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NSGA-II with Local Search for a Multi-Objective VAR Planning Problem

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Abstract

This paper presents the results of an Elitist Non-dominated Sorting Genetic Algorithm (NSGA II) enhanced with local search for computing solutions to a multi-objective VAR planning model. NSGA II has revealed a good performance in comparison with other multi-objective evolutionary algorithms also tested to tackle this problem. This performance is still improved using a local search scheme within NSGA II, which is specially tailored to the problem characteristics. The use of local search within the NSGA II operational framework in this complex combinatorial problem improves convergence towards the non-dominated front and ensures that the solutions attained are well spread over it. A comparative study is presented between the results obtained using a standard NSGA II based approach and the enhanced NSGA II approach with local search, to provide decision support in the VAR planning problem in radial distribution systems. The model explicitly considers two objective functions concerning economical and operational evaluation aspects.

Keywords: Multi-objective models, Distribution electric networks planning, Multi-objective Genetic Algorithms (MOGA), Elitist Non-dominated Sorting Genetic Algorithm (NSGA II)

1. Introduction

Reactive power compensation (VAR planning) is an important issue in electric power systems, because it is directly related with efficient delivery of active power to loads, releasing electric system capacity, improving voltage bus profile and reducing losses.

In radial distribution networks, the most widely used device for reactive power compensation is the shunt capacitor (source of reactive power). The problem of optimal capacitor placement can be considered as follows: identifying the network nodes to install capacitors, the dimension of each capacitor to be installed, the utilization of existing capacitors and the operation of the capacitors at different load levels. The common objectives to be achieved are: minimizing costs, reducing system losses and improving bus voltage profile.

Since operational, economical and quality of service aspects are at stake, these multiple, conflicting and incommensurate axes of evaluation must be explicitly addressed by mathematical models for decision support, thus leading to multi-objective models. The essential concept in a multi-objective setting is the one of non-dominated (Pareto optimal) solutions, that is feasible solutions for which no improvement in all objective functions is possible simultaneously (in order to improve an objective function it is necessary to accept worsening at least another objective function value). In real-world problems, a high number of non-dominated solutions generally exist. Therefore, it is important to characterize as extensively as possible the Pareto optimal front, namely in order to grasp the trade-offs between the objective functions that are at stake in different regions, which are relevant for decision support purposes.

Multi-objective mathematical programming models generally require the optimization of so-called scalarizing functions. These are surrogate scalar functions which (temporarily) aggregate the multiple objective functions in such a way that an optimal solution to the scalarizing function is a non-dominated solution to the multi-objective problem. Therefore, obtaining a characterization (more or less exhaustive) of the non-dominated region requires optimizing different versions of the scalarizing function, generally by changing some technical or preference information parameters therein included.

Evolutionary algorithms (EAs) have gained a growing importance to tackle multi-objective models (particularly, for hard combinatorial problems) due to their capability of working with a population of individuals (solutions). Since they deal with a population of solutions and the aim is generally the characterization of a Pareto optimal front, EAs endowed with techniques to maintain diversity of solutions present advantages with respect to the use of scalarizing functions as in traditional mathematical programming approaches. A Pareto optimal front can be identified throughout the evolutionary process, which hopefully converges to the true non-dominated front for the problem under study. It must be noticed that, in real-world problems, this is, in general, a potential Pareto optimal front, classified as such because no other solutions dominating it could be found but no theoretical tools exist guaranteeing their true Pareto optimality, as it is the case for scalarizing functions in some types of mathematical programming models. EAs can incorporate techniques aimed at guaranteeing the diversity of the Pareto optimal front in order to display the trade-offs between the conflicting objective functions in different regions of the search space. This feature is usually important to offer the Decision Maker (DM) a wider view of the compromises that can be established in different regions of solutions with distinct characteristics. These advantages of using EAs are not just related with the computational effort required but also with the difficulty of using mathematical programming algorithms in most high-dimensional combinatorial multi-objective problems.

The reactive power planning problem has been studied using conventional mathematical programming techniques and widely reported in the scientific literature since the 1960's. However, for the sake of computer tractability, most of these approaches use mathematical models with unrealistic assumptions. Reactive power compensation is commonly addressed as a constrained single objective optimization problem. Usually, the objective function is a combination of several components, including investment costs as well as monetized transmission losses and voltage deviations. It is also common to find power losses as an objective function to minimize, the other evaluation aspects, such as voltage deviation and investment costs, being considered as constraints. The first works published in the literature proposed analytical methods [1], [2], [3], [4], [5], [6] and [7], and mathematical programming algorithms using mathematical models with many simplifications [8], [9] and [10]. A single objective function was considered in these models.

More recently meta-heuristics such as Simulated Annealing [11], [12], [13], Ant Colony [14], Particle Swarm [15], and Tabu Search [16] have been used to address the problem. Because of the characteristics of the VAR planning problem, which is a combinatorial non-linear multi-objective mixed integer problem, meta-heuristics can cope with model complexity and tractability, reduce the exhaustive search in large spaces and lead to (near) Pareto optimal solutions.

Because of their appealing characteristics referred to above, Multi-Objective Evolutionary Algorithms (MOEAs) have been applied to the VAR planning problem, [17], [18], [19], [20] and [21], in order to tackle its intrinsic complexity due to its multi-objective combinatorial nature, also involving continuous, integer and binary decision variables as well as non-linearities (such as the constraints associated with physical laws in networks).

This paper presents the results of a NSGA II based approach enhanced with local search for computing solutions to a multi-objective VAR planning model. The rationale for this study is as follows: some standard multi-objective evolutionary algorithms have been implemented and tested; NSGA II has revealed the best performance; the particular characteristics of this problem suggested the investigation whether its performance could still be enhanced with the addition of a local search scheme within NSGA II, which is tailored to the main features of this problem. The use of local search within the NSGA II operational framework in this complex combinatorial problem improved convergence towards the non-dominated front and ensured that the solutions attained were well spread over it. A comparative study is presented between the results obtained using a standard NSGA II based approach and the enhanced NSGA II approach with local search, to provide decision support in the VAR planning problem in radial distribution systems.

The reactive power planning problem has been formulated as a non-linear mixed integer problem with two objective functions to be minimized: power losses and investment costs. The main constraints include voltage limits at each bus, impossibility to locate capacitors in certain nodes, budget restrictions, operational constraints due to the required load to supply at each node, and the power flow equations in the network.

The interest and motivation of the study have been provided in this Introduction. The multi-objective model for VAR planning is presented in Section 2. In sections 3 and 4, NSGA II and NSGA II enhanced with local search are described. The results obtained are presented in section 5. Some conclusions are drawn in section 6.

2. Multi-Objective Mathematical Model

Dealing with the reactive power planning problem through the installation of shunt capacitors implies answering the following questions, taking in consideration the objectives to be optimized:

- Where to locate the capacitors (in which network nodes)?
- How many capacitors should be placed?
- What should be the size of each capacitor?

The approach herein proposed formulates the VAR planning problem using a multi-objective mathematical programming model with two objectives: minimize branch resistive losses and minimize the costs of installing capacitors. The mathematical model respects the operation of the distribution network, its topology and quality requirements. A consequence of reducing the reactive power flow in the system is the improvement of the voltage profile. This approach also takes into account this quality requirement explicitly by modeling it as a set of constraints.

2.1. Terminology

SB – Substation;

k – iteration number;

t – next bus index;

m – previous bus index;

M – network bus number;

B_m – bus m;

Y – maximum number of capacitors that can be installed;

$\overline{S}_{\mathrm{m}}$ - apparent power vector entering bus m;

P_m - active power vector entering bus m;

Q_m - reactive power vector entering bus m;

$\overline{S}_{Cm}$ - apparent compensation power vector installed in bus m;

Q_{Cm} - reactive compensation installed in bus m;

$\overline{S}_{Lm}$ - apparent power demand vector at bus m;

P_{Lm} - active power demand vector at bus m;

Q_{Lm} - reactive power demand vector at bus m;

$\overline{S}_{losses(m)}$ - total apparent power losses vector in all branches subsequent to bus m;

$P_{losses(m)}$ - total active power losses vector in all branches subsequent to bus m;

$Q_{losses(m)}$ - total reactive power losses vector in all branches subsequent to bus m;

$\overline{V}_m$ - root mean square (rms) voltage vector of bus m;

δ_m –voltage angle in bus m;

$\overline{I}_m$ - current vector that enter bus m;

$\overline{Z}_{mt}$ - impedance of the connection branch from bus m to bus j branch;

r_{mt} - resistance of the connection branch from bus m to bus j branch;

x_{mt} - reactance of the connection branch from bus m to bus j branch;

Q_{Fu} –capacity of capacitor of type u;

C_u – cost of capacitor of type u;

$\overline{X}^*$ - conjugate vector of a generic vector $\overline{X}$.

The vectors structure is described in equations (1) to (6):

$$\overline{Z}_{mt} = r_{mt} + jx_{mt} \qquad (1)$$

$$\overline{S}_m = P_m + jQ_m \qquad (2)$$

$$\overline{V}_m = d_m + je_m \qquad (3)$$

$$\overline{S}_{Lm} = P_{Lm} + jQ_{Lm} \qquad (4)$$

$$\overline{S}_{Cm} = -jQ_{Cm} \qquad (5)$$

$$\overline{S}_{losses(m)} = P_{losses(m)} + jQ_{losses(m)} \qquad (6)$$

2.2. Power flow model

The mathematical model explicitly considers the power flow equations that must be satisfied. The power flow calculations are made using an iterative method specifically developed for this purpose.

The need of taking advantage of the particular structure of radial distribution networks has been recognized by several authors to simplify the calculations and decrease the memory requirements (see [9], [10], [22] and [23]). In our case the power flow is calculated with a different algorithm herein described. The power flow algorithm was implemented with MATLAB, maintaining complex numbers to achieve more accurate results.

A generic radial distribution system is illustrated in Figure 1.

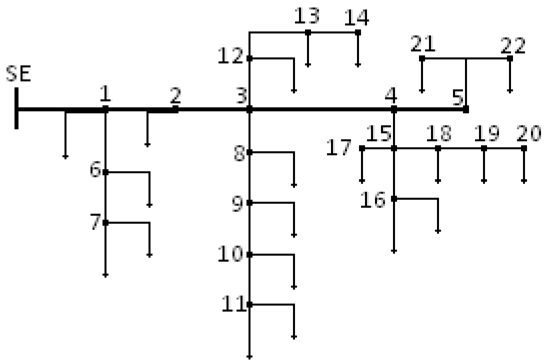

Figure 1- Radial distribution system.

Figure 2 illustrates a generic bus *m*, with compensation and load directly connected to the bus. This bus is fed by a preceding bus and supplies several buses following it (j, $j+1$,... , $j+n$). Connecting branches are characterized by their impedance (resistance and reactance).

A network bus can be seen as a circuit node, receiving power from a previous circuit node, through an interconnecting branch, and delivering power to other points of the circuit, subsequent to it. The proposed power flow equations and algorithm are easier to describe and understand seeing the radial distribution network in this way. Figure 2 shows that each bus of the network has just one way of receiving power, but it can deliver the received power from zero to multiple exits: directly to demanding loads and compensation devices and through branches to subsequent buses.

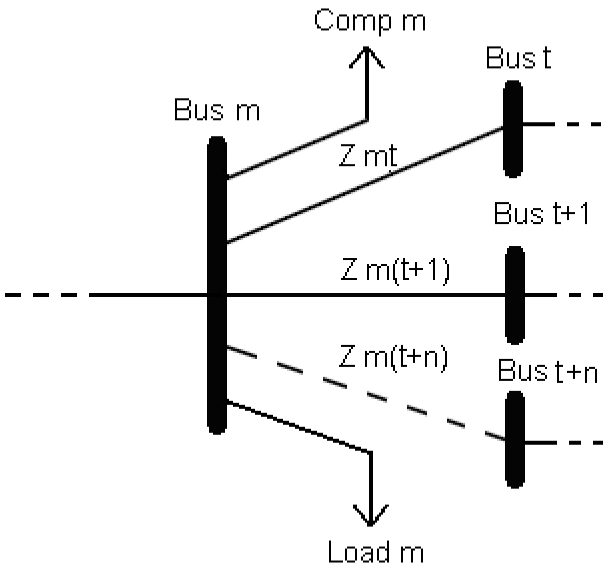

Figure 2 – Generic connection between buses.

The apparent power that arrives to bus *t* can be calculated using equation (7).

$$\overline{S}_t = \overline{V}_t \overline{I}_t^* \tag{7}$$

From equation (7) it is possible to state the current equation

$$\overline{I}_t = \left(\frac{\overline{S}_t}{\overline{V}_t} \right)^* \tag{8}$$

Considering the branch impedance between buses *m* and *j*, $\overline{Z}_{mj}$, Figure 2, equation (9) is derived.

$$\overline{V}_t = \overline{V}_m - \overline{Z}_{mt} \times \overline{I}_t \tag{9}$$

From equations (8) and (9) the final equation to calculate bus *j* voltage (10) can be written.

$$\overline{V}_t = \overline{V}_m - \overline{Z}_{mt} \times \left(\frac{\overline{S}_t}{\overline{V}_t} \right)^* \tag{10}$$

The apparent power equation can be obtained by applying the law of conservation of energy to each bus of the network. Taking into consideration the generic buses, *m* and *t*, see Figure 2, and considering equation (11)

$$\overline{V}_{mj} = \overline{V}_m - \overline{V}_t \tag{11}$$

the power loss equation of the connecting branch (between bus *m* and bus *t*) (12) is written as

$$\overline{S}_{losses(mt)} = \overline{V}_{mt} \times \left(\frac{\overline{S}_t}{\overline{V}_t} \right)^* \tag{12}$$

Taking into consideration all the buses subsequent to bus *m*, equations (13) and (14) are obtained.

$$\overline{S}_{losses(m)} = \overline{V}_{mt} \times \left(\frac{\overline{S}_t}{\overline{V}_t} \right) + \overline{V}_{m(j+1)} \times \left(\frac{\overline{S}_{(t+1)}}{\overline{V}_{(t+1)}} \right) + \ldots + \overline{V}_{m(t+n)} \times \left(\frac{\overline{S}_{(t+n)}}{\overline{V}_{(t+n)}} \right) \tag{13}$$

$$\overline{S}_{losses(m)} = \sum_{i=0}^{n} \left(\overline{V}_{m(t+i)} \times \left(\frac{\overline{S}_{t+i}}{\overline{V}_{t+i}} \right)^* \right) \tag{14}$$

The index *t* denotes the next bus following m and n, that is, it is the identification of the first bus of the lateral.

Finally, the apparent power equations at bus *m* can be written since the apparent power that enters bus *m* must be equal to the sum of all power that goes out of it.

$$\overline{S}_m = \sum_{i=0}^{n} \overline{S}_{t+i} + \sum_{i=0}^{n} \left(\overline{V}_{m(t+i)} \times \left(\frac{\overline{S}_{t+i}}{\overline{V}_{t+i}} \right)^* \right) + \overline{S}_{Lm} + \overline{S}_{Cm} \tag{15}$$

Active and reactive powers can be obtained by calculating the real and imaginary parts of $\overline{S}_m$ respectively, (16) and (17).

$$\overline{P}_m = \mathrm{Re}(\overline{S}_m) \tag{16}$$

$$\overline{Q}_m = \mathrm{Im}(\overline{S}_m) \tag{17}$$

To perform these computations, it is necessary to know the network topology, the characteristics (resistance and reactance) of the connecting branches and the load profile. This type of data is usually supplied by electric distribution companies.

Network buses are numbered by sequential order, beginning with the main feeder from the first to the last bus, and the lateral branches after the last bus, as in Figure 1.

The iterative algorithm may be described in the following steps.

Consider a radial network with *M* buses and *M*-1 branches. For the first iteration consider that losses are null and all the bus voltages are 1 p.u. In this way it is possible to obtain, after the first iteration, an estimated value for branch losses and apparent power that enters each bus of the network, $\overline{S}_{losses(m)}$ and $\overline{S}_m$, m =1,…,M.

Since the apparent power that arrives to any bus of the network is the sum of all apparent power that is possible to distribute from this bus to the remaining network, (15), then the power that enters the first bus of the network, the substation, is the total apparent power necessary for the normal operation of the network. It can also be stated that the power that enters the last bus of a branch it is just the necessary to supply the load demand and/or the power compensation supplied from that bus.

Therefore, apparent power is calculated from the last bus of the branch to the first bus, in all branches. This is done until the first bus of the network, i.e. the substation, is reached. The new power values calculated are immediately used in their predecessors' equations.

Voltages are calculated from the first bus to the last one of the branches, using equation (10), considering that $\overline{V_1} = 1$ p.u. and $\delta_1 = 0^o$.

Voltages are calculated from the first bus to the last one of the network. The new voltage values calculated are immediately used in their successors' equations.

The iterative process ends when the voltage difference, for all buses, between two consecutive iterations is less then a given value, ε_0, usually less than 10^{-4}, as in (18)

$$(V_m)^k - (V_m)^{(k-1)} \le \varepsilon_0 \;\forall\; m,\; m = 1, ..., M \tag{18}$$

This iterative algorithm may be illustrated using an example. Let us consider a simple radial network, with only two branches and three buses, Figure 3.

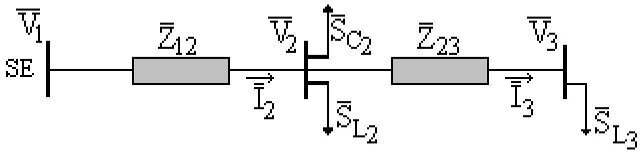

Figure 3 – Simple radial network with three buses.

Apparent power supplied from bus 1, 2 or 3 are calculated using equation (15), as described in (19), (20) and (21).

$$\overline{S}_3 = \overline{S}_{L3} \tag{19}$$

$$\overline{S}_2 = \overline{S}_{L2} + \overline{S}_{C2} + \overline{S}_3 + \overline{V}_{23} \times \left(\frac{\overline{S}_3}{\overline{V}_3} \right)^* \tag{20}$$

$$\overline{S}_1 = \overline{S}_{L1} + \overline{S}_2 + \overline{V}_{12} \times \left(\frac{\overline{S}_2}{\overline{V}_2} \right)^* \tag{21}$$

To calculate bus voltages equation (10) is used. Considering Figure 3, with $\delta_1 = 0^o$ and $\overline{V}_1 = 1\,\text{p.u.}$ all bus voltages can be computed:

$$\overline{V}_1 = V_1 \tag{22}$$

$$\overline{V}_2 = \overline{V}_1 - \overline{Z}_{12} \left(\frac{\overline{S}_2}{\overline{V}_2} \right)^* \tag{23}$$

$$\overline{V}_3 = \overline{V}_2 - \overline{Z}_{23} \left(\frac{\overline{S}_3}{\overline{V}_3} \right)^* \tag{24}$$

As stated above, apparent power is calculated from the last bus of the branch to the first bus, and voltages are calculated from the first bus to the last one of the network. The new power values calculated are immediately used in their predecessors' equations and the new voltage values calculated are immediately used in their successors' equations.

This iterative algorithm has been implemented using MATLAB.

2.3. Objective functions and constraints

The decision variables are the power magnitudes (real and reactive) flowing in the network (P_m and Q_m variables), the voltage magnitudes (V_m variables), and the binary variables encoding the decision whether a new capacitor of a certain type is installed in a given node. New capacitors are characterized by their capacity and the acquisition cost. Standard units, generally used in distribution systems, are considered.

Two objective functions have been considered associated both with the minimization of the system resistive losses (25) and the capacitor deployment cost (26):

$$\text{Min} \sum_{m=1}^{M} \left\{ \text{Re}\left[\sum_{i=0}^{n} \left(\overline{V}_{m(t+i)} \times \left(\frac{\overline{S}_{t+i}}{\overline{V}_{t+i}} \right)^{*} \right) \right] \right\} \tag{25}$$

The index t has the same role as in expression (13), denoting the identification of the first bus of the lateral.

$$\text{Min} \sum_{m=0}^{M} \sum_{j=1}^{Y} a_m^j c_j \tag{26}$$

with:

$$a_m^u = \begin{cases} 1 & \text{if the new capacitor } Q_{Fu} \text{ is installed in } B_m \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

$$b_m = \begin{cases} 1 & \text{if it is possible to locate a capacitor at } B_m \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

The coefficients b_m represent the technical feasibility of installing capacitors at B_m.

$$Q_{Cm} = b_m \sum_{j=1}^{Y} a_m^j \, Q_{Fj} \quad \forall\, m \tag{29}$$

$$\sum_{j=1}^{Y} a_m^j = 1, \forall m \tag{30}$$

The constraints consist of power flow equations (10) and (15), restrictions guaranteeing that at most one capacitor can be placed in each node, (30), and upper and lower bounds for the node's voltage magnitude (31).

$$\overline{V}_{\min} \leq \overline{V}_{m} \leq \overline{V}_{\max} \qquad \forall\, m \tag{31}$$

This model is hard to solve by mathematical programming algorithms because it is nonlinear and contains both discrete and continuous variables.

3. An Elitist Non-Dominated Sorting Genetic Algorithm (NSGA II)

The Elitist Non-Dominated Sorting Genetic Algorithm was presented in [24]. Unlike the majority of elitist multi-objective EAs, NSGA-II uses not just an elite-preserving strategy but also an explicit diversity-preserving mechanism.

3.1 Elitism

In NSGA II, an elite-preserving operator favors the best individuals in the population by giving them the opportunity to be directly carried over to the next generation. There are several ways of introducing elitism, but no matter how elitism is established, the aim is to ensure that the fitness of the best solution in the population does not deteriorate. Therefore, a good solution found early on in the evolutionary process will never be lost unless a better solution is discovered. Normally elitism improves convergence to the global optimal solution, in single objective problems, or to the Pareto optimal front, in multi-objective optimization problems, because the presence of elites enhances the probability of creating better offspring. Elitism can be implemented in different degrees. Better results are generally obtained when an intermediate degree of elitism is introduced. In most single-objective implementations, the best α solutions of the population are used as the elites. The choice of an appropriate α value becomes important in the successful working of the algorithm [25].

In single-objective optimization, elites are easy to identify: the best elite is the solution with the best objective function value. In multi-objective optimization, since multiple and conflicting objective functions are at stake, and therefore there is no prominent solution, it is not as straightforward to identify the best solutions to become the elite members. In multi-objective optimization, a solution can be evaluated as “good” or “bad” based on its non-dominance rank in the population. However, in general, there is more than one solution in each non-dominated set, and all these best Pareto optimal solutions would become elites of identical importance. Since the number of Pareto optimal solutions in any front is not controllable, it becomes difficult to introduce elitism in a restricted manner in multi-objective optimization [25].

Although the presence of elitism should improve the performance of a multi-objective EA, care must be taken to control the effective degree of elitism introduced into the process because it may cause loss of diversity. Therefore, the balance between the convergence to the true Pareto front and solution diversity requires a controlled elitism in multi-objective evolutionary optimization.

3.2 NSGA II

NSGA II provides an efficient procedure for introducing elitism into an MOEA while guaranteeing a diversity-preserving mechanism, assuring in this way a good convergence towards the Pareto-optimal front without loosing solution diversity. In this algorithm, in generation (iteration) t, the offspring population E_t is created by using the parent population D_t both of size N. However, instead of finding the non-dominated front of E_t only, first the two populations are combined together to form a population R_t of size 2N. This population is classified with a non-dominated sorting algorithm. Although this requires more effort compared with performing a non-dominated sorting on E_t alone, it allows a non-dominance check among offspring and parent solutions. After this procedure the new population is filled by solutions of different non-dominated fronts, one at a time. The process starts with the best non-dominated front and continues with solutions of the second non-dominated front (that is, the non-dominated front after the solutions of the first front have been removed), and so on. Since the size of R_t is 2N, not all fronts may be accommodated in the N slots available in the new population, and they are simply deleted. When the last front is being considered, there may be more solutions in the last front than the remaining slots in the new population. Instead of arbitrarily discarding some members from the last front, a niching strategy is used to choose the members of the last front that reside in the least crowded region in that front.

The standard NSGA II algorithm is outlined below (see also [24] [25]). Initially, a random population D_0 is created. The population is sorted into different non-dominance levels. Each solution is assigned a fitness equal to its non-dominance level (1 will be assigned to the first non-dominated front). Accordingly, it will be assumed the minimization of the fitness. Binary tournament selection, recombination and mutation operators are used to create an offspring population E_0, of size N. The stopping criterion is the number of generations (iterations).

Step 1 Combine parent and offspring populations to create $R_t = D_t \cup E_t$. Perform a non-dominated sorting in R_t and identify different fronts F_i, $i = 1, 2, \ldots$.

Step 2 Set a new population $D_{t+1} := \emptyset$. Set counter i=1.

While $|D_{t+1}|+|F_i| < N$, do $D_{t+1} := D_{t+1} \cup F_i$ and $i := i + 1$.

Step 3 Perform the Crowding-sort (F_i < c) procedure (mentioned below) and include the most widely spread (N - $|D_{t+1}|$) solutions into D_{t+1}, by using the crowded distance values in the sorted F_i.

Step 4 Create an offspring population E_{t+1} from D_{t+1} by using the binary crowding tournament selection, crossover and mutation operators.

The process of non-dominated sorting and filling the population D_{t+1} steps can be performed together, so that every time a non-dominated front is found its size can be used to check if it can be included in D_{t+1}. If it is not possible, no more sorting is needed.

In Step 3, the crowding-sorting of the solutions in front F_i, which is the last front that could not be completely accommodated, is performed by using a crowded-distance metric. The crowding comparison operator compares two solutions and returns the winner of the tournament. The winner is selected based on two attributes: the non-dominance ranking r_i and the local crowding distance d_i, in the population. This crowding distance attribute of a solution i is a measure of the search space around i, which is not occupied by any other solution in the population. d_i is an estimate of the perimeter of the cuboid formed by using the nearest neighbours as the vertices (which is called the crowding distance). Based on r_i and d_i the binary crowding tournament selection operator works as follows: a solution i wins a tournament over another solution j if any of the following conditions are true

1. If $r_i < r_j$ (this makes sure that the solution chosen lies on a better non-dominated front).
2. If $r_i = r_j$ and $d_i > d_j$ (this is applied when both solutions lie on the same front and the condition above cannot be applied; in this case the solution residing in a less crowded area, with a larger d_i, wins).

A detailed explanation of NSGA II can be found in [24]and [25].

4. NSGA II with local search

The enhanced NSGA II approach couples a local search scheme to the standard NSGA II, which is adapted to the characteristics of the VAR planning problem in radial networks. In this problem the main decision variables refer to the identification of the network nodes to install capacitors and the dimension of each capacitor to be installed aimed at minimizing system losses and the amount of investment in the equipment, while keeping an adequate voltage profile and satisfying physical laws in electrical networks.

The chromosome structure is divided in two parts both with the same length. The first part of the chromosome keeps the identification of the nodes where capacitors should be located for that specific solution. The second part of the chromosome holds the capacitor types that are located on the network nodes described in the first part. An example of the chromosome structure with the corresponding physical meaning is illustrated in Figure 4. For this example, a small radial network with 28 nodes and three types of capacitors has been used. A capacitor of type 2 is installed on nodes 1 and 3, of type 3 on node 5, and of type 1 on node 21.

As described in the previous section, initially a random population D_0 is created. The population is sorted into different non-dominance levels. Each solution is assigned a fitness equal to its non-dominance level (1 will be assigned to the first non-dominated front, and so on).

A new population E_0 is obtained using binary tournament selection, crossover and mutation. Before combining together the parent and offspring populations the local search procedure is performed on all offspring solutions, trying to adjust the capacity installed and each node selected. The local search operates on both parts of the chromosome.

In this local search scheme, a move leading to a neighbour solution is defined by changing the capacitor location in the network to a neighbour location, or the capacitor type corresponding to a capacity value.

In the first phase of the local search scheme, the best buses to install the capacitor are determined. The procedure then attempts to improve this solution by moving the capacitors to the neighbour buses without changing their capacity values. In the second phase, all possible types of capacitors are tested, for each node chosen in the first part of the chromosome. For example, considering the chromosome illustrated in Figure 5 and technical possibilities to install three types of capacitors, the combinations that form the neighbourhood to be exploited is displayed in Figure 6.

After the neighbourhood search, the resulting offspring population, E_0, is then combined with the parent population D_0. The non-dominated sorting is performed as in the standard NSGA II procedure described in section 3.2. In this way, the combined population (parents and modified offspring) is classified with the NSGA II non-dominated sorting algorithm. The crowding-sort procedure is performed, which allows the most diverse solutions to be included into the new population D_1. Then the offspring population E_1 is created from D_1 by using the crowding tournament selection, crossover and mutation operators. The local search scheme is applied to this new population again. The NSGA II algorithm enhanced with local search repeats until a given number of iterations is attained.

As it is illustrated in the next section, this overall procedure not just improves the convergence towards the non-dominated front but also ensures that the solutions obtained are well spread over it. With minimal changes in the standard NSGA II algorithm, the local search procedure fine-tunes the amount of reactive power supplied in the selected nodes greatly improving the results obtained.

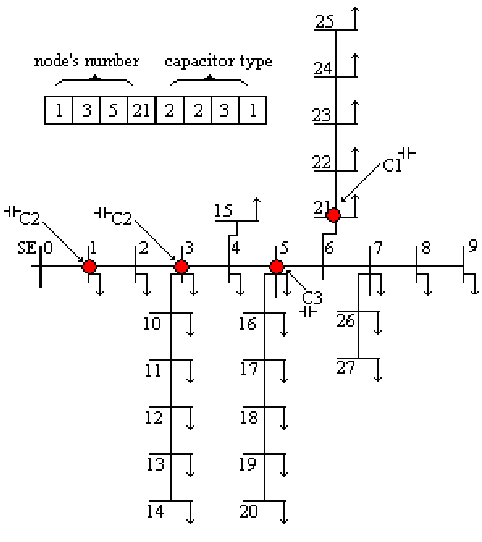

Figure 4 – Chromosome structure and network physical example.

7	25	67	80	3	2	2	1

Figure 5 – Original solution.

6	25	67	80	3	2	2	1
8	25	67	80	3	2	2	1
7	**24**	67	80	3	2	2	1
7	**26**	67	80	3	2	2	1
7	25	**66**	80	3	2	2	1
7	25	**68**	80	3	2	2	1
7	25	67	**79**	3	2	2	1
7	25	67	**81**	3	2	2	1
7	25	67	80	**1**	2	2	1
7	25	67	80	**2**	2	2	1
7	25	67	80	3	**1**	2	1
7	25	67	80	3	**3**	2	1
7	25	67	80	3	2	**1**	1
7	25	67	80	3	2	**3**	1
7	25	67	80	3	2	2	**2**
7	25	67	80	3	2	2	**3**

Figure 6 – Example of neighbour solutions of a given original solution (Figure 5).

5. Results and Analysis

The proposed methodology has been applied to an actual Portuguese radial distribution system with 94 nodes. The network layout is displayed in Figure 7, and its physical characteristics are summarized in Table 1. The detailed description of the network – line data and load data - is given in appendix (for the sake of result replicability).

The average power factor of this network is 0.9 and the voltage at the substation is 15KV+5%.

Table 1- Network Characteristics

	Minimum	Maximum	Average	S.D.
Line length (m)	256	4027	856	559.6
Resistance (Ω/Km)	0.213	1.5	0.745	0.393
Inductance (Ω/Km)	0.356	0.395	0.379	0.011

The capacitors are characterized by their capacity and the acquisition cost (Table 2). Standard units, from Merlin Gerin/Schneider Electric (2007 Portuguese Catalogue prices), are considered. Capacitor cost does not include installation or maintenance costs.

Table 2- Capacitor dimension and acquisition costs

	Maximum capacity (kVAr)	Composition	Cost (Euros)
C1	50	10+20+20	2035
C2	100	20+40+40	2903
C3	140	20+40+80	4545
C4	200	40+80+80	4875
C5	240	40+80+120	5716
C6	300	60+120+120	6578
C7	360	40+80+120+120	7337
C8	400	40+40+80+120+120	9395

This electric distribution network is located in a rural area and it has some demanding characteristics: the load is heavy and the length of the distribution branches is high; the voltage profile without compensation does not respect the quality requirements – lower and upper voltage bounds, see equation (30) in the mathematical model presented in section 2 - so the zero cost solution is not feasible. Therefore, it is necessary to install capacitors, which locally provide the required reactive power, to have feasible solutions with respect to the voltage profile constraints (1 p.u ± 10%). So, even before any results were obtained, it was expected to have a challenging compensation scheme to this distribution network, with several compensated nodes and an associated high cost.

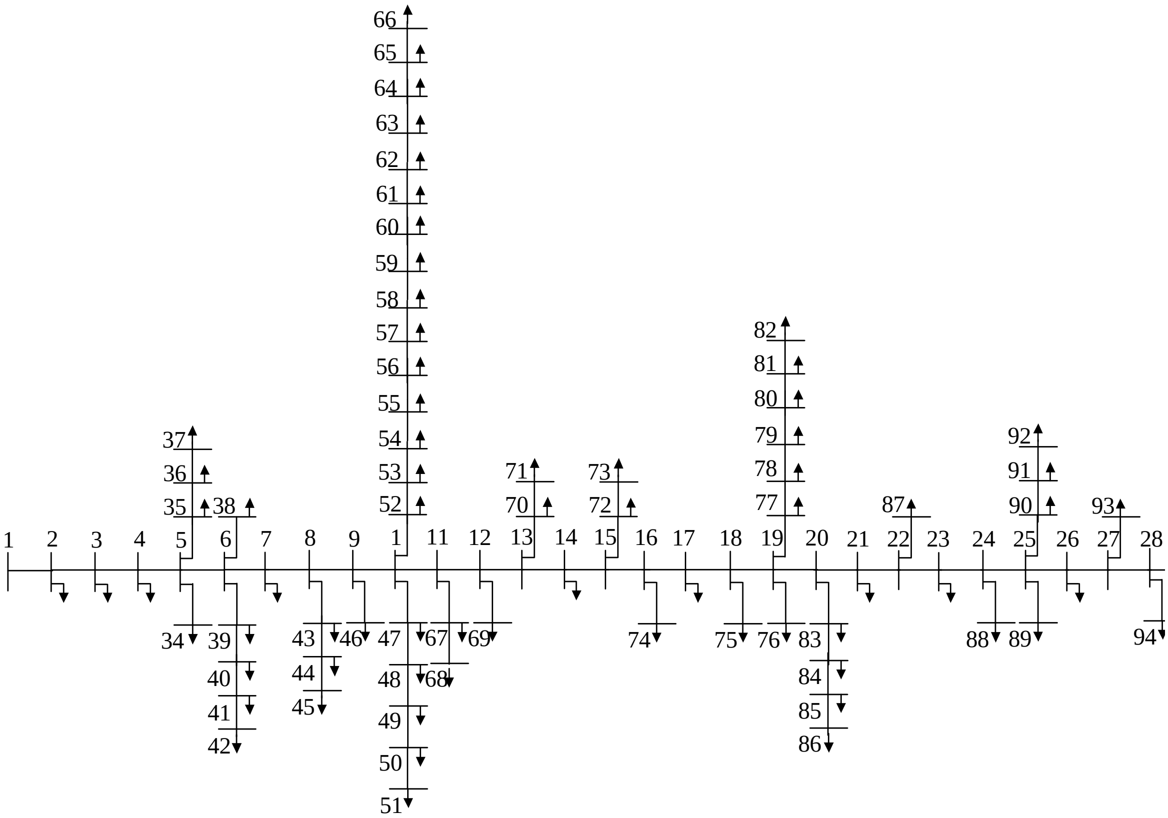

Figure 7 – Actual Portuguese radial electrical distribution network.

The results herein presented were obtained with five hundred iterations and eight types of capacitors in Table 2, after tuning the main parameters (mutation and crossover probabilities, etc.) controlling the run of the algorithms.

NSGA II has revealed a good performance in this reactive power compensation problem in comparison with other MOEAs. The authors used other MOEAs to solve this problem, in particular Vector Evaluated Genetic Algorithm (VEGA) [26], Strength Pareto Evolutionary Algorithm (SPEA) [27], and Multi-Objective Genetic Algorithm (MOGA) [28]. NSGA II revealed the best performance in comparison with the other algorithms. This motivated the research on how to improve the global solutions obtained with NSGA II with a local search scheme tailored to the particular characteristics of this problem and the encoding of solutions.

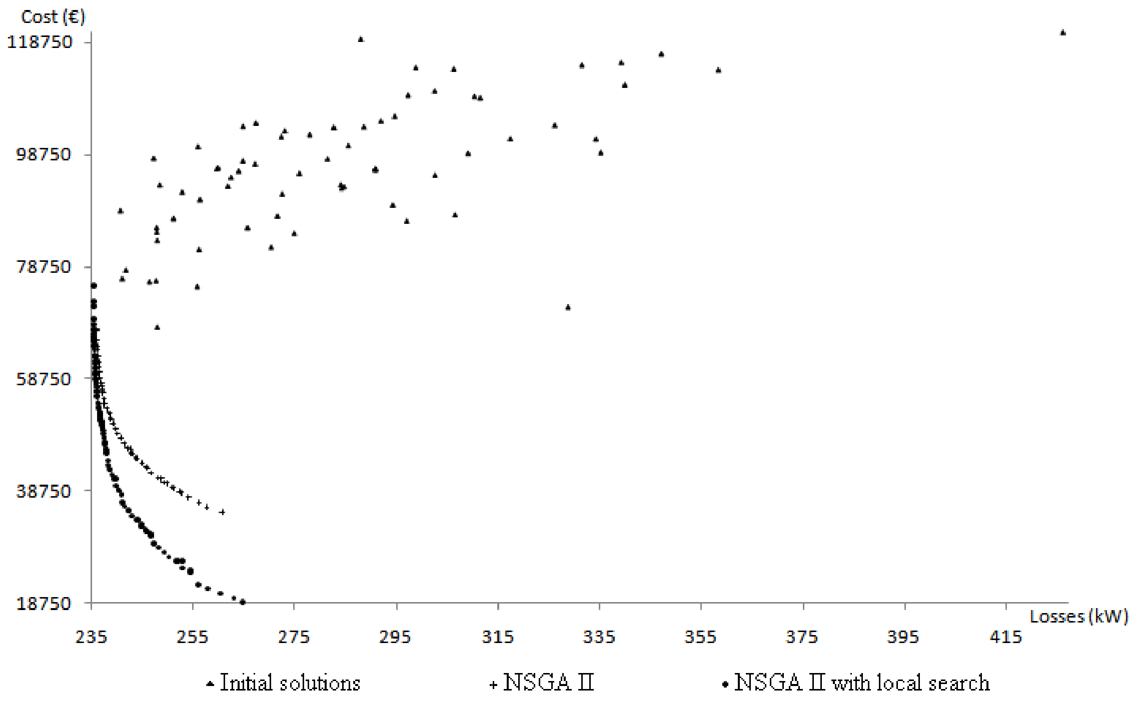

Figure 8 – Initial solutions and Pareto Frontiers obtained with NSGA II and NSGA II with local search.

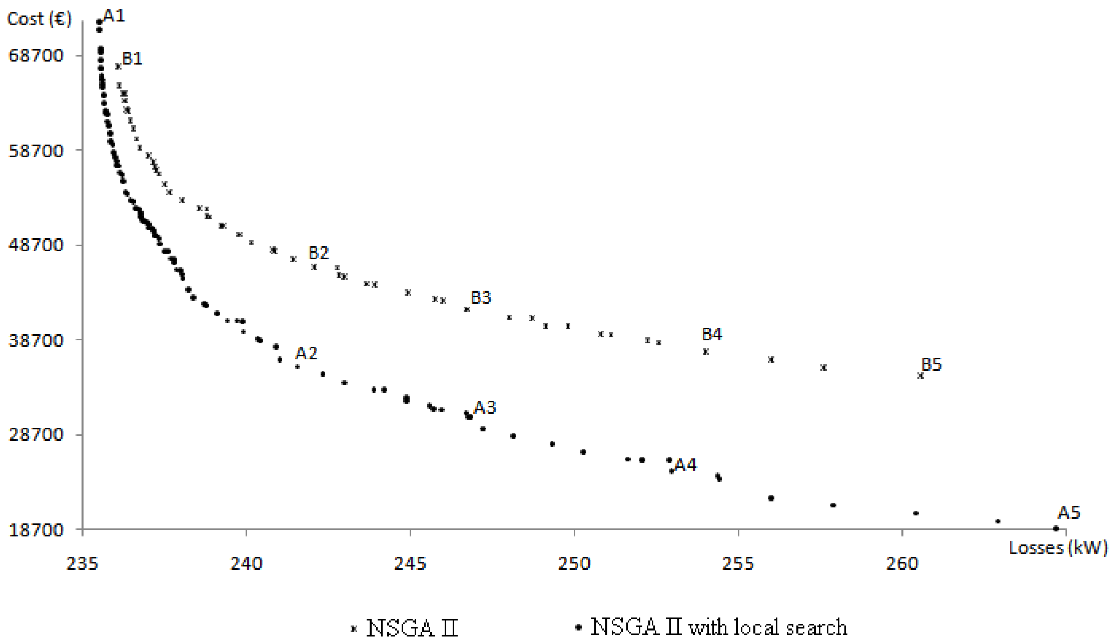

Figure 9 – Pareto Frontier obtained with NSGA II and NSGA II with local search.

The local search scheme within NSGA II exploits different compensation magnitudes for the locations (network nodes) identified by NSGA II acting as a global search tool. The initial solutions were the same for both algorithms.

Figure 8 presents the final results for both algorithms: NSGA II and NSGA II enhanced with local search, as well as the initial population. As it is depicted in Figure 8, the Pareto-optimal frontiers obtained with the standard and the enhanced versions of NSGA II display a considerable improvement regarding the initial (random) solutions. The Pareto frontiers are generally well defined and the solutions are spread all over them for both approaches. It is important to notice that the zero cost solution is not feasible. This means that without compensation this electrical distribution network had a very bad voltage profile, violating the voltage upper and lower bounds, as explained before. Similarly, there are several low cost solutions that are not feasible for the same reasons.

Each point displayed in Figure 9 corresponds to a physical solution, with power losses and cost computed through expressions (25) and (26), and a compensation profile associated with capacitors installed along the network. The objective function values of some selected representative solutions are presented in Table 3, and the physical characterization of three solutions are represented in Table 4, Table 5 and Table 6.

Table 3 - Sample of Pareto Optimal Solutions.

Solution	Losses (kw)	Cost (Euros)
A1	**235,4565**	75261
B1	236,0764	67593
A2	241,0107	36685
B2	241,4282	47205
A3	246,7696	30630
B3	246,7247	41903
A4	252,9622	24914
B4	254,0151	37469
A5	264,7107	**18790**
B5	260,5941	34865

Solutions A1 and A5 are the extreme solutions, which optimize individually the active power losses and the cost objective functions, respectively (values in bold in Table 3), computed with NSGA II enhanced with local search. Solutions B1 and B5 are the corresponding extreme solutions for the standard NSGA II. As the NSGA II enhanced with the local search procedure discovers new solutions, these dominate clearly the Pareto Frontier found with the standard NSGA II. In particular, in the region with higher cost/lower losses solutions, the local search found new solutions defining a larger and more diverse Pareto front. The results make clear the advantage of tuning the capacitor locations and sizes assigned to the compensated nodes.

Table 4 –Physical description, node number and chosen capacitor, of solution A1.

A1																			
Node Number	14	18	28	35	38	40	43	47	52	56	58	59	65	67	70	79	84	88	91
Capacitor	3	4	2	5	4	3	2	4	4	4	4	4	1	1	1	2	6	2	1

Table 5 –Physical description, node number and chosen capacitor, of solution A4.

A4				
Node Number	17	24	59	83
Capacitor	2	7	7	7

Table 6 –Physical description, node number and chosen capacitor, of solution A5.

A5			
Node Number	26	77	83
Capacitor	4	6	7

6. Conclusions

This work addresses a relevant problem in electrical distribution power systems. Since most loads are inductive, there is an interest in improving power factor, because the reactive power flow in power system branches impairs the system capacity and can adversely affect voltage level. The actual electrical power system used for this comparative case study is a challenging one due to its physical characteristics. It illustrates that problem, because without compensation the quality requirements for the voltage profile are violated for the load conditions in which the system works.

NSGA II revealed the best performance among some of the most representative MOEAs applied to this problem. The local search incorporated into this algorithm provides even better results than the standard NSGA II alone. The benefits of finding the best nodes to install the capacitors and choosing the compensation values are illustrated. Both power losses and voltage profile, which can be understood as surrogates for improving system capacity respecting quality of service constraints, benefit from the installation of capacitors in specific nodes in the system. Care has been taken in the number of allowed capacitors to be installed, and all operating constraints were respected.

The characterization of the Pareto front can be used as a valuable planning tool by planning engineers when choosing an adequate compromise solution for power factor compensation, unveiling information on the trade-offs between the conflicting objective functions to reach a well-founded and technically feasible compensation configuration.

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Table 7 – Load data[1].

Bus number	Active Power (kW)	Reactive Power (KVAr)	Bus number	Active Power (kW)	Reactive Power (KVAr)
2	22.5	10.9	57	31.5	15.3
3	240.3	116.4	58	521.1	252.4
4	24.3	11.8	59	212.4	102.9
7	28.8	14	60	39.6	19.2
14	57.6	27.9	61	45	21.8
17	18.9	9.2	62	17.1	8.3
20	55.8	27	63	21.6	10.5
21	40.5	19.6	64	35.1	17
23	54	26.2	65	70.2	34
26	46.8	22.7	66	34.2	16.6
29	13.5	6.5	67	22.5	10.9
30	3.6	1.7	68	45.9	22.2
31	18	8.7	69	33.3	16.1
32	21.6	10.5	70	36.9	17.9
33	9	4.4	71	45	21.8
34	64.8	31.4	72	75.6	36.6
35	65.7	31.8	73	67.5	32.7
36	59.4	28.8	74	27.9	13.5
37	13.5	6.5	75	38.7	18.7
38	161.1	78	76	53.1	25.7
39	26.1	12.6	77	65.7	31.8
40	134.1	65	78	63	30.5
41	85.5	41.4	79	67.5	32.7
42	41.4	20.1	80	45	21.8
43	41.4	20.1	81	9	4.4
44	41.4	20.1	82	16.2	7.8
45	21.6	10.5	83	67.5	32.7
46	25.2	12.2	84	296.1	143.4
47	45.9	22.2	85	72	34.9
48	36.9	17.9	86	76.5	37.1
49	63.9	31	87	90.9	44
50	68.4	33.1	88	72	34.9
51	27.9	13.5	89	63	30.5
52	81	39.2	90	21.6	10.5
53	69.3	33.6	91	36.9	17.9
54	62.1	30.1	92	20.7	10
55	35.1	17	93	17.1	8.3
56	205.2	99.4	94	90	43.6

[1] Buses without consumer loads (P= 0 W; Q= 0 VAr) are not showed in this table.

Table 8 – Line data.

S.N.[2]	R..N.[3]	R (Ω)	X (Ω)	S.N.[2]	R..N.[3]	R (Ω)	X (Ω)	S.N.[2]	R..N.[3]	R (Ω)	X (Ω)
1	2	0.112	0.1873	40	41	0.5177	0.2892	79	80	1.1738	0.6556
2	3	0.0763	0.1274	41	42	0.7148	0.3992	80	81	0.619	0.3457
3	4	0.1891	0.3161	8	43	1.0575	0.2785	81	82	0.5684	0.3174
4	5	0.2243	0.3749	43	44	0.5198	0.2903	20	83	0.8393	0.3011
5	6	0.2571	0.4297	44	45	0.3341	0.1866	83	84	0.2133	0.1191
6	7	0.134	0.2239	9	46	0.349	0.1949	84	85	0.3645	0.2036
7	8	0.2986	0.4991	10	47	0.5771	0.3223	85	86	0.3206	0.1791
8	9	0.1953	0.3265	47	48	0.3598	0.2009	22	87	0.7675	0.4286
9	10	0.5097	0.8519	48	49	0.7688	0.4294	24	88	1.5914	0.5709
10	11	1.5303	1.5101	49	50	0.2599	0.1451	25	89	0.702	0.3921
11	12	0.1889	0.1864	50	51	0.8654	0.4833	25	90	20.743	0.7441
12	13	0.1816	0.1793	10	52	0.5248	0.5179	90	91	0.678	0.2432
13	14	0.0661	0.0653	52	53	0.1737	0.1714	91	92	0.5738	0.3205
14	15	0.4115	0.4061	53	54	0.6148	0.6068	27	93	0.5913	0.3303
15	16	0.2584	0.255	54	55	0.198	0.1954	28	94	1.1865	0.3124
16	17	0.2033	0.2006	55	56	0.198	0.1954				
17	18	0.7243	0.7148	56	57	0.285	0.2813				
18	19	0.2162	0.2134	57	58	0.1429	0.141				
19	20	0.35	0.3454	58	59	0.3409	0.1904				
20	21	1.4775	0.3891	59	60	0.3679	0.2055				
21	22	0.45	0.1185	60	61	0.3591	0.2006				
22	23	0.771	0.203	61	62	0.3503	0.1957				
23	24	0.885	0.2331	62	63	0.4219	0.2356				
24	25	0.9915	0.2611	63	64	1.538	0.5517				
25	26	0.384	0.1011	64	65	0.9788	0.3511				
26	27	0.7245	0.1908	65	66	1.4911	0.5349				
27	28	1.185	0.3121	11	67	0.969	0.2552				
28	29	1.2353	0.6899	67	68	0.6705	0.1766				
29	30	0.3557	0.1987	12	69	0.4354	0.2432				
30	31	0.9494	0.3406	13	70	0.4631	0.2586				
31	32	0.6899	0.3853	70	71	0.2707	0.1512				
32	33	1.5707	0.8773	15	72	0.6683	0.3732				
5	34	1.2655	0.454	72	73	0.8525	0.4762				
5	35	0.1688	0.0943	16	74	0.3314	0.1851				
35	36	0.2741	0.1531	18	75	0.405	0.2262				
36	37	0.2552	0.1425	19	76	0.4367	0.2439				
6	38	0.4165	0.2326	19	77	0.3416	0.1908				
6	39	1.4835	0.3907	77	78	0.2113	0.118				
39	40	1.8	0.474	78	79	1.1249	0.4035				

[2]*S.N.: Sending Node;* [3]*R.S.: Receiving Node*