

# An Evolutionary Algorithm for Reactive Power Compensation in Radial Distribution Networks

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## **Abstract**

In this paper, the problem of locating and sizing capacitors for reactive power compensation in electric radial distribution networks is modeled as a multi-objective programming problem. An evolutionary approach consisting of an elitist genetic algorithm with secondary population is used to characterize the Pareto Optimal (non-dominated) frontier and provide decision-makers and planning engineers insightful information about the tradeoffs to be made. Two objective functions of technical and economical nature are explicitly considered in this model: minimization of system losses and minimization of capacitor installation costs. Constraints refer to quality of service (related with the node voltage profile), power flow (associated with physical laws in electric networks), and technical requirements.

## **Keywords**

Genetic algorithms, Multi-objective models, Power factor compensation, capacitor location, Distribution networks

## **1. Introduction**

Reactive power compensation is an important issue in electric power systems, involving operational, economical and quality of service aspects. Consumer loads (residential, industrial, service sector, etc.) impose active and reactive power demand, depending on their characteristics. Active power is converted into “useful” energy, such as light or heat. Reactive power must be compensated to guarantee an efficient delivery of active power to loads, thus releasing system capacity, reducing system losses, and improving system power factor and bus

voltage profile. The achievement of these aims depends on the sizing and allocation of shunt capacitors (sources of reactive power).

Since operational, economical and quality of service aspects are at stake, these multiple, conflicting, and incommensurate evaluation aspects must be explicitly addressed by the mathematical model. Therefore, a multi-objective model has been developed incorporating the objectives of distinct nature that are weighed by decision-makers / planning engineers to select acceptable solutions having in mind their practical implementation. Multi-objective models enable to grasp the conflicting nature of the objectives and the tradeoffs to be made in order to identify satisfactory compromise solutions by providing a basis to rationalize the comparison between non-dominated solutions. A non-dominated solution is a feasible solution for which no improvement in all objective functions is simultaneously possible; that is, an improvement in an objective function can only be achieved by degrading, at least, another objective function value. Besides contributing to make the model more realistic vis-a-vis actual problems, a multi-objective approach intrinsically possesses a value-added role in the modeling process and in model analysis, supporting reflection and creativity of decision makers in face of a larger universe of potential solutions.

This paper deals with the problem of optimal capacitor placement in radial distribution systems considering two objective functions: minimizing capacitor installation cost and minimizing system losses. Constraints are related with requirements of acceptable node voltage profile (quality of service), power flow (physical laws in electric networks), and impossibility of capacitor location at certain nodes (technical restrictions). The aim is to identify non-dominated solutions to the multi-objective model, involving the determination of the locations where to install the capacitor banks, and the size of each capacitor bank to be installed.

A large range of models and methodological approaches has been proposed in the scientific literature devoted to the reactive power compensation problem. Broadly in chronological order, approaches to tackle this problem have ranged from analytical methods to mathematical programming algorithms and, more recently, to heuristics and meta-heuristics. In particular, special attention has been paid to genetic algorithms and evolutionary approaches whose characteristics are well-suited for solution representation in networks: Iba (1994), Lee, Bai, and Park (1995), Kim and You (1999), Levitin et al. (2000), Delfanti et al. (2000), Baran et al. (2001).

It must be noticed that, due to the problem complexity, some unrealistic assumptions are often adopted to make models manageable (uniform load distribution, uniform feeder size, constant loads, etc.) or a heavy computational burden is required to deal with actual real-world networks. In this work, an evolutionary approach has been developed to compute non-dominated solutions to the problem of reactive power compensation in electric radial distribution networks. The motivation for the use of this methodological approach is twofold. The ability to work in each

generation with a population of potential solutions makes evolutionary approaches well suited for multi-objective optimization problems in which a set of non-dominated solutions must be identified rather than a single optimal solution. On the other hand, this combinatorial problem is very complex to be tackled by mathematical programming tools since, besides its multi-objective nature, it is non-linear with continuous, integer and binary variables.

In the realm of optimization, evolutionary approaches present a behavior similar to natural evolution, in which a population of potential solutions evolves over successive generations with survival of the fittest. An elitist strategy has been implemented aimed at increasing the performance, both accelerating the convergence speed towards the non-dominated frontier and ensuring the solutions attained are indeed non-dominated ones and are well-spread over the frontier. This is an important issue in real-world problems since it is necessary to provide the DM with well-distributed and diverse solutions for a well-informed final decision to be made upon. The suitability of genetic algorithms and evolutionary approaches to deal with multi-objective problems has been widely recognized (Deb, 2001; Coello, Veldhuizen and Lamont, 2002; Osyczka, 2002).

In this section the interest and motivation of the study have been provided. The mathematical model for the problem of reactive power compensation with two objective functions is presented in section 2. The evolutionary approach developed to tackle the problem is described in section 3. In section 4, illustrative results are presented which have been obtained with the evolutionary approach applied to the model instantiated with real-world data of a Portuguese distribution network in a sparsely populated area (which imposes heavier operating conditions to the network). Finally, some conclusions are drawn in section 5.

## **2. Mathematical model**

The problem of reactive power compensation involves to determine the number, location, and sizes for shunt capacitors (sources of reactive power) to be installed, in this case in a distribution feeder, to reduce losses and improve the voltage profile. Therefore, a balance is needed between costs (associated with installing new capacitors) and technical and quality of service evaluation aspects. The model described in this section explicitly assumes the multi-objective nature of the problem by considering two objective functions: minimizing (resistive) losses and minimizing costs of new sources of reactive power. Quality of service requirements of an acceptable voltage profile in load buses are included as constraints resulting from legislation.

The terminology is as follows:

- SE      Substation.
- $m$       Lateral bus index.
- $n$       Lateral feeder index.

$k$	Main feeder bus index.
$L$	Index that indicates the variable is of load type.
$C$	Index that indicates the variable is of compensation type.
$F$	Index that indicates the variable is of a standard capacitor type.
$c_j$	Cost of capacitor $Q_{Fj}$ .
$P_{L_{nm}}^k$	Load (real power) at bus $m$ (lateral $n$ , derived from bus $k$ at the main feeder).
$Q_{L_{nm}}^k$	Load (reactive power) at bus $m$ (lateral $n$ , derived from bus $k$ at the main feeder).
$P_{nm}^k$	Real power which flows at feeder $m$ , i.e. that goes from bus $m$ to bus $m+1$ (lateral $n$ , derived from bus $k$ at the main feeder).
$Q_{nm}^k$	Reactive power which flows at feeder $m$ , i.e. that goes from bus $m$ to bus $m+1$ (lateral $n$ , derived from bus $k$ at the main feeder).
$Q_{C_{nm}}^k$	Reactive power injection from capacitor $Q_C$ at node $m$ (lateral $n$ , derived from bus $k$ at the main feeder).
$B_{nm}^k$	Bus $m$ of lateral $n$ (lateral $n$ derived from bus $k$ at the main feeder).
$V_{nm}^k$	Bus voltage's magnitude at node $m$ (lateral $n$ , derived from bus $k$ at the main feeder).
$T_{nm}^k$	Branch $m$ of lateral $n$ (lateral $n$ , derived from bus $k$ ); this branch connects bus $B_{nm}^k$ to bus $B_{n(m+1)}^k$ .
$Q_{Fj}$	Capacity of capacitor $j$ .
$M_n^k$	Lateral $n$ number of buses (lateral $n$ , derived from bus $k$ at the main feeder).
$N^k$	Number of laterals which derives from bus $k$ at the main feeder.
$K$	Number of buses of the main feeder.
$Y$	Number of capacitors considered for installation.
$r_{nm}^k$	Resistance of branch $m$ (lateral $n$ , derived from bus $k$ at the main feeder).
$x_{nm}^k$	Inductance of branch $m$ (lateral $n$ , derived from bus $k$ at the main feeder).

A set of recursive equations (1–5) describe the physical requirements associated with power flow through each branch in a radial distribution system (Fig. 1).

The load flow calculation imposes a significant computational burden in the assessment of the merit of each solution. The procedure used for this purpose (recursive equations 1-5) is adapted to radial networks, such as the ones used in the Portuguese electrical distribution system (see also Baran and Wu (1989a, 1989b), Das et al. (1994) and Pires et al. (2003)).

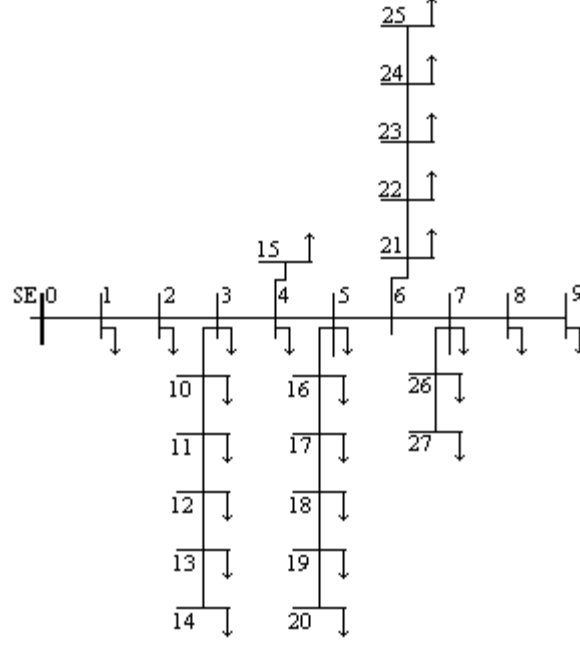


Fig. 1 - Example of a radial distribution system.

$$P_{n(i+1)}^k = P_{ni}^k - r_{ni}^k \frac{P_{ni}^{k2} + Q_{ni}^{k2}}{V_{ni}^{k2}} - P_{Ln(i+1)}^k, \quad \forall n \neq 0 \text{ and } k \neq 0 \quad (1)$$

$$P_{0(i+1)}^0 = P_{0i}^0 - r_{0i}^0 \frac{P_{0i}^{02} + Q_{0i}^{02}}{V_{0i}^{02}} - P_{L0(i+1)}^0 - \sum_{n=1}^{N^{i+1}} P_{n0}^{(i+1)} \quad (2)$$

$$Q_{n(i+1)}^k = Q_{ni}^k - x_{ni}^k \frac{P_{ni}^{k2} + Q_{ni}^{k2}}{V_{ni}^{k2}} - Q_{Ln(i+1)}^k + Q_{Cn(i+1)}^k, \quad \forall n \neq 0 \text{ and } k \neq 0 \quad (3)$$

$$Q_{0(i+1)}^0 = Q_{0i}^0 - x_{0i}^0 \frac{P_{0i}^{02} + Q_{0i}^{02}}{|V_{0i}^0|^2} - Q_{L0(i+1)}^0 - \sum_{n=1}^{N^{i+1}} Q_{n0}^{i+1} + Q_{C0(i+1)}^0 \quad (4)$$

$$V_{n(i+1)}^{k2} = V_{ni}^{k2} - 2 \times \left( r_{ni}^k P_{ni}^k + x_{ni}^k Q_{ni}^k \right) + \left( r_{ni}^{k2} + x_{ni}^{k2} \right) \times \left( \frac{P_{ni}^{k2} + Q_{ni}^{k2}}{V_{ni}^{k2}} \right) \quad (5)$$

The main feeder has index  $n = 0$ , i.e. it is considered the 0<sup>th</sup> lateral, and  $k = 0$ , i.e. it begins at the substation (SE). Besides power flow equations there are other conditions to be satisfied for each lateral (including the feeder). From the last bus of each branch, there is no power (real or reactive) flowing to other branches:

$$P_{nm}^k = Q_{nm}^k = 0 \quad (6)$$

Two objective functions are considered, dealing with the minimization of the network resistive loss (7) and the minimization of the cost associated with the capacitors (8).

$$\text{Min} \sum_{k=0}^K \sum_{n=0}^{N^k} \sum_{m=0}^{M_n^k} r_{nm}^k \frac{P_{nm}^k{}^2 + Q_{nm}^k{}^2}{V_{nm}^k{}^2} \quad (7)$$

$$\text{Min} \sum_{k=0}^K \sum_{n=0}^{N^k} \sum_{m=0}^{M_n^k} \sum_{j=1}^Y j a_{nm}^k c_j \quad (8)$$

New capacitors are characterized by their capacity and the installation cost. Standard units, generally used in distribution systems, are considered.  $c_j$  is the capacitor's ( $Q_{Fj}$ ) cost ( $j = 1, \dots, Y$ ) and:

$$j a_{nm}^k = \begin{cases} 1 & \text{if the new capacitor } Q_{Fj} \text{ is installed in } B_{nm}^k \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

$$Q_{C_{nm}}^k = b_{nm}^k \sum_{j=1}^Y j a_{nm}^k Q_{Fj} \quad \forall m, n, k \quad (10)$$

Constraints (1-6) - load flow equations - are of physical nature.

Constraints (11) impose that, at most, one capacitor can be placed in each node  $B_{nm}^k$ .

$$\sum_{j=1}^Y j a_{nm}^k \leq 1 \quad \forall m, n, k \quad (11)$$

Constraints (12) are related with quality of service, regarding the upper and lower bounds of node voltage magnitude.

$$V_{nm_{\min}}^k \leq V_{nm}^k \leq V_{nm_{\max}}^k \quad \forall m, n, k \quad (12)$$

The multi-objective problem herein formulated is nonlinear and involves both discrete and continuous variables.

### 3. The evolutionary approach

An evolutionary algorithm has been developed aimed at characterizing the Pareto Optimal (non-dominated) front for this problem of determining the size and location of capacitors for compensation of reactive power in electric radial distribution networks. The problem has been modeled as a multi-objective programming problem with two objective functions of economic and technical nature: minimization of system losses and minimization of capacitor installation costs (see section 2).

The ability to work in each generation with a population of potential solutions makes evolutionary algorithms (EAs) well suited for multi-objective optimization problems, namely combinatorial ones, in which a set of non-dominated solutions must be identified rather than a

single optimal solution. The evolution process, in which a population of potential solutions evolves over successive generations, takes place in several phases. Generally, the first one is the evaluation of individuals according to some objectives. A selection phase follows in which some potential solutions are chosen from the population, using a probability that is a function of individual performance. Usually, a higher selection probability is given to the individuals with higher performance. Crossover and mutation operators are then applied to the selected solutions to generate the next generation. The process continues until some stopping condition is reached. These operations carried out in the realm of optimization problems are inspired on natural evolution, allowing for the exploration of the search space and striving for good solutions to the problem under study. The operators (selection, crossover and mutation) and an adequate fitness function are generally sufficient for the evolution process to take place. Usually, the fitness of every individual in the population encompasses the evaluation in the several objectives under analysis. However, other issues may be taken into consideration, such as the sharing of the fitness within their neighborhood or changing the selection pressure by using an appropriate scaling of the fitness.

In the implementation of the genetic algorithm devoted to the reactive power compensation problem, an elitist strategy is used with a secondary population of constant size, consisting of the following main steps:

- the fitness of the individuals composing the main population is computed;
- from the main population (consisting of  $P$  individuals)  $P-K$  individuals are selected by using a tournament technique;
- a new population is formed by the  $P-K$  offspring generated by crossover and mutation;
- $K$  individuals (elite) are randomly selected from the secondary population;
- the evaluation of individuals by a dominance test is carried out, which defines an approximation to the non-dominated frontier;
- the non-dominated solutions are computed and they are processed to update the secondary population using a sharing technique (aimed at favoring a well-spread distribution of the secondary population throughout the objective space), if necessary.

### **Individuals coding**

The population used in the algorithm implementation consists of individuals represented by an array of  $N$  integer values ( $N$  being the number of network nodes where it is possible to install a new capacitor or change the capacity of a capacitor already installed). The index of the array corresponds to a network node and the value therein denotes the type of capacitor to install in that node (0 denotes no capacitor to be installed).

## **Fitness**

The computation of the fitness of each solution involves determining various solution fronts in the following way:

- A niche is defined by a radius *dist* around a solution, where *dist* is the maximum distance between solutions necessary to obtain a well-spread front and is equal to  $\sqrt{2} / P$ ; P is the size of the main population and  $\sqrt{2}$  is the normalized distance between the pseudo-solutions obtained by considering the best and the worst values for each objective function in the main population;
- The first front consists of all non-dominated solutions, a performance level equal to P being assigned to them;
- This performance level of each one of these solutions is divided by a quantity proportional to the number of solutions belonging to a niche defined by a radius *dist* around that solution;
- The solutions in the first front are temporarily ignored and the remaining feasible solutions (the dominated solutions) are processed by applying them a dominance test;
- A fitness value is obtained by subtracting 1 to the minimum fitness value of the solutions in the first front, being then assigned to the second level;
- For each solution in the second front, the fitness value is divided by a quantity which is proportional to the number of solutions belonging to the same niche;
- This process continues until all feasible solutions are assigned a fitness value;
- The same process is repeated for the non-feasible solutions until all non-feasible solutions are assigned a fitness value.

## **Sharing**

The sharing mechanism used in updating the secondary population uses a niche scheme whose radius is a dynamic value. This mechanism is applied after computing all non-dominated solutions candidates to becoming part of the secondary population. These are all the solutions already belonging to the secondary population which are not dominated by any solution in the main population and the non-dominated solutions of the main population which are not dominated by solutions in the secondary population. This mechanism is only applied in case the number of solutions candidates to becoming part of the secondary population (NCandPS) is greater than the size of this population (NPS). The sharing mechanism consists in the following steps:

1. Insert the two extreme solutions (those with the best values for each objective function);
2. Compute the first niche radius (*dist*) as the ratio: normalized distance between extreme solutions / NPS (that is,  $\sqrt{2} / NPS$ );

3. Insert solutions located at a distance greater than *dist* from the ones already belonging to the secondary population;
4. Update the value of niche radius, *dist*, by reducing it by 10%;
5. If the secondary population is not complete then return to step 3.

### **Initial population**

The strategy used to determine the initial population consists of randomly generating non-dominated feasible solutions only. This strategy produced better results when compared with strategies that randomly generate solutions of any type (feasible or non-feasible) or feasible solutions (dominated or non-dominated) only.

### **Crossover**

Two-point crossover has been used, because it produced better results than one-point and uniform crossover. The two crossover points are selected randomly with the restriction that they have to be apart for at least 1/4 of chromosome size.

### **Mutation**

The capacitor type is indexed by a variable ranging from 0 to 3 (where 0 means no capacitor). The mutation consists in modifying (with a probability *pm*) the current index value to one of other possible values.

### **Algorithm**

The algorithm associated with this approach consists in the following steps:

1. Initialization: randomly generate the initial population with P non-dominated solutions;
2. Evaluation: compute the fitness value of each individual in the initial population (it is not necessary to apply the dominance test, because the initial population contains only feasible non-dominated solutions);
3. Determine the initial secondary population of maximum size NPS from the initial population: if  $NPS \geq P$  then copy all non-dominated solutions from the initial population to the secondary population; else apply the sharing mechanism to the initial population to select NPS solutions;
4. Current population  $\leftarrow$  initial population;

#### Repeat

5. Build up the population associated with the next generation (main population) of size P:
  - (a) Introduce directly K individuals from the secondary population (elite) into the main population;

- (b) Select 2 individuals of the current population by tournament (in each tournament, 10% of the individuals in the current population are used to produce the selected one);
  - (c) Apply genetic operators crossover and mutation to the 2 individuals selected;
  - (d) Insert the new individuals into the main population;
  - (e) If the main population does not yet contain P individuals then return to step (b);
6. Evaluation: apply the dominance test and compute the fitness value of each individual in the main population;
  7. Determine the NCandPS solutions candidates to becoming part of the secondary population;
  8. Update the secondary population: if  $NPS \geq NCandPS$  then copy all candidate non-dominated solutions to the secondary population else apply the sharing mechanism to all solutions found in step 7 to select NPS solutions;
  9. Current population  $\leftarrow$  main population;
- Until the pre-specified number of iterations is attained.

#### **4. Illustrative results of a real-world case study**

The methodology described in section 3 to characterize the Pareto Optimal front and provide decision support in the multi-objective model presented in section 2, has been applied to an actual Portuguese radial distribution system with 94 nodes and 24 lateral buses. Three types of capacitors are considered for possible installation.

Several runs have been done with different sets of parameters. The best results were obtained with:  $P = 30$ ,  $NPS = 40$ ,  $K = 4$ , 7500 generations,  $pm = 0.1$ , and  $pc=1$ .

Figure 2 displays the Pareto front, in the objective function space, regarding the final secondary population (that is, the output of the process to be presented to DMs/planning engineers). Figure 3 enables a comparison to be made between the final secondary population and the initial population. Each solution is associated with a compensation scheme defined by: number and size of the capacitors, the network nodes where they are installed, and the corresponding cost and resistive losses.

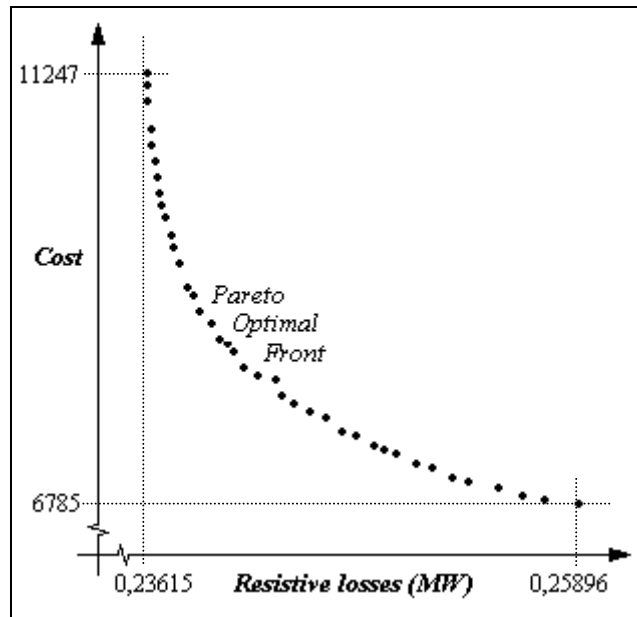


Fig. 2 – Final secondary population with 40 solutions for the 94 node network.

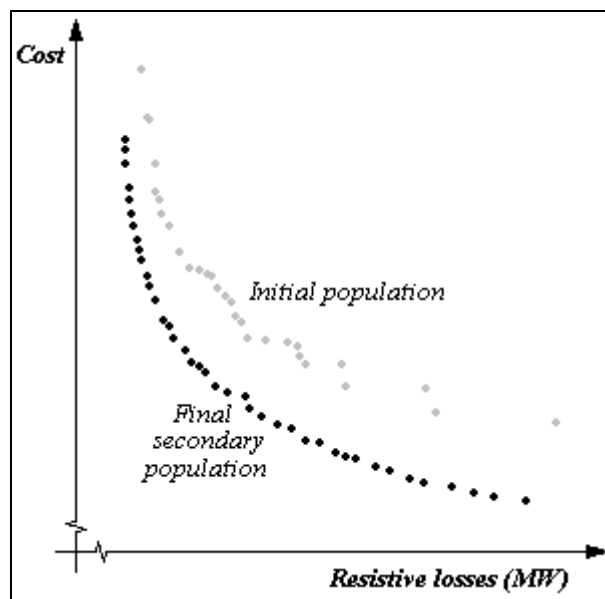


Fig. 3 – Final secondary population versus initial population.

In order to fine tune the algorithm to the characteristics of the case study, some variants of the techniques used were also implemented and tested. In particular, experiments have been made using different strategies to build the initial population and distinct types of crossover.

### Crossover types

For this specific problem, three crossover types were analyzed: (A) one-point, (B) two-point, and (C) uniform with a randomly generated mask. The experiments carried out indicate that the two-point crossover produces better results than the other approaches (fig. 4).

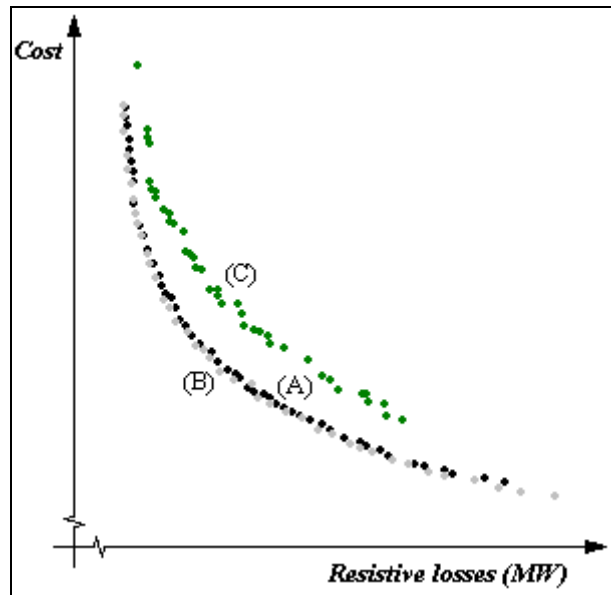


Fig. 4 – Final secondary populations associated with the best results for each crossover type.

### Initial population

Three strategies to determine the initial population were analyzed which are related to the type of solutions that belong to the population:

- (A) feasible and non-feasible solutions;
- (B) feasible (dominated and non-dominated) solutions only;
- (C) feasible non-dominated solutions only.

The initial populations consisting of feasible non-dominated solutions only (type C) produced the best results. The Pareto Optimal front determined by the algorithm using this type of initial population dominates the fronts determined by using initial populations of types (A) and (B), as shown in figures 5 and 6.

Figure 5 shows the three Pareto Optimal fronts associated with the three types of initial population (for the same set of parameters:  $P = 30$ ,  $NPS = 40$ ,  $K = 4$ , number of generations = 7500,  $pm = 0.1$ , and  $pc=1$ ).

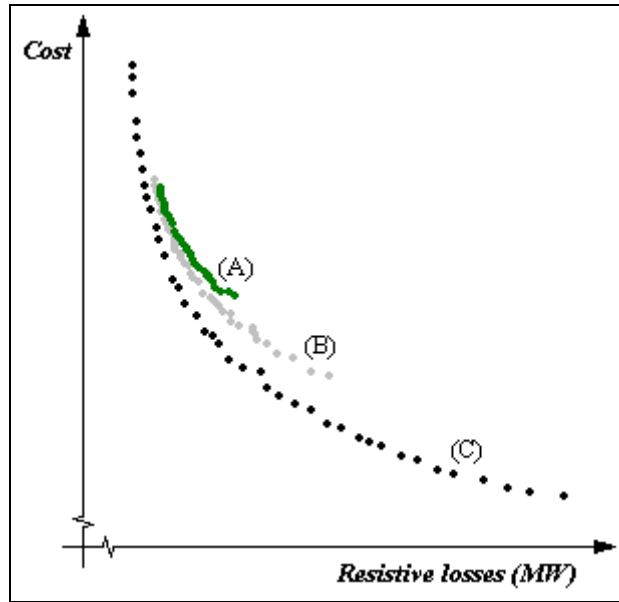


Fig. 5 – Final secondary populations (using the same set of parameters).

Figure 6 displays the three Pareto Optimal fronts associated with the best results for each type of initial population (for all sets of parameters tested):

- type A:  $P = 100$ ,  $NPS = 40$ ,  $K = 6$ , number of generations = 10000,  $pm = 0.1$ , and  $pc=1$ ;
- type B:  $P = 40$ ,  $NPS = 50$ ,  $K = 4$ , number of generations = 7500,  $pm = 0.1$ , and  $pc=1$ ;
- type C:  $P = 30$ ,  $NPS = 40$ ,  $K = 4$ , number of generations = 7500,  $pm = 0.1$ , and  $pc=1$ .

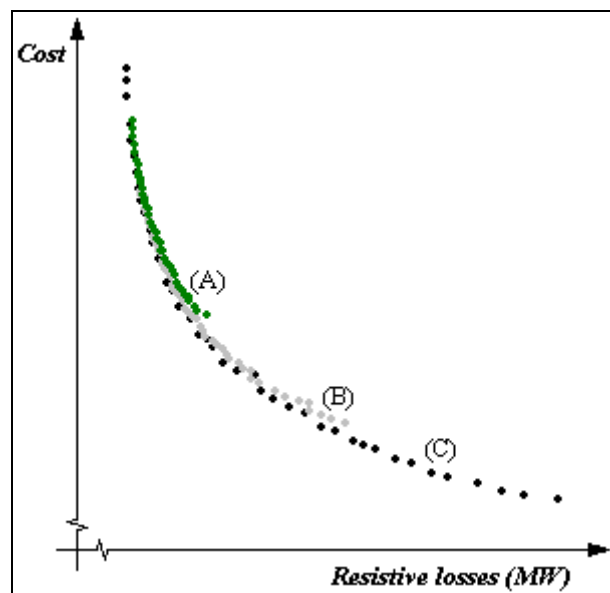


Fig. 6 – Final secondary populations associated with the best results.

For the 94 node distribution network, the run times (Pentium 3, 1 GHz, 512 Ram) associated with each type of initial population are the following (two-point crossover was adopted for all runs, and 12 run averages are presented):

- using the same set of parameters: 51 sec. (A), 57 sec. (B), and 387 sec. (C) (fig. 5);
- for the best results obtained: 267 sec. (A), 100 sec. (B), and 387 sec. (C) (fig. 6).

However, though the run times associated with initial population of type (C) have been the longest, the average run time for this type was 260 sec. (the best run time was 96 sec. and the worst run time was 406 sec.). The run times include the load flow calculation (specially adapted to radial networks in electrical distribution systems such as the ones typically found in the Portuguese network; see section 2)

## **Conclusions**

In this paper, the problem of locating and sizing capacitors for reactive power compensation in electric radial distribution networks has been modeled as a multi-objective programming problem. Two objective functions of technical and economical nature are considered in the model: minimization of system losses and minimization of capacitor installation costs.

An evolutionary approach consisting of an elitist genetic algorithm with secondary population has been presented. This algorithm is mainly aimed at characterizing the Pareto Optimal frontier by computing well-distributed solutions which are representative of distinct compromises to be faced by DMs/planning engineers in the selection of practical plans. Work is currently underway along this direction, namely regarding the development of a decision support system devoted to the selection problem which uses the output of the evolutionary approach as an input. The results of a large set of experiments have been briefly reported regarding the performance of the evolutionary approach, namely concerning its behavior with respect to different ways of building the initial population and distinct crossover types. These results enabled to fine tune the methodological approach for achieving a better performance in real-world studies in the Portuguese distribution network.

## **Acknowledgments**

This work was supported in part by the Foundation for Science and Technology (FCT) under Grants POCTI/ESE/38422/2001 and POSI/SRI/37346/2001.

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