similar study.

Let us suppose that the DM also wants to study the behavior of initial basic efficient solutions if some of the RHS coefficients are triangular fuzzy numbers. He/she can dynamically visualize the changes in size of the indifference regions, by changing the grade of membership t .

For instance, with $t=0.3367_R$ (figure 6 and table 4) the efficient solutions 1, 3, 5, 6 and 7 become infeasible and it is possible to calculate 8 new basic efficient solutions (A, D, E, F, G, H, I and J).

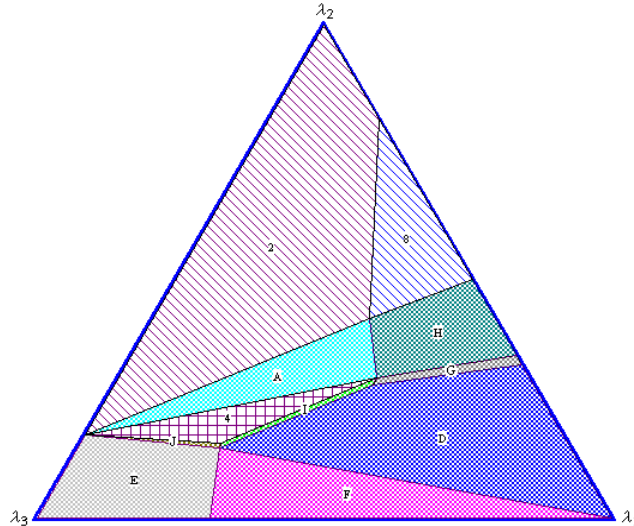


Figure 6 – Fuzzy analysis of RHS coefficients using indifference regions ($t=0.3367_R$)

Table 4 – Fuzzy analysis of the RHS coefficients ($t=0.3367_R$)

Basis ($t = 0.3367_R$)	Energy imports	Self-production of electricity	CO ₂ emissions	Area (%)	Starting basis	Number of iterations
2	118 603 000	31 654 500	42 445 400	35.5831	-	-
4	13 138 500	1 668 290	3 243 940	3.6413	-	-
8	115 908 000	31 484 400	48 484 100	7.0706	-	-
A	14 906 300	2 597 670	4 458 920	6.2980	5	1
D	12 252 700	1 274 350	3 269 700	21.1289	1 or 6	5 or 3
E	12 728 800	166 182	1 280 220	8.4277	3	2
F	12 252 700	181 764	1 634 470	9.9112	1	5
G	12 798 400	1 598 550	3 754 920	0.8651	6	4
H	14 624 400	2 592 540	5 242 590	6.3779	7	2
I	13 132 000	1 665 570	3 244 120	0.4887	1 or 6	5 or 4
J	13 137 500	1 665 990	3 240 930	0.2028	3	6

The grade of membership t can be changed from 0.0_L to 0.0_R and the results presented in table 5 are obtained. Considering those membership functions no changes happen for $t \in [0.0_L; 1.0[$.

As for the case of the objective function coefficients, the DM may also be interested in knowing which of the computed efficient bases are obtained if the grade of membership t is at least α . For instance, considering a grade of membership above 0.6 the bases associated with solutions E, F, J, L and 1 to 8 can be reached (ranges (i) to (iii) on table 5). For a grade of

membership above 0.9333 only the bases associated with the crisp efficient solutions 1 to 8 are obtained (range (i) on table 5).

Table 5 – Fuzzy analysis of the RHS coefficients

$t \in$	Initial Efficient Basis	New Efficient Basis
(i) [0.0000 _L ; 0.9300 _R]	1, 2, 3, 4, 5, 6, 7, 8	
(ii) [0.9300 _R ; 0.9100 _R]	1, 2, 4, 5, 6, 7, 8	E, J, M
(iii) [0.9100 _R ; 0.5333 _R]	2, 4, 5, 6, 7, 8	E, F, J, L
(iv) [0.5333 _R ; 0.4933 _R]	2, 4, 5, 6, 8	E, F, H, J, L
(v) [0.4933 _R ; 0.4867 _R]	2, 4, 5, 8	D, E, F, G, H, I, J
(vi) [0.4867 _R ; 0.3333 _R]	2, 4, 8	A, D, E, F, G, H, I, J
(vi) [0.3333 _R ; 0.0000 _R]	2, 8	A, B, C, D, E, F, G, H

Similarly, if the DM wants to proceed the fuzzy analysis of the RHS coefficients he/she could change some of the triangular membership functions associated with these parameters (by modifying the corresponding c_L and c_R values).

An analogous study can be performed if, for example, the DM is interested in considering new fuels or other forms of electricity production, by means of the introduction of a new decision variable in the initial problem, to study how it would affect the computed basic efficient solutions.

6. Conclusions

Visual interactive techniques based on the analysis of indifference regions related to the weights have been developed and computationally implemented as the core of a DSS to deal with uncertainty in MOLP models by using fuzzy set theory. It must be noticed that weights are used as an operational and information exchange means and not as a way for eliciting the DM's preferences.

The DM has the possibility of interactively changing the membership functions associated with constraints and objective functions as well as regarding the introduction of a new decision variable into the model. Special attention has been paid to the computational simplicity and graphical interactivity, in order to visualize dynamically the behavior of the efficient solutions according to changes in the initial model coefficients, by displaying the indifference regions. The grade of membership functions can also be interactively changed.

These techniques have been illustrated in a multiple objective input-output model, supplied with actual data for Portugal, aimed at studying the interactions between the energy sector and the economy on a national level.

Acknowledgements

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