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Hierarchical Multiobjective Routing in MPLS Networks with Two Service Classes – Heuristic Resolution and Sensitivity Analysis*

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Abstract

Modern multiservice network routing functionalities have to deal with multiple, heterogeneous and multifaceted QoS (Quality of Service) requirements. A heuristic approach devised to find "good" solutions to a hierarchical multiobjective alternative routing optimisation problem in MPLS networks with two service classes (and different types of traffic flows in each class), namely QoS and Best Effort (BE) services, formulated within a hierarchical network-wide optimisation framework, is presented. This heuristic resolution is based on a bi-objective constrained shortest path model and is applied to a test network used in a benchmarking case study. An experimental study based on analytic and discrete event simulation results is presented, allowing for an assessment of the quality of results obtained with this new heuristic resolution for various traffic matrices. A dynamic version of the routing method is formulated and its performance with the same case study network is analysed. A sensitivity analysis is performed by using relaxation and floating barrier techniques, so as to evaluate if the proposed heuristic treats the objective functions in a balanced way. The results of these tests led to the formulation of variants of the heuristic. A description and an experimental study of these variants are also presented.

Keywords Routing models, Multiobjective optimisation, Communication networks, Internet.

1 Introduction and Motivation

Modern multiservice network routing functionalities often deal with multiple and heterogeneous QoS (Quality of Service) requirements. This has led to routing models with the

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aim of calculating and selecting one (or more) sequences of network resources (designated as routes, which correspond to loopless paths in the network representation) satisfying certain QoS constraints and the optimisation of route related metrics. Therefore there are potential advantages in formulating the routing problems in these types of networks as multiple objective optimisation problems. These formulations enable the trade-offs among different performance metrics and other network cost function(s) to be pursued in a consistent manner.

In the emergent MPLS (Multiprotocol Label Switching) technology for the Internet, connection-oriented services from origin to destination can be implemented. The LSRs (Label Switching Routers) in the MPLS network forward the packets (grouped in FECs, Forward Equivalence Classes), through LSPs (Label Switched Paths) using a specific packet label switching technique. Due to this feature and other functional capabilities of MPLS, advanced QoS-based routing mechanisms can be implemented, namely through the definition of "explicit routes" (determined at the originating node) for each traffic flow of a given FEC.

The analysis of routing in communication networks can be conducted in different perspectives at different levels. Taking into account the features and capabilities of MPLS routing, a significant number of routing models have been proposed in the literature in recent years (see a brief overview in [4, 8]), which often differ in key instances of the modelling framework. The differences between the different models are concerned with: i) the routing optimisation framework, which includes the scope and nature of the formulation of the routing calculation problem (network-wide optimisation models[1] or flow-oriented optimisation models[2]); ii) the nature of the optimisation model in terms of the objective functions (single or multiobjective) and constraints (the type of constraints, for instance); iii) the level of representation of the traffic flows, that can be made at the level of micro-flows of packet streams carried on a certain LSP or at the level of the "traffic trunks", corresponding to the aggregation of traffic streams of the same class placed on the same LSP.

A brief overview of multiobjective routing models in communication networks is presented in [8] where models for MPLS networks and other multiservice networks which may be applicable (with some adaptations) to MPLS, are reviewed. Different assumptions as well as different objectives and constraints may be used in the formulation of routing models, which lead to different approaches to the routing problem. The references [15, 16, 17] formulate a routing optimisation problem in MPLS networks of the mixed integer type to be solved off-line, where the traffic is represented at the traffic trunk level in a deterministic way. Three different objectives subject to constraints are considered in this problem: the minimisation of the expected delay experienced by the packets in the traffic trunks along the chosen paths, the optimisation of the utilisation of resources (which is achieved by optimising the load distribution in the network) and the minimisation of the number of LSPs used. In [21, 41], the routing problem in a MPLS network is formulated as a bi-objective problem, of the mixed integer type with constraints, and it is solved off-line. The traffic is again represented at the traffic trunk

[1] In network-wide optimisation models, the objective functions are formulated at the network level and depend explicitly on all traffic flows in the network. Examples of these objective functions are the total traffic carried, the total expected revenue, the average packet delay or a function which seeks an optimisation of the use of all links in terms of their occupation levels.

[2] In flow-oriented optimisation models the objective functions are formulated at the level of each node-to-node traffic flow or call. Examples of these objective functions are the number of arcs or cost of the path, the mean packet delay on each packet stream or the end-to-end blocking probability.

level in a deterministic way. The two objectives are the minimisation of the maximum link utilisation (which intends to achieve an optimal load distribution in the network) and the minimisation of the number of hops in a path (which intends to achieve the minimisation of the delay and of the fault probability in the path). This objective is secondary, in the sense that the pursuit of the minimisation of the number of hops in a path should be made without affecting the primary objective of load balancing. The order in which the requests are serviced is important for the quality of the final results, as analysed in [20]. The reference [37] formulates the routing problem in an MPLS network as a bi-criteria optimisation problem. This model will be fully described in subsection 5.1, as it is the benchmark used for performance evaluation of our model. In [28] a multiobjective routing model, of the mixed integer type, for packet switched networks is proposed, with the traffic represented at the traffic trunk level in a deterministic way, to be solved off-line. The three considered objectives are the minimisation of the deviation between the bandwidth required by the traffic routed in a link and the capacity of that link, the minimisation of the deviation between the utilisation of the links and a pre-defined value of utilisation and the minimisation of the costs of communication, associated with the utilisation of each link. The authors address the resolution of this problem by an evolutionary method. In [42] a multiobjective routing model with constraints, of the mixed integer type, for packet networks is also proposed. The traffic is represented at the packet level in a stochastic form and the problem is solved on-line. The two objectives are the optimisation of the utilisation of the network (related to the efficient management of the network resources) and the minimisation of the number of hops in a path. For the resolution of the routing problem, the author uses a heuristic algorithm based on a constrained minimum cost tree (constrained Steiner tree). The reference [38] formulates a bi-objective routing problem with constraints in networks supporting multimedia applications, of the mixed integer type. The two objectives are the minimisation of the number of hops in a path (so that the number of resources used is minimum) and the minimisation of the cost of a path (where the cost of a link is defined in terms of the used bandwidth in that link). The problem is solved using a heuristic procedure, based on the composition of shortest paths obtained when only one of the objectives is minimised. An exact algorithm to compute, in an efficient manner, the whole Pareto solution set, in this type of bi-objective routing problems, is given in [5]. In [13], a multiple objective dynamic routing model with alternative routing of periodic type is proposed. The solution of the formulated multiobjective network-wide optimisation routing problem is based on a bi-objective shortest path model using two conflicting path metrics, the implied costs and the blocking probabilities. A heuristic devised to solve this problem is proposed in [33], and its performance is compared with reference dynamic routing methods. The references [30, 31] propose and describe MODR-S, a multiobjective dynamic alternative routing model for multiservice networks equivalent in the traffic plane to multirate loss traffic networks. The traffic flows are represented as multirate Poisson processes. Multiple types of service, with different QoS requirements, are considered. The optimisation is performed at two hierarchical levels: at the network level, the objectives are the maximisation of the revenue of the total traffic carried in the network for a given routing solution and the minimisation of the maximal average blocking probability for the different services; at the services level, the objectives are the minimisation of the average blocking and the minimisation of the maximal point-to-point blocking for each service.

Based on the analysis of the differences observed in the models proposed in this area, some key conceptual issues involved in the various modelling approaches were discussed and a generic hierarchical multiobjective network-wide routing optimisation framework was proposed by the authors in [8]. A major motivation for the present work was the application of this modelling framework to a MPLS type network, as outlined in [8], where two classes of service, QoS traffic (first priority traffic) and Best Effort (BE) traffic (second priority traffic), are considered.

In a previous work [9], the authors have presented in detail, a model for multiobjective routing in MPLS networks formulated within the framework developed in [8], considering that there are two classes of services (and different types of traffic flows in each class), which are QoS and BE services. This is a network-wide multiobjective routing optimisation approach of a new type, with flows of different priority. The flows of QoS type (first priority flows), when accepted by the network, have a guaranteed QoS level, related to the required bandwidth. As for BE traffic flows, which are treated in the model as second priority flows, they are carried by the network in order to obtain the best possible QoS level for the current network routing solution. Alternative routing is used in the routing model: when a first choice route assigned to a given micro-flow[3], belonging to a specific traffic flow (corresponding to a "traffic trunk") is blocked a second choice route may be attempted. An important feature of this model is the use of hierarchical optimisation with two optimisation levels, including fairness objectives: the first priority objective functions are related to the network level objectives of QoS type flows, and they are the total expected revenue and the maximal value of the mean blocking of all types of QoS flows; the second priority objective functions are related to performance metrics for the different types of QoS services and the total expected revenue for the BE traffic flows. The traffic flows in the network are represented in an approximate stochastic form, based on the use of the concept of effective bandwidth for macro-flows and on a generalised Erlang model for estimating the blocking probabilities in the arcs, as in the model used in [36, 31]. A specialised heuristic strategy for finding "good" compromise solutions to the very complex bi-level routing optimisation problem was presented in [9]. That heuristic is based on a constrained bi-objective shortest path model with QoS or BE marginal path implied costs, depending on the class of the routed traffic, and path blocking probabilities as objective functions.

This work presents a new heuristic approach devised to find "better" solutions to the mentioned hierarchical multiobjective routing optimisation problem, based on a constrained bi-objective shortest path model with QoS or BE marginal path implied costs and path blocking probabilities as objective functions. The main contributions of this paper are: i) the proposal of a new heuristic resolution of the hierarchical multiobjective alternative routing optimisation problem in MPLS networks with two service classes, which is more effective than the previous heuristic resolution MOR-S2 [9]; ii) the application of this new heuristic resolution to a test network used in a benchmarking case study and the comparison with the results obtained with the previous heuristic resolution MOR-S2 [9]; iii) the presentation of an experimental study based on analytic and discrete event simulation results, that allows for an assessment of the quality of results obtained with the new heuristic resolution for various traffic matrices; iv) the formulation of a dynamic version of the routing method and the analysis of its performance with the same case study network; v) the description of sensitivity tests performed on the heuristic that allow for a study of the balance in the treatment of the different objective functions; vi) the proposal of variants of the heuristic, which are applied to the same test network; vii) the presentation of analytic and discrete event simulation results for the most promising

[3]A μ-flow corresponds in our model to a 'call', that is, a connection request with certain features.

variants.

The report is organised as follows. The two-level hierarchical multiobjective alternative routing model with two service classes is reviewed in section 2. In section 3, the foundations of the resolution approach are described including a bi-objective constrained shortest path problem used at the core of the resolution approach and the stochastic traffic model underlying the formulated multiobjective optimisation problem. The proposed specialised heuristic is described in section 4. The results of an experimental study for a test network used in a benchmarking model, are revealed in section 5. These results were obtained with analytic and simulation experiments. In this same section a dynamic version of the routing model is formulated and simulation results obtained with this version and with the static routing model are presented. In section 6, the sensitivity analysis performed in the heuristic is presented. The reasoning behind the sensitivity tests led to the formulation of variants of the basic heuristic approach. In this section, two different variants are proposed and experimentally studied, one with the imposition of a floating relaxation on one of the first level objective function values, and the other with the imposition of a floating barrier on one of those values. Finally, conclusions are drawn and future work is outlined in section 7.

2 Review of the Routing Model

This model is an application of the multiobjective modelling framework for MPLS networks described by the authors in [8], where the underlying concepts and methodological considerations are discussed in detail. The framework (or “meta-model”) in [8] considers hierarchical optimisation with up to three optimisation levels: the first priority objective functions are related to the global network level; the second priority objective functions are related to performance metrics for the different types of services supported by the network; the third priority functions refer to performance metrics for the micro-flows of packet streams of the same FEC. Traffic flows in the network are represented in a stochastic form, considering two levels of representation: ‘macro’ level or traffic flow level (corresponding to the MPLS concept of ‘traffic trunk’), and ‘micro’ level (corresponding to micro-flows of packet streams).

Two classes of services are considered: QoS, corresponding to services with guaranteed QoS levels, and BE, where the traffic flows are routed in order to obtain the best possible quality of service but not at the expense of the QoS of the QoS traffic flows (first priority traffic flows). The service types in each class are grouped in the sets $\mathcal{S}_Q$ (for QoS service types) and $\mathcal{S}_B$ (for BE service types), and the traffic flows of each service type $s \in \mathcal{S}_Q$ or $s \in \mathcal{S}_B$ may differ in important attributes, such as the required bandwidth.

The model reviewed here is a simplification of the general model for QoS and BE service classes outlined in [8, sec.3.3] since only the macro level traffic representation was considered. With this simplification, the additional complexity which would result from the inclusion of a third optimisation level in the routing model (associated with the average delays experienced by accepted packet streams), as well as the corresponding additional computational burden, can be avoided. Therefore, a hierarchy of objective functions, with two levels of optimisation with several objective functions in each level, was considered. The first level (first priority) includes objective functions formulated at the network level for the QoS traffic, taking into account the combined effect of all types of traffic flows in the network. The second level is concerned with average performance metrics of the QoS traffic flows associated with the different types of QoS services supported by the network and the expected revenue of the BE traffic.

It is a network-wide routing optimisation approach, which takes into account the nature and relations between the objective functions related to the different types of traffic flows associated with different services. Several multiobjective models previously proposed use objective functions chosen to reflect only indirectly network technical-economic objectives. This sort of approach is just a rough approximation to the implicit objective function the model seeks to reflect, especially taking into account the random nature of traffic patterns, even in stationary or quasi-stationary network working situations. Instead, in this model, the most relevant technical-economic objectives in a network-wide routing optimisation, such as the total expected revenue (expressed in terms of the traffic carried of all service types), are explicitly represented. This aspect of the modelling approach pursues the line of thought adopted by [26, 27, 36, 37], in the context of single-objective routing models.

In this model, 'fairness' objectives are explicitly considered as objective functions at the two levels of optimisation. These are objectives of min-max type and they seek to make the most of the proposed multiobjective formulation. In other formulations of routing models for these networks, such type of aims related to fairness are usually not considered explicitly in any form or just represented through constraints on certain performance metrics.

The traffic flows in the network are represented in a stochastic way, as described in [8]. A traffic flow can be specified by $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}_s)$ for $s \in \mathcal{S} = \mathcal{S}_Q \cup \mathcal{S}_B$ and corresponds to a stochastic process, in general a marked point process. This process describes the arrivals and basic requirements of μ-flows, with their origin at the MPLS ingress node v_i and their destination at the MPLS egress node v_j, using the same LSP and characterised by the vectors of 'attributes' $\overline{\gamma}_s$ and $\overline{\eta}_s$ for service type s. The vector $\overline{\gamma}_s$ describes the traffic engineering attributes of flows of service type s and the vector $\overline{\eta}_s$ enables the representation of the mechanism(s) of admission control to all links l_k in the network by calls of flow f_s. These attributes include information on the required effective bandwidth d_s and the mean duration $h(f_s)$ of each μ-flow in f_s. The concept of effective bandwidth [25] used in the present context (MPLS networks with explicit routes) was earlier proposed in [36] and employed in [30, 31]. The effective bandwidth can be thought of as a stochastic measure of the utilisation of network resources allowing for the representation (in an approximate manner) of the effects of the variability of the rates of different traffic sources, as well as the effects of statistical multiplexing of different traffic flows in a network.

Consider an approximate teletraffic model that enables the calculation of the blocking probabilities $B(f_s)$ for all flows f_s of all service types, from which one can calculate the average blocking probability B_{ms}, for all traffic flows of type s, for a given set of routes for all node to node traffic flows. The maximal average blocking probability among all QoS service types, $B_{Mm|Q}$, is

$$B_{Mm|Q} = \max_{s \in \mathcal{S}_Q} \{B_{ms}\}$$

and it represents the fairness objective at the network level.

The total expected network revenues, W_Q and W_B referring to QoS and BE traffic flows respectively, can be expressed in terms of the expected revenues $w(f_s)$ associated with calls of flows $f_s, \forall s \in \mathcal{S}$ and of the values of carried traffic A_s^c for all service types,

$$W_{Q(B)} = \sum_{s \in \mathcal{S}_{Q(B)}} W_s = \sum_{s \in \mathcal{S}_{Q(B)}} A_s^c w_s$$

A usual simplification will be considered, such that $w(f_s) = w_s, \forall f_s \in \mathcal{F}_s$, where $\mathcal{F}_s$ is the set of traffic flows of type s. The total expected revenue for the traffic flows of QoS type W_Q will be a first priority objective function together with the maximal blocking probability for all QoS service types, $B_{Mm|Q}$, but the total expected revenue for the traffic flows of BE type W_B will be a second level objective function. Therefore, the routing of BE traffic, in a quasi-stationary situation, will not be made at the expense of the decrease in revenue or at the expense of an increase in the blocking probability of QoS traffic flows. Nevertheless, note that while QoS and BE traffic flows are treated separately in terms of objective functions so as to take into account their different priority in the optimisation model, the interactions among all traffic flows are fully represented in the model. This is achieved by the traffic modelling approach underlying the optimisation model, since the traffic model used to estimate the blocking probabilities $B(f_s)$ integrates the contributions of all traffic flows which may use every link of the network. This is a major difference in comparison to other routing models that have been proposed for networks with two service classes.

At the second level of optimisation, apart from the BE traffic revenue, there are $2|\mathcal{S}_Q|$ objective functions related to QoS service types, to be minimised: the mean blocking probability for flows of type $s \in \mathcal{S}_Q$,

$$B_{ms|Q} = \frac{1}{A_s^o} \sum_{f_s \in \mathcal{F}_s} A(f_s) B(f_s)$$

where A_s^o is the total traffic offered by the flows of type s and $A(f_s)$ is the mean traffic offered associated with f_s (in Erlang), and the maximal loss $B_{Ms|Q}$, defined over all flows of type $s \in \mathcal{S}_Q$,

$$B_{Ms|Q} = \max_{f_s \in \mathcal{F}_s} \{B(f_s)\}$$

These constitute the fairness objectives defined for every service type $s \in \mathcal{S}_Q$.

A two-level hierarchical optimisation problem for two service classes can then be formulated:

Problem P-M2-S2

$$\boxed{\begin{array}{ll}
\text{1st level} & \left\{ \begin{array}{ll} \text{QoS - Network objectives:} & \min_{\overline{R}}\{-W_Q\} \\ & \min_{\overline{R}}\{B_{Mm|Q}\} \end{array}\right. \\
\text{2nd level} & \left\{ \begin{array}{ll} \text{QoS - Service objectives:} & \min_{\overline{R}}\{B_{ms|Q}\} \\ & \min_{\overline{R}}\{B_{Ms|Q}\} \\ & \forall s \in \mathcal{S}_Q \\ \text{BE - Network objective:} & \min_{\overline{R}}\{-W_B\} \end{array}\right. \\
\multicolumn{2}{c}{\text{subject to the equations of the underlying traffic model.}}
\end{array}}$$

The acronym P-M2-S2 stands for '**P**roblem - **M**ultiobjective with **2** optimisation hierarchical levels - with **2** **S**ervice classes'. The parameter $\overline{R}$ represents the network routing plans.

The basic calculation sub-model allows for the blocking probabilities B_{ks}, for connections of service type s in link l_k, to be obtained in the form

$$B_{ks} = \mathcal{L}_s \left(\overline{d_k}, \overline{\rho_k}, C_k\right) \tag{2.1}$$

where $\mathcal{L}_s$ is the basic function (implicit in the analytical model) that gives the marginal blocking probabilities, B_{ks}, in terms of $\overline{d_k} = (d_{k1}, \cdots, d_{k|\mathcal{S}|})$ (vector of equivalent effective bandwidths), $\overline{\rho_k} = (\rho_{k1}, \ \cdots, \rho_{k|\mathcal{S}|})$ (vector of reduced traffic loads ρ_{ks} offered by flows of type s to l_k) and the link capacity C_k.

This approximation was suggested in [36] for off-line single-objective multiservice routing optimisation models and was also used in the multiobjective dynamic alternative routing model proposed in [31]. Very efficient and robust approximations have to be used in a routing optimisation model of this type, for tractability reasons.

3 Foundations of the Resolution Approach

Since the dedicated heuristic resolution procedure proposed in this paper uses the theoretical foundations described by the authors in the report [9], these will now be reviewed.

In this hierarchical multiobjective network-wide optimisation routing problem P-M2-S2 the routing principle of alternative routing is used i.e. the decision variables are the network routing plans $\overline{R} = \cup_{s=1}^{|\mathcal{S}|} R(s)$ for all the network services, where $R(s) = \cup_{f_s \in \mathcal{F}_s} R(f_s), s \in \mathcal{S}_Q \cup \mathcal{S}_B$ and $R(f_s) = (r^p(f_s)), p = 1, \cdots, M$ with $M = 2$. Therefore, for each flow f_s the first choice route $r^1(f_s)$ will be used unless it is blocked because one of its links l_k has not got the required available bandwidth d_s (or as prescribed by a general probabilistic availability function ψ_{ks}). If $r^1(f_s)$ is blocked then the second choice route $r^2(f_s)$ will be attempted by the connection request. This request will be blocked only if $r^2(f_s)$ is also blocked.

The high 'complexity' of the routing problem P-M2-S2 stems from two major factors: all objective functions are strongly interdependent (via the $\{B(f_s)\}$), and the objective function parameters and the (discrete) decision variables $\overline{R}$ (network route plans) are also interdependent. All these interdependencies are defined through the underlying traffic model. Regarding overall complexity, note that the simplest, 'degenerated' single objective version of the problem, considering the single objective function W_Q, one single service and no alternative routing ($M = 1$) is NP-complete in the strong sense, as shown in [14]. The present problem is a bi-level, multiobjective extension of this type of problem.

Regarding the possible conflict between the objective functions in P-M2-S2, in many situations, the maximisation of W_Q leads to a deterioration on $B(f_s), s \in \mathcal{S}_Q$, for traffic flows $A(f_s)$ with low intensity, which tends to increase $B_{Ms|Q}$ and, as a result, $B_{Mm|Q}$. In single-objective routing models this effect is usually dealt with by imposing upper bounds on the values $B(f_s)$. The relations between objective functions of this type were analysed in [34]. This is a major factor to justify the interest and potential advantage in using multiobjective approaches when dealing with this type of routing methods.

The resolution (in a multicriteria analysis sense) of the routing problem P-M2-S2 will be performed by a heuristic approach, which is presented in the next section. This heuristic is a new form of the heuristic approach described in [9] and it is based on the calculation of solutions of a bi-objective shortest path problem, formulated for every end-to-end flow f_s:

$$\text{Problem } \mathcal{P}_{s2}^{(2)} : \min_{r(f_s) \in \mathcal{D}(f_s)} \left\{ m^n(r(f_s)) = \sum_{l_k \in r(f_s)} m_{ks}^n \right\}_{n=1;2} \tag{3.1}$$

where the path metrics to be minimised are the marginal implied costs $m_{ks}^1 = c_{ks}^{Q(B)}$ (as defined according to the following analysis) and the marginal blocking probabilities $m_{ks}^2 = -\log(1 - B_{ks})$; $\mathcal{D}(f_s)$ is the set of all feasible loopless paths for flow f_s, which satisfy the traffic engineering constraints for flows of type s, namely imposing a maximal number of arcs per path depending on the type of service s. The logarithmic function is necessary to transform the blocking probability into an additive metric. The link cost coefficients $m^1_{ks} = c^{Q(B)}_{ks}$ are used when one intends to obtain candidate solutions to improve the revenue of the QoS(BE) traffic, depending on the step of the heuristic procedure. According to [13, 31], the comparison of the efficiency of different routes in the context of a multicriteria routing framework of the present type should include both the loss probabilities in the routes and the knock-on effects upon the other routes in the network associated with the acceptance of a call on a given route, which can be assessed by the implied costs.

This auxiliary constrained bi-objective shortest path problem will be used as a basis for the resolution approach to the network problem P-M2-S2 because the metric blocking probability tends (at a network level) to minimise the maximal node-to-node blocking probabilities $B(f_s)$ while the metric implied cost tends to maximise the total average revenue W_T (see [13, 34]). Stating that using the minimisation of path implied cost 'tends' to maximise W_T would be truly only valid if the choice of such 'optimal' path, for a given f_s, would not change in any form the network working conditions concerning all the remaining traffic flows, an assumption that is not true, due to the interdependencies among $\{c_{ks}\}, \{B_{ks}\}$ and $\overline{R}$. This is yet another source of difficulty in devising a heuristic based on this principle, as outlined in the next section.

The implied cost c_{ku} resulting from the acceptance of a call of flow f_u in link l_k is an important mathematical concept in routing optimisation in loss networks and was originally proposed by Kelly [26] for single-rate traffic networks. It was later extended to single route (i.e. without alternative routing) multirate traffic networks in [18, 36]. It can be viewed as the expected value of the loss of revenue in all traffic flows which may use link l_k as a result of the acceptance of a call from f_u associated with the decrease on the capacity of this link. The implied cost measures the knock-on effects on all network routes (of all traffic flows) as a consequence of the acceptance of a call from f_u in a link l_k. In [12, 31], the definition of c_{ku} was adapted to multirate loss networks with alternative routing by extending the expression given for single-service networks in [26]. The extension of this definition to a multi-rate network with alternative routing and two service classes was proposed in the report [9]. For this purpose the following definition of marginal implied costs associated with QoS (BE) traffic was considered. The marginal implied cost for QoS traffic, $c^{Q(B)}_{ku}$, associated with the acceptance of a connection (or "call") of traffic f_u of any service type $u \in \mathcal{S}$ on a link l_k is defined as the expected value of the traffic loss induced on all QoS(BE) traffic flows resulting from the capacity decrease in link l_k.

In [9], a conjecture is presented, where the marginal implied costs for QoS (BE) traffic can be estimated by solving a system of equations

$$c^{Q(B)}_{ku} = \sum_{s \in \mathcal{S}_{Q(B)}} \frac{1}{1 - B_{ks}} \zeta_{kus} \quad \Big[\textstyle\sum_{f_s \in \mathcal{F}_s : l_k \in r^1(f_s)} \lambda_{r^1(f_s)} \left(s^{Q(B)}_{r^1(f_s)} + c^{Q(B)}_{ks} \right) + \\ + \textstyle\sum_{f_s \in \mathcal{F}_s : l_k \in r^2(f_s)} \lambda_{r^2(f_s)} \left(s^{Q(B)}_{r^2(f_s)} + c^{Q(B)}_{ks} \right) \Big] \qquad (3.2)$$

with

$$\begin{aligned}
s^{Q(B)}_{r^2(f_s)} &= w^{Q(B)}(f_s) - \sum_{l_j \in r^2(f_s)} c^{Q(B)}_{js} \\
s^{Q(B)}_{r^1(f_s)} &= w^{Q(B)}(f_s) - \sum_{l_j \in r^1(f_s)} c^{Q(B)}_{js} - (1 - L_{r^2(f_s)}) s^{Q(B)}_{r^2(f_s)} \\
\zeta_{kus} &= \mathcal{L}_s \left(\overline{d_k}, \overline{\rho_k}, C_k - d_{ku}\right) - \mathcal{L}_s \left(\overline{d_k}, \overline{\rho_k}, C_k\right)
\end{aligned}$$

where $s^{Q(B)}_{r^p(f_s)}$ is the surplus value of a call on route $r^p(f_s)$, $\lambda_{r^p(f_s)}$ is the marginal traffic carried on $r^p(f_s)$ by flow f_s, $L_{r^p(f_s)}$ is the blocking probability for calls of f_s on route $r^p(f_s)$ $(p = 1; 2)$ (considering that $r^1(f_s)$ and $r^2(f_s)$ are arc-disjoint paths) and ζ_{kus} is the increase in the congestion for type s calls on link l_k as a result of a decrease in the arc capacity because of the acceptance of a type u call. The marginal expected revenues per call of f_s, $w^{Q(B)}(f_s)$ such that

$$w^Q(f_s) + w^B(f_s) = w(f_s)$$

are given by

$$w^{Q(B)}(f_s) = \alpha^{Q(B)} w(f_s)$$

with coefficients $\alpha^{Q(B)} \in]0.0; 1.0[$ which satisfy the normalisation condition

$$\alpha^Q + \alpha^B = 1.0$$

These marginal revenues may be interpreted as the part of the expected revenue $w(f_s)$ associated with a connection of f_s that is accepted by the network (for a given choice of the pair of routes $(r^1(f_s), r^2(f_s)))$ that corresponds to the calculation of the sensitivity of the revenue from the point of view of traffic losses induced either in the QoS traffic flows or in the BE flows. As a first approach, $\alpha^Q = \alpha^B = 0.5$ will be considered so that no bias is induced in the calculation of the marginal costs through the choice of these factors.

A system of implicit non-linear equations can be formulated in order to calculate the B_{ks} in terms of link capacities (expressed through matrix $\overline{C} = [C_k]$), the offered traffic matrix $\overline{A} = [A(f_s)]$, and the current network routing solution $\overline{R}$,

$$B_{ks} = \beta_{ks}(\overline{B}, \overline{C}, \overline{A}, \overline{R})$$

with $k = 1, \cdots, |\mathcal{L}|; s = 1, \cdots, |\mathcal{S}|$ and $\overline{B} = [B_{ks}]$. As for the calculation of $c^{Q(B)}_{ks}$ through (3.2), it implies the solution of a system of equations of the form

$$c^{Q(B)}_{ks} = \kappa^{Q(B)}_{ks}(\overline{c}, \overline{B}, \overline{C}, \overline{A}, \overline{R})$$

with $\overline{c} = [c^{Q(B)}_{ks}]$. The numerical resolution of the two systems in B_{ks} and $c^{Q(B)}_{ks}$ (in this order) is performed by fixed point iterators, given the matrices $\overline{C}, \overline{A}$ and $\overline{R}$.

The auxiliary constrained shortest path problem $\mathcal{P}^{(2)}_{s2}$ (3.1) will be solved by using the bi-objective constrained shortest path, algorithm MMRA-S2 [9], which is an adaptation of a previously developed algorithmic approach MMRA-S or Modified Multiobjective Routing Algorithm for multiservice networks (described in [12, 31]). Generally speaking, there is no feasible solution which minimises both objective functions of $\mathcal{P}^{(2)}_{s2}$ simultaneously. Since this ideal optimal solution may be unfeasible, the resolution of this routing problem aims at finding a 'best' compromise path from the set of non-dominated solutions, according to some relevant criteria previously defined. Since path computation and selection have to be fully automated, such criteria are embedded in the working of the algorithm MMRA-S2 via preference regions in the objective function space [13, 34].

Solutions to the problem $\mathcal{P}_{s2}^{(2)}$ can therefore be calculated with the MMRA-S2. This version is a variant of the algorithm proposed in [1] for a bi-objective shortest path problem of the same type, with the necessary adaptations to the requirements and specifics of the routing model. The approach presented in [1] was inspired by the one given in [40, 6], which is a procedure to search interactively non-dominated paths. With that approach, non-dominated paths in the framework of a routing control mechanism can be calculated and selected. The procedure integrates the K-shortest paths algorithm [29] and a special concept designated as "soft constraints", which are constraints not directly incorporated into the mathematical model. The main features of this approach are: i) QoS requirements are represented through soft constraints corresponding to requested and acceptable thresholds for each QoS metric; ii) this type of thresholds allows for the definition of priority regions in the objective function space where non-dominated solutions are searched for; iii) the non-dominated paths are calculated with the MPS algorithm, an extremely efficient K-shortest path algorithm [29], in the improved version adapted to paths with a maximum number of arcs, given in [19]; iv) the preference thresholds adapt to the network working conditions.

To review the main features of this algorithmic approach (MMRA-S2) an illustrative example is in figure 3.1, where the metrics $m^1(r(f_s))$ (path implied cost for QoS or BE traffic associated with s type connections) and $m^2(r(f_s))$ (path marginal blocking probability for s type connections) are given according to (3.1)-(3.2).

Firstly the vertex solutions which optimise each metric are computed by solving the two corresponding shortest path problems, leading to the ideal solution O (usually unfeasible) in the objective function space. Dynamic preference thresholds, in the form of requested values (aspiration levels) and acceptable values (reservation levels) are then calculated for both objective functions. These thresholds, also designated as 'soft constraints' (that is, constraints not directly incorported in the mathematical formulation), are obtained from the current average and minimal values of the link blocking probabilities B_{ks} and the marginal link implied costs $c_{ks}^{Q(B)}$ according to the expressions given in [31], and they enable the definition of priority regions in the objective function space, in which non-dominated solutions are searched for. The search for solutions uses, as previously mentioned, a very efficient K-shortest path algorithm with a length constraint (associated with pre-defined QoS requirements, for each service type), given in [19], that is applied to a convex combination of the two metrics.

Region A in figure 3.1 is the first priority region where both requested values for the two metrics are satisfied. In the second priority regions B_1 and B_2 only one of the requested values is satisfied, while the acceptable value for the other function is also met. A further distinction was made between these two regions by establishing a preference order on the objective functions, in this case giving prefence to m^1 (path implied cost) over m^2. This means that if no solutions were found in A, then non-dominated solutions in B_1 would be given preference over those in B_2. For the example in figure 3.1 the first solution (solution 3) found in A would be selected. In other situations, solutions within regions B_1 and B_2 could be found first. In this case, these solutions should be stored but not reported until the entire region A is completely scanned. If there are no non-dominated solutions in A the search should proceed to B_1 and B_2 and solutions previously

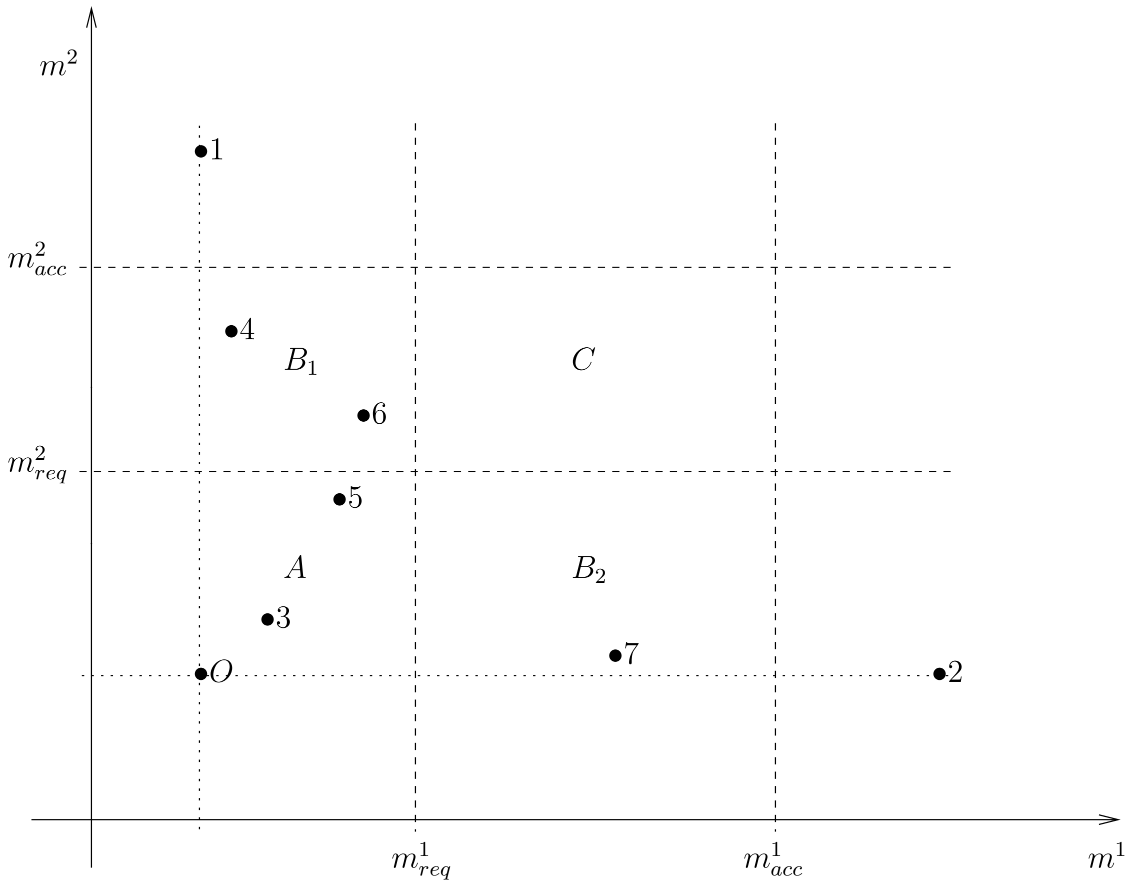

Figure 3.1: Priority regions defined by 'soft constraints'

computed in these regions, if any, should now be reconsidered. Region C, where only the acceptable values are satisfied, is the last priority region to be possibly searched. Finally, note that if MMRA-S2 is used to calculate first and second choice paths $(r^1(f_s), r^2(f_s))$ (at an intermediate step of the heuristic) then, assuming that $r^1(f_s)$ corresponds to solution 3 in the example, it would be justified to consider solution 5 (instead of the non-dominated solutions 4 or 7) as $r^2(f_s)$, although this is dominated by the solution 3, since it is still located in the first priority region.

Finally note the dynamic nature of the preference thresholds, which reflects current network conditions and enables candidate solutions $(r^1(f_s), r^2(f_s))$ to be always found in the defined priority regions.

Another important component of the considered routing optimisation model is the underlying traffic model. This traffic model involves all the sub-models and procedures that are necessary for calculating all traffic related parameters, namely implied costs and blocking probabilities, under certain simplifying assumptions.

A first important element of the traffic modelling approach is the use of the concept of effective bandwidth in association with the definition of MPLS explicit routes. The effective bandwidth of a given traffic flow can be interpreted as a stochastic measure of the consumption of network transmission resources, capable of representing, in a condensed approximate form, the effects of statistical multiplexing of different traffic flows and the variability of the rates of different types of traffic sources. This enables the essential features of packet streams, in MPLS networks using explicit routes, to be 'encapsulated' in a simplified manner. Note that the 'hiding' of packet dynamics in a traffic flow level representation ("macro" level representation) is essential in stochastic network-wide routing optimisation approaches, for tractability reasons. The theoretical foundation and typical applications of the concept of effective bandwidth can be seen in [25]. Examples of the application of this concept in routing optimisation models of multiservice communication networks, are in [36, 31]. In the present context the use of this conceptual tool enables the network to be represented in the traffic plane by a multiclass loss traffic network, equivalent to a multirate traffic circuit-switched network, and it is the basis for the calculation of blocking probabilities in the links. The simplified stochastic model used for calculating the blocking probabilities B_{ks} on a link l_k (equation (2.1)) is a multidimensional Erlang system $M_1 + M_2 + \cdots + M_n/M/C_k/0$ where the input traffic is the superposition of n independent multirate marked Poisson processes associated with all n traffic flows f_s which may use l_k according to the current network routing plan $\overline{R}$. The numerical calculation of the B_{ks} through the implicit function $\mathcal{L}_s$ (equation (2.1)) associated with this model can be made iteratively through the Kaufman/Roberts algorithm [24, 39]. However, for large values of the link capacities C_k, the calculation procedure used for the B_{ks} is based on the uniform asymptotic approximation [35] which is much more efficient, as noted in [36, 31]. It is important to note that the use of very efficient and numerically robust stochastic approximations for traffic related calculations is absolutely critical in a network-wide optimisation model of this type, for tractability reasons, in spite of the errors/inaccuracies inherent to the associated simplifying assumptions.

Further details on the traffic modelling approach used in the present context can be seen in [8].

4 An Improved Heuristic Approach

In this section the features of a heuristic procedure for solving the routing model described in the previous sections are presented. This heuristic (HMOR-S2 or Hierarchical Multiobjective Routing considering 2 classes of service) aims to be an improvement with respect to a previous heuristic procedure, described in [9], based on the same methodological approach.

In this heuristic, a routing solution $\overline{R}(s)$ is sought for each service $s \in \mathcal{S}$, so as to lead to a better performance in terms of W_B, $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$ while respecting the optimisation hierarchy. Network resources are left available for traffic flows of other services so that the solutions selected at each step may improve the higher level objective functions W_Q and $B_{Mm|Q}$. Therefore the heuristic is devised in order to seek, firstly for each QoS service and beginning by the higher bandwidth services (considering the numbering of s, $s = 1, \cdots, |\mathcal{S}_Q|$) and, secondly, for each BE service and beginning by the higher bandwidth services ($s = |\mathcal{S}_Q| + 1, \cdots, |\mathcal{S}|$), solutions which dominate the current one, in terms of $B_{ms|Q}$ and $B_{Ms|Q}$ for QoS services and in terms of W_B for BE services, while not worsening any of the network metrics W_Q and $B_{Mm|Q}$ (taking into account the optimisation priorities in P-M2-S2).

A core idea of the heuristic is the generation of candidate solutions $(r^1(f_s), r^2(f_s))$ for each f_s using MMRA-S2, and their selection (or rejection) according to specific criteria, to be 'tuned' throughout the heuristic execution. A maximal number of arcs D_s per route for each service type s is imposed and a feasible route set $\mathcal{D}(f_s)$ is obtained for each f_s. For real-time QoS services, D_s is equal to the network diameter; for the non-real time QoS services, D_s is the network diameter + 1; for the BE services, no limits are imposed on D_s, so $D_s = N - 1$, where N is the total number of nodes in the network.

Special rules were defined for the selection of candidate first choice routes $r^1(f_s)$ because of the network topology and the need to make a further distinction between real-time QoS services (video and voice services) and non-real time QoS services (such as 'premium data' service). In general, for QoS services, $r^1(f_s)$ is chosen as the direct arc whenever it exists; for BE traffic flows, the direct path, whenever it exists, is treated exactly as any other path, i.e. no preference is given to the direct path over the other feasible paths. For real-time QoS services, if no direct path exists then one of the feasible paths with the least number of arcs is chosen. If there is more than one of these paths, the selection is made according to MMRA-S2, by using the priority regions defined in the objective function space of $\mathcal{P}_{s2}^{(2)}$. These choice criteria are due to the more stringent constraints on delay and jitter of this type of services, and to the need to increase the connections reliability. For the other QoS services the choice of $r^1(f_s)$ is made by using the algorithm MMRA-S2 and its priority regions. A similar procedure is applied for obtaining $r^1(f_s)$ for BE traffic flows. If there are several non-dominated solutions in the "best" possible priority region, the solution with the smallest QoS(BE) implied cost of the path is chosen.

As for the calculation of the second choice routes $r^2(f_s)$ for QoS or BE traffic, the MMRA-S2 algorithm is used. In order to prevent performance degradation in overload conditions, these alternative routes may be eliminated through a mechanism designated as Alternative Path Removal (APR) originally proposed in [32, 31]. Therefore, $r_s = r^2(f_s)$ is eliminated whenever

$$\begin{cases} m^1(r_s) & > & \alpha^{Q(B)} \cdot d_s \cdot z_{APR} \\ m^2(r_s) & > & -\log(0.7) \cdot z_{APR} \end{cases}$$

where $z_{APR} \in [0.0; 1.0]$ is an empirical parameter which was chosen through extensive experimentation with the model. Two cycles for solution search were considered, with $z_{APR} = 1.0$ in the first cycle and $z_{APR} = 0.01 \cdot nPaths$ (this parameter will be defined later). In situations where the traffic load is high, smaller values of z_{APR} give better results; when the traffic load is smaller, higher values of z_{APR} are preferable.

The analysis of the model and experimentation have shown that successive application of MMRA-S2 to every traffic flow does not lead to an effective or robust resolution approach to the network routing problem P-M2-S2. This is because of an instability phenomenon that arises in such path selection procedure, as the route sets $\overline{R}$ tend to oscillate between certain solutions some of which may lead to poor global network performance under the prescribed criteria. This instability phenomenon occurs due to the complexity and interdependencies in the network problem P-M2-S2, namely the interdependencies between $\{c_{ks}^{Q(B)}\}$ and $\{B_{ks}\}$ and between these two sets of parameters and the current network route set $\overline{R}$.

Therefore, at the basis of the heuristic approach (similarly to MODR-S [31]) the subset of the path set $\overline{R}^a = \cup_{s=1}^{|S|} \overline{R}^a(s) : \overline{R}^a(s) = \{(r^1(f_s), r^2(f_s)), f_s \in \mathcal{F}_s\}$ the elements of which should be possibly changed in the next route improvement cycle, is searched for. After detailed analysis and extensive experimentation with the heuristics, a new criterion for choosing candidate paths for possible routing improvement is proposed, by increasing order of a function $\xi(f_s)$ of the current $(r^1(f_s), r^2(f_s))$. The criterion depends explicitly on the first choice path $r^1(f_s)$ and on the second choice path $r^2(f_s)$. The adaptation of this criterion to the present model considers two search cycles, where $\xi(f_s) = F_L(f_s)$ in the first cycle and $\xi(f_s) = F_C^{Q(B)}(f_s)$ in the second cycle, if the effect over QoS(BE) traffic is being considered, and where

$$\begin{aligned} F_C^{Q(B)}(f_s) &= (n_2 - n_1) c_1'^{Q(B)} + c_{r^1(f_s)}^{Q(B)} - c_{r^2(f_s)}^{Q(B)} \\ c_{r(f_s)}^{Q(B)} &= \sum_{l_k \in r(f_s)} c_{ks}^{Q(B)} \\ c_1'^{Q(B)} &= \frac{1}{n_1} \sum_{l_k \in r^1(f_s)} c_{ks}^{Q(B)} = \frac{1}{n_1} c_{r^1(f_s)}^{Q(B)} \\ F_L(f_s) &= 1 - L_{r^1(f_s)} L_{r^2(f_s)} \end{aligned}$$

The aim of the function $F_C^{Q(B)}(f_s)$ is to give preference (concerning the interest in changing the second choice route when seeking to improve W_Q or W_B) to the flows for which the first choice route has a low implied cost and the second choice route has a high implied cost. The factor $(n_2 - n_1)$ was introduced for normalisation purposes as $r^1(f_s)$ has n_1 arcs and $r^2(f_s)$ n_2 arcs, in the considered network. The aim of the function $F_L(f_s)$ is to give preference to the choice of the flows with worse current blocking probability. When overload conditions led to the elimination of the alternative path (see explanation above),

$$\begin{aligned} F_C^{Q(B)}(f_s) &= c_{r^1(f_s)}^{Q(B)} \\ F_L(f_s) &= 1 - L_{r^1(f_s)} \end{aligned}$$

The two metrics (marginal implied cost and blocking probability) should be treated separately, because their ranges are very different.

Another point to be tackled by the heuristic procedure is to specify $nPaths$, which represents the number of routes with smaller values of $\xi(f_s)$ that should possibly be changed by applying MMRA-S2 once again. In order to do so, the effect of each candidate route on the relevant objective functions is anticipated by solving the corresponding analytical model.

4.1 Generic Description of the HMOR-S2 Heuristic

An overview of the present heuristic is presented in this subsection. For a complete formalisation of this heuristic, see the Appendix.

The initial solution has to be carefully chosen, as the quality of the final solution obtained with the heuristic is highly dependent on the quality of the initial one. At the start of the heuristic a set of paths ($r^1(f_s)$) that is typical of Internet routing conventional algorithms is considered. As explained in [9] the path chosen for every flow f_s is the shortest one (that is, the one with minimum number of arcs); if there is more than one shortest path, the one with maximal bottleneck bandwidth (i.e. the minimal capacity of its arcs) is chosen; if there is more than one shortest path with equal capacity, the choice is arbitrary. The initial solution is the same for all the services and the paths are symmetrical.

There are two main (“`for`”) cycles in the heuristic, “`for (`$nPaths$`)`” and “`for (`s`)`”, which are improvement cycles of the objective functions. The heuristic starts off with a “`for (`$nPaths$`)`” cycle, where $nPaths$ defines the current number of paths which are candidates for possible improvements in the improvement cycles of the objective functions. The initial value of $nPaths$ is the total number of flows in the network and the final value is 1. Therefore, the initial strategy is one of ‘diversification’, as the paths of many flows are liable to change, and the final strategy is one of ‘intensification’, as the "good" paths are kept and only a few flows (which still have not got "good" paths) can have their paths changed. One variable within the "`for` ($nPaths$)" cycle, ape, determines the value of z_{APR} to be used in the inner main ("`for`") cycle, which covers all the services, beginning with the QoS services $s = 1, \cdots, |\mathcal{S}_Q|$ and ending with the BE services $s = |\mathcal{S}_Q| + 1, \cdots, |\mathcal{S}|$. As previously mentioned, QoS services are treated in the model as first priority traffic, and BE services as second priority traffic. Within each class of service, the procedure begins with the types of services that have higher bandwidth demands. Experiences have shown that this ordering of the services usually leads to a better performance of the heuristic. Therefore, the heuristic is set up to find, for each service, and starting with the most demanding services, solutions that dominate the current one with respect to the first level objective functions W_Q and $B_{Mm|Q}$, without worsening the partial criteria for each service, $B_{ms|Q}$ and $B_{Ms|Q}$ for QoS services and W_B for BE services. One variable, $nCycles$, determines the expression of $\xi(f_s)$ to be used within this inner main ("`for`") cycle.

Note that the ordering of the execution of the cycles in HMOR-S2 is different from the one in the previous heuristic approach, MOR-S2. As explained in [9], the outer cycle in the MOR-S2 heuristic is "`for` (s)" and for each value of the service s, the value of $nPaths$ changes throughout the inner cycle. This approach did not seem adequate for the P-M2-S2 problem because of the interdependencies among the different services that try to access the network resources. For instance, while an attempt to improve the paths for the first service flows is carried out, the paths for the flows of all the other services remain as in the initial solution, which is hardly the "best" solution that can be found. In the present heuristic HMOR-S2, an attempt to improve the routing plan for each service (one at a time, in a specified order) for a specific value of $nPaths$ is conducted; afterwards, the value of $nPaths$ decreases and a new attempt to improve the routing plan for each service is made, considering as an initial solution the global routing plan that is the "best" up to this point in the algorithm execution.

Concerning the numerical complexity of this heuristic, it can be said that the instructions in the inner cycle of the procedure are executed $C_i^{\text{HMOR-S2}} = 4|\overline{\mathcal{F}}||\mathcal{S}|$ times, where $|\overline{\mathcal{F}}| = \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}} |\mathcal{F}_s|$ is the average number of traffic flows per service. The numerical complexity of the heuristic in terms of the number of solutions that are analysed is $C_s^{\text{HMOR-S2}} = 2|\mathcal{S}||\overline{\mathcal{F}}|\left(|\overline{\mathcal{F}}| + 1\right)$. For comparison, the corresponding numerical complexities of the MOR-S2 heuristic approach (see [10]) are $C_i^{\text{MOR-S2}} \geq |\mathcal{S}|(6|\overline{\mathcal{F}}| - 1)$ and $C_s^{\text{MOR-S2}} \geq |\mathcal{S}||\overline{\mathcal{F}}|(3|\overline{\mathcal{F}}| - 4)$. This means that the HMOR-S2 heuristic involves a significantly lower number of calculations than MOR-S2.

These complexity measures are an indication of the heuristic numerical complexity just at the level of the 'optimisation' procedures. One should note that each calculation of the objective functions and marginal costs (which are used as coefficients in the auxiliary bi-objective shortest path model) involves the numerical resolution of a large system of non-linear equations in $\{B_{ks}\}$ and $\{c_{ks}^{Q(B)}\}$ and such calculations have to be repeated whenever a candidate pair of paths $(r^1(f_s), r^2(f_s))$ is recalculated and analysed, in terms of its impact on the objective function values.

5 Application of the Model

In this section, the computational results obtained through the HMOR-S2 heuristic in a network case study analogous to the one in [37] are presented. To obtain these results, both analytical and simulation experiments were conducted. In the analytical study, the heuristic was run only once, starting with an initial solution typical of Internet routing conventional algorithms, where only shortest path first choice routing is used. For the routing plan obtained at the end of this single run, values for all the objective functions are computed and if the first level objective function values dominate the corresponding values for the initial solution, then this routing plan will be the final solution.

In the discrete event stochastic simulation study, two different approaches are considered. The first one is a simulation applied to a static routing model, where the routing plan is the final solution obtained at the end of the analytical study. After an initialization phase, information on the number of offered calls and effectively carried calls in the network for each flow $f_s, s \in \mathcal{S}$, is gathered, until the end of the simulation. With this information, $B(f_s), \forall s \in \mathcal{S}$ and subsequently, the values of the upper and lower level objective functions related to blocking probabilities can be calculated. As for the revenues, knowing the effectively carried calls in the network allows for the calculation of the carried traffic estimates and the revenues. The second approach is a simulation applied to a dynamic version of the routing model, described in subsection 5.3.2. The routing plan at the beginning of this simulation is the final solution obtained at the end of the analytical study. The initialization phase has two different stages: in the first stage, the estimates of the offered traffic are periodically updated with period τ; in the second stage, this update is performed along with periodical updates of the routing plan, with period $\mathcal{T}$. After the initialization phase, both updates are still performed with the indicated period and information on the number of offered calls and effectively carried calls in the network for each flow $f_s, s \in \mathcal{S}$, is gathered, until the end of the simulation. Taking this information into account, $B(f_s), \forall s \in \mathcal{S}$ and subsequently, the values of the upper and lower level objective functions related to blocking probabilities can be calculated. As for the revenues, knowing the effectively carried calls in the network allows for the calculation of the carried traffic estimates and the revenues.

The analytical results concerning W_Q were compared to results obtained with the previous heuristic MOR-S2 [9] and with the model proposed in [37] for MPLS networks with two service classes that uses a lexicographic optimisation formulation based on a deterministic MCF (Multicommodity Flow) model, which provides an upper bound to our objective function W_Q in P-M2-S2. The network case study for two service classes in MPLS addressed in [37], was also considered in the simulation study using a discrete-event simulation platform, which enabled the validation of the routing model results and the evaluation of the errors intrinsic to the analytical model which provides the estimates for the objective functions.

5.1 Application of the Model to a Network Case Study

In [37] a model for traffic routing and admission control in multiservice networks supporting traffic with different QoS requirements is proposed. For a better understanding of the case study an overview of the relevant features of this model is given. Important properties of the proposed techniques are their scalability and quick response to the changing traffic and network conditions. For this purpose, instead of using stochastic traffic models in the calculation of paths, deterministic models are used, in particular mathematical programming models based on MCFs. These models constitute only a rough approximation in this context and they tend to under-evaluate the blocking probabilities. Therefore, an adaptation of the original model was introduced in [37] to obtain more 'correct' models, that is models which give a better approximation in a stochastic traffic environment. A simple technique is proposed in [37] to adapt the MCF model to a stochastic environment: the requested values of the flows bandwidths in the MCF model are compensated with a parameter $\alpha \geq 0.0$, in order to model the effect of the random fluctuations of the traffic that are typical of stochastic traffic flows. The higher the variability of the point processes of the stochastic traffic model, the higher is the need for compensation and as a result, the higher should α be.

The objective functions of the problem in [37] to be maximised are the revenues W_Q and W_B associated with QoS and BE flows. A bi-criteria lexicographic optimisation problem is formulated, concerning the revenues W_Q and W_B, so that the improvements in W_B are to be found under the constraint that the optimal value of W_Q is kept. To solve this problem a two-stage heuristic procedure based on a MCF formulation, enabling to find the optimal value W_Q, was developed. An admission control mechanism is applied in the first stage of the heuristic. Initially only QoS traffic in the original network $\mathcal{N}$ is taken into account. In the application example presented in [37], all the offered QoS traffic is carried, i.e. all the bandwidth demands are carried with QoS guarantees and an optimal value of W_Q is obtained under these conditions. The remaining capacities in the network arcs are the residual capacities and a residual network $\mathcal{N}'$ is obtained, to which the BE traffic is offered. In the case study example, about 20.67% of the BE traffic bandwidth is not carried. A routing plan for each service is obtained at the end of this first stage. In the second stage of the procedure the MCF-based model is adapted to a stochastic environment. There are two main ideas in this adaptation: the flow rates (input data for the flow-based problem) are inflated to make up for the variabilities in traffic intensities in a stochastic environment, and the MCF-based result is mapped into the adapted model, maintaining the relations between traffic intensities invariant. Traffic splitting is used in this traffic routing problem. This means that the required bandwidth of each flow may be divided by multiple paths from source to destination, allowing for a better load balancing in the network.

In the deterministic flow-based model, a base matrix $T = [T_{ij}]$ with offered bandwidth values from node i to node j [Mbps] is presented. A multiplier $m_s \in [0.0; 1.0]$ with $\sum_{s \in \mathcal{S}} m_s = 1.0$ is applied to these matrix values to calculate the offered bandwidth of each flow f_s to the network,

$$T(f_s) = m_s T_{ij}$$

The adaptation of the MCF model to a stochastic model is based on a compensation mechanism that models the effect of random fluctuations of traffic that are typical of a stochastic traffic model. This compensation mechanism is proposed in [37, eq.(5.1)],

$$\frac{T(f_s)}{d'_s} = A(f_s) + \alpha\sqrt{A(f_s)}$$

with the compensation factor $\alpha \geq 0$. This expression establishes a relation between the bandwidth demand $T(f_s)$ for the MCF model and the parameters $A(f_s)$ and d'_s of the stochastic model. From [37, eq.(5.2)], the average number of μ-flows of f_s offered during the average duration of a μ-flow, can be obtained:

$$A(f_s) \approx \frac{T(f_s)}{d'_s} - \alpha\sqrt{\frac{T(f_s)}{d'_s}} = \frac{m_s T_{ij}}{d_s u_0} - \alpha\sqrt{\frac{m_s T_{ij}}{d_s u_0}} \text{ [Erl]}$$

if

$$\frac{T(f_s)}{d'_s} = \frac{m_s T_{ij}}{d_s u_0} > \alpha^2$$

and both $T(f_s)$ and $A(f_s)$ are high. Otherwise,

$$A(f_s) \approx \frac{T(f_s)}{d'_s} = \frac{m_s T_{ij}}{d_s u_0} \text{ [Erl]}$$

where u_0 is a basic unit of transmission [bit/s].

As an illustrative example, the routing model in [37] is applied to the test network depicted in figure 5.1. In this application example, results for the QoS flows revenue W_Q

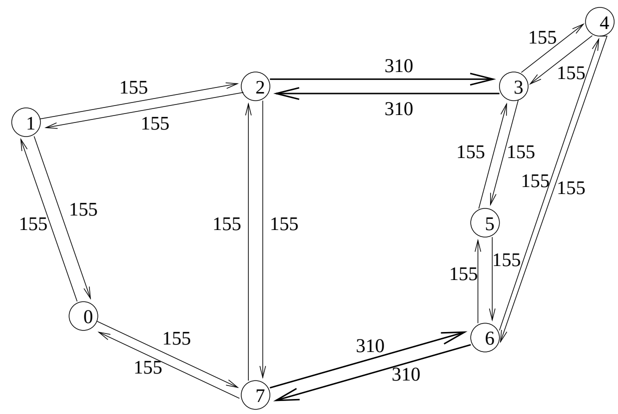

Figure 5.1: Test network M1 [37], with the indication of the bandwidth of each arc C'_k, in Mbps

are presented for three values of α: $\alpha = 0.0$ corresponds to the deterministic situation; $\alpha = 0.5$ is the compensation parameter when calls arrive according to a Poisson process, service times follow an exponential distribution and the network is critically loaded; and $\alpha = 1.0$ is used for traffic flows with higher 'variability'. The results for the BE trsffic revenue W_B are not given, but for $\alpha = 0.0$ the maximum value is 79.33% of the ideal value W_B^{ideal}, because 20.67% of the traffic is not even admitted to the network due to the admission control scheme. The revenue values W_Q in the model [37] should be viewed as upper bounds on the QoS traffic revenue values of the problem P-M2-S2, because of the differences between the two optimisation problems and the differences in the underlying routing control and traffic models. As a matter of fact an important feature of the resolution approach of the routing problem in [37] is the admission control of BE traffic flows at the first stage of resolution so the BE traffic that is actually offered to the network is the fraction of traffic that was not rejected by the admission control. Also note there is no alternative routing, but traffic splitting is used. As a result, for a specific traffic matrix, the model in [37] tends to obtain smaller values of blocking probability by comparison with a situation without admission control, hence it tends to favour somehow higher global revenues. Also the traffic representation, even in the approximated stochastic model is a bit rough, which tends to under-evaluate the blocking probabilities and to over-estimate the revenues.

5.2 Analytical Results

The test network M1 proposed in [37] is represented in figure 5.1. It has $N = 8$ nodes, with 10 pairs of nodes linked by a direct arc and a total of $|\mathcal{L}| = 20$ unidirectional arcs. The bandwidth of each arc C'_k [Mbps] is shown in figure 5.1. The number of channels C_k is $C_k = \left\lceil \frac{C'_k}{u_0} \right\rceil$, with basic capacity $u_0 = 16$ kbps. There are $|\mathcal{S}| = 4$ service types with

Table 5.1: Service features on the test network M1

Service	Class	d'_s [kbps]	d_s [channels]	w_s	h_s [s]	D_s [arcs]	m_s
1 - video	QoS	640	40	40	600	3	0.1
2 - Premium data	QoS	384	24	24	300	4	0.25
3 - voice	QoS	16	1	1	60	3	0.4
4 - data	BE	384	24	24	300	7	0.25

Table 5.2: Initial solution for HMOR-S2 on the test network M1, $\forall s \in \mathcal{S}$

Node	0	1	2	3	4	5	6	7
0	–	0-1	0-1-2	0-1-2-3	0-7-6-4	0-7-6-5	0-7-6	0-7
1	1-0	–	1-2	1-2-3	1-2-3-4	1-2-3-5	1-0-7-6	1-0-7
2	2-1-0	2-1	–	2-3	2-3-4	2-3-5	2-7-6	2-7
3	3-2-1-0	3-2-1	3-2	–	3-4	3-5	3-4-6	3-2-7
4	4-6-7-0	4-3-2-1	4-3-2	4-3	–	4-3-5	4-6	4-6-7
5	5-6-7-0	5-3-2-1	5-3-2	5-3	5-3-4	–	5-6	5-6-7
6	6-7-0	6-7-0-1	6-7-2	6-4-3	6-4	6-5	–	6-7
7	7-0	7-0-1	7-2	7-2-3	7-6-4	7-6-5	7-6	–

the features displayed in table 5.1. The value of $d_s = \frac{d'_s}{u_0}$ [channels] $\forall s \in \mathcal{S}$ in the table (where d'_s is the required bandwidth in kbps) is calculated with $u_0 = 16$ kbps. Note that $w_s = d_s, \forall s \in \mathcal{S}$. The values of $A(f_s)$, the average number of offered μ-flows of f_s, during the average service time of a μ-flow can be calculated as explained earlier.

In HMOR-S2, an initial solution has to be provided as input data to the heuristic. The initial solution is chosen to be the same as in MOR-S2 [9], which is a solution with only one path for each flow, i.e. without an alternative path, leaving it up to the heuristic to find an adequate solution with second choice paths. The initial solution is the same for all the services $s \in \mathcal{S}$ and the paths are symmetrical. The path for every flow f_s is the shortest one (that is, the one with minimum number of arcs); if there is more than one shortest path, the one with maximal bottleneck bandwidth (i.e. the minimal capacity of its arcs) is chosen; if there is more than one shortest path with equal bottleneck bandwidth, the choice is arbitrary. This initial solution is displayed in table 5.2.

The HMOR-S2 heuristic was used to solve the P-M2-S2 problem. The analytical results obtained with the experiments are in table 5.3. In this table the objective function values for both the initial and the final solution for MOR-S2 and HMOR-S2 are displayed, with the best value for each objective function in bold. The value of the QoS traffic revenue in the final solution is also given as a percentage of the optimal value obtained in [37]: $W^{\max}_{Q|\alpha=0.0} = 65156.00$; $W^{\max}_{Q|\alpha=0.5} = 60829.72$; $W^{\max}_{Q|\alpha=1.0} = 56338.65$. The revenue values have 2 decimal places and the blocking probability values have 3 significant figures. These results were obtained in 45 seconds on average in a Linux environment on a Pentium 4 processor with 3GHz CPU and 1GB of RAM.

The HMOR-S2 heuristic starts off with an initial solution (using only shortest path first choice routing, typical of Internet routing conventional algorithms) with poor values for the objective functions and manages to finish with a solution with significantly better

Table 5.3: Objective function values for the initial and the final solution for MOR-S2 and HMOR-S2 on the test network M1, for different values of α

Objective Functions	Initial solution	Final solution MOR-S2	Final solution HMOR-S2
		$\alpha = 0.0$	
W_Q	54803.69	64330.56*	**64731.51**$\diamond$
$B_{Mm\|Q}$	0.413	0.135	**0.0898**
$B_{m1\|Q}$	0.413	0.135	**0.0898**
$B_{m2\|Q}$	0.314	**0.0159**	0.0199
$B_{m3\|Q}$	0.0198	0.00489	**0.00216**
$B_{M1\|Q}$	0.912	0.848	**0.691**
$B_{M2\|Q}$	0.766	**0.0427**	0.0723
$B_{M3\|Q}$	0.0585	0.0456	**0.0287**
W_B	15106.57	**17391.44**	17007.15
		$\alpha = 0.5$	
W_Q	51785.21	60097.78†	**60569.09**•
$B_{Mm\|Q}$	0.413	0.0962	**0.0424**
$B_{m1\|Q}$	0.413	0.0962	**0.0424**
$B_{m2\|Q}$	0.296	0.00811	**0.00534**
$B_{m3\|Q}$	0.0174	0.00263	**0.00119**
$B_{M1\|Q}$	0.882	**0.628**	**0.628**
$B_{M2\|Q}$	0.722	**0.0305**	0.0432
$B_{M3\|Q}$	0.0517	**0.0241**	0.0243
W_B	13787.49	**17031.62**	16904.99
		$\alpha = 1.0$	
W_Q	49010.41	55978.80‡	**56100.60**□
$B_{Mm\|Q}$	0.405	0.0582	**0.0263**
$B_{m1\|Q}$	0.405	0.0582	**0.0263**
$B_{m2\|Q}$	0.275	**0.00279**	0.00515
$B_{m3\|Q}$	0.0150	**0.000436**	0.000560
$B_{M1\|Q}$	0.841	**0.440**	0.544
$B_{M2\|Q}$	0.667	**0.0111**	0.0185
$B_{M3\|Q}$	0.0446	**0.0143**	0.0193
W_B	12445.64	**16509.86**	16479.60

MOR-S2: *)98.73%; †)98.80%; ‡)99.36% of the optimal revenue $W_Q^{\max}$ in [37];
HMOR-S2: $\diamond$)99.35%; •)99.57%; □)99.58% of the optimal revenue $W_Q^{\max}$ in [37]

values, as the values for all the objective functions for all values of α are substantially improved through the heuristic. The QoS traffic revenue of the final solutions are slightly worse than those of the optimal solution in [37], as expected, but these QoS traffic revenues can still be considered very good as they stand for approximately 99% of the optimal values in [37] (upper bounds for the QoS traffic revenue in our model). Note that no admission control is performed in HMOR-S2 and the routing problem is hierarchical multiobjective, as opposed to the two-level lexicographic formulation in [37]. Therefore, HMOR-S2 has managed to find a "good" compromise routing solution to the routing problem P-M2-S2. Comparing the results obtained with the MOR-S2 and the HMOR-S2, the present heuristic clearly produces final solutions with better values for the first level objective functions. Therefore, the heuristic approach HMOR-S2 is considered to be better suited to the P-M2-S2 problem than the MOR-S2.

5.3 Simulation Results

A more realistic assessment of the performance of the HMOR-S2 heuristic can be achieved with the analysis of simulation results. For this purpose, a discrete event stochastic simulator originally described in [22] and further applied in [30, 23] was used. Two types of simulation were performed, one corresponding to a static routing model where the routing plan calculated by the heuristic is never changed regardless of the random variations in offered traffic throughout the simulation, for a given matrix of mean traffic offered in statistical equilibrium. The other corresponds to a periodic type state-dependent dynamic routing model, where the routing plans are updated periodically as a function of real-time traffic measurements, by using the heuristic HMOR-S2 repeatedly, as described in subsection 5.3.2.

5.3.1 Static routing model

Initially, the information on the network topology and the arc capacities of the different arcs, together with the mean offered traffic values $A(f_s)$ for each flow $f_s, s \in \mathcal{S}$, the effective bandwidth d_s and the average service time h_s for a μ-flow of f_s, are loaded onto the simulator. A routing solution, corresponding to the routing plan obtained with the proposed heuristic HMOR-S2 for the three traffic matrices $\overline{A}$ corresponding to the three values of α, is also loaded. Note that this routing plan never changes throughout the simulation for each α.

A first phase of simulation, the initialization phase, lasts for a time designated as 'warm-up' time. Afterwards, information on the number of offered calls and effectively carried calls in the network for each flow $f_s, s \in \mathcal{S}$, is gathered, until the end of the simulation. Using this information, a calculation of $B(f_s)$ estimates $\forall s \in \mathcal{S}$ can be made, as well as a calculation of the values of all the upper and lower level objective functions related to blocking probabilities. As for the revenues, the knowledge of the effectively carried calls in the network allows for the calculation of the carried traffic estimates and hence the calculation of revenues follows straightforwardly. A total of 6 seed files was used, so the number of independent runs that were performed is $R = 6$, for each α. To illustrate the way in which the 95% confidence interval is calculated, take the example of the QoS traffic revenue, W_Q. An estimate of its average value is

$$\hat{W}_Q = \frac{1}{R}\sum_{i=1}^{R} W_{Q|i}$$

and an estimate of its variance is

$$\hat{\sigma}^2(\hat{W}_Q) = \frac{\sum_{i=1}^{R}(W_{Q|i} - \hat{W}_Q)^2}{R(R-1)}$$

where $W_{Q|i}$ is the QoS traffic revenue value for the i-th run, $i = 1, \cdots, R$. Considering a two-sided t-Student distribution, the confidence interval for W_Q is

$$\hat{W}_Q \pm t_{0.025;R-1}\hat{\sigma}(\hat{W}_Q)$$

where the critical value of t is $t_{0.025;R-1} = t_{0.025;5} = 2.57$ (see [3, Table A.4]).

The total simulation time should be long enough to assure the data gathering phase allows for an accurate estimate of the steady state behaviour of the system. The total time considered in these experiments was t_{total} = 48h. Different values were tested for the warm-up time, which should be long enough to guarantee that the system state at the end of the initialization phase is representative of the steady state behaviour of the system. The results displayed in table 5.4 were obtained with $t_{warm-up}$ = 8h. It took about 1h56m to get these results in the same computer mentioned earlier.

5.3.2 Dynamic routing model

Dynamic routing in a telecommunications network is a well known routing principle where the most recent information on the network conditions is taken into account in order to find appropriate paths for the connection requests in the network. This is especially important when there are significant fluctuations of the offered traffic in various parts of the network, in particular as a result of overload or network failures. The dynamic version of the routing method used here is periodic and state-dependent. This means that the network state is assessed periodically and the gathered information on that state is used to periodically choose the most appropriate paths in the network, according to specific routing algorithms. An example of periodic state-dependent routing method is DCR (Dynamically Controlled Routing). Surveys on dynamic routing schemes in different types of networks can be found for example in [2, 7].

A dynamic version of the described routing method of periodic, state-dependent time, is now formulated. The decision variables in the dynamic version of the problem P-M2-S2 $\overline{R}_t$ represent the network routing plans to be used at the time period $t = n\mathcal{T}$ (where $n \in \mathbb{N}$ and $\mathcal{T}$ is the update period of the network routing plans) and are recalculated periodically by the route updating system through application of the heuristic HMOR-S2, and as a function of the network measurement updates. In our dynamic routing model the recalculation of $\overline{R}_t$ is based on new estimates of the mean offered traffic matrix $\overline{A}_t = [A_t(f_s)] = [\tilde{x}_{f_s}(n)]$ with $t = n\tau$ and τ is the update period of the estimates of the offered traffic. A value of $\tau = \mathcal{T}$ has been considered. The routing method seeks to obtain new routing plans adapted to the changing network working conditions resulting from the random fluctuations of traffic intensities. Taking into account the features and great complexity of the routing model, the choice of the "start-up" routing solution of the dynamic method is of great importance concerning its performance. The start-up solution $\overline{R}_{t_0}$ is the routing plan given by the heuristic for the traffic matrix $\overline{A} = [A(f_s)]$ of the static routing model that is the traffic matrix for which the network was initially engineered, assuming stationary traffic conditions. The availability of a good estimate of this initial nominal traffic matrix is a necessary requirement of this dynamic routing method. Having in mind the periodically updated characteristics of the offered traffic, the "best" possible set of paths are chosen so as to improve the multidimensional network performance, as specified by the multiobjective routing model.

To perform the simulations for this multiobjective dynamic routing method, the same simulator as in the static routing method was used. Initially, the information on the network topology and the arc capacities of the different arcs, together with the nominal traffic matrix $\overline{A} = [A(f_s)]$ for each flow $f_s, s \in \mathcal{S}$, the effective bandwidth d_s and the average service time h_s for a μ-flow of f_s, are loaded onto the simulator. The initial routing plan (obtained from the static routing model) is then updated throughout the simulation, in accordance with the periodically updated measures of the traffic offered by all the flows.

A first phase of simulation, the initialization phase, lasts for a time designated as 'warm-up' time, $t_{warm-up} = t_0 + t_1$. In a first stage that lasts t_0, only periodical updates of the estimate of the offered traffic are performed, with period τ. After that time t_0, the offered traffic estimates are assumed to be representative of a steady state behaviour. Afterwards, during a time t_1, the estimate of the offered traffic is still performed with a period τ, along with periodical updates of the routing plan, with period $\mathcal{T}$. The considered time τ between the calculation of estimates of the offered traffic and the time $\mathcal{T}$ between the calculation of routing plans were the same, in particular $\tau = \mathcal{T} = 15$m, as in [30]. After the warm-up time, both updates are still performed with the indicated period and based on information on the number of offered calls and effectively carried calls in the network for each flow $f_s, s \in \mathcal{S}$ gathered from real-time measurements, until the end of the simulation. Using this information, a calculation of $B(f_s)$ estimates $\forall s \in \mathcal{S}$ can be made, as well as a calculation of the values of all the upper and lower level objective functions related to blocking probabilities. As for the revenues, the knowledge of the effectively carried calls in the network allows for the calculation of the carried traffic estimates, hence the calculation of revenues follows straightforwardly. Different values for the times t_0 and t_1 were tried, and the results displayed in table 5.4 were obtained with $t_0 = t_1 = 4$h for a total simulation time of 48h. It took 4h13m on average to get these results in the same computer mentioned earlier.

The offered traffic estimate is calculated according to the model in [12]. In the time interval $[n\tau; (n+1)\tau[$, the estimate of the average traffic offered to the network by the flow f_s, is given by

$$\tilde{x}_{f_s}(n) = (1-b)\tilde{x}_{f_s}(n-1) + b\tilde{X}_{f_s}(n-1)$$

where $\tilde{X}_{f_s}(n-1)$ is an estimator of the average value of the traffic offered by f_s during the previous interval $[(n-1)\tau; n\tau[$, and $b \in]0.0; 1.0[$ is a parameter for which a value has to be adequately chosen, having in mind a compromise between the stability of the estimate and the quick response to changes in the partial estimate in the previous interval, $\tilde{X}_{f_s}$ [26]. Different values of b were tested, and the results displayed in table 5.4 are the ones obtained with $b = 0.7$. Further details on the estimation procedures are in [12].

The calculation of the routing plan in an update instant $n\mathcal{T}$ is made by the heuristic HMOR-S2, considering as an initial solution the routing plan used in the previous time interval $[(n-1)\mathcal{T}; n\mathcal{T}[$. The routing plan determined in $n\mathcal{T}$ will remain valid during the n-th interval $[n\mathcal{T}; (n+1)\mathcal{T}[$. Note that the process of path choice should take a short time, when compared to $\mathcal{T}$.

5.3.3 Simulation results

In the table 5.4, the analytical values of each objective function are displayed, together with the simulation results (average value ± half length of the 95% confidence interval) for these functions. The best simulation results (statistical estimate of the function value) obtained for each function are indicated in bold. Furthermore, if some simulation result is better than the corresponding analytical value, this is indicated in italic. The revenue values have 2 decimal places and the blocking probability values have 3 significant figures. In order to know the uncertainty level associated with the objective function values, the ratio $\frac{\text{half length of the 95\% confidence interval}}{\text{average value}} \cdot 100\%$ is also displayed in the table.

Generally speaking, the analytical results obtained with the HMOR-S2 in most cases are not inside the 95% confidence interval of the static routing model simulation results, although they are of similar magnitudes. The analytical results tend to be better than the corresponding static routing model simulation results. This is especially noticeable in situations of lower traffic loads (i.e. for higher values of α in our routing problem application example). In fact, only for $\alpha = 0.0$ did we get a result where a first level objective function analytical value was in the corresponding confidence interval and had a worse value than the corresponding static routing model simulation result. The differences between the analytical and the simulation results for the static routing model are mainly due to the inaccuracies intrinsic to the analytic/numerical resolution, namely those associated with the simplifications of the traffic model, and the associated error propagation. In fact, the analytical model is a simplification which tends to underestimate the blocking probabilities in the network (and to overestimate the revenues), because the overflow traffic is treated as Poisson traffic, an error that is propagated throughout the complex and lengthy numerical calculations associated with the resolution of the traffic model, involving the resolution of large systems of implicit non-linear equations. Also in the stochastic model for the traffic in the links, a superposition of independent Poisson flows and an independent occupation of the links were assumed as a further simplification. A more accurate and realistic representation of the traffic flows for example based on their average and variance values (see, for example, [11]) would allow for better estimates of the blocking probabilities. However, the approximations in our model can be deemed appropriate in this context for practical reasons: if more complex models were used to represent the traffic and to calculate the blockings in overflow conditions, the computational burden would be too heavy as the analytical model has to be numerically solved many times in the course of the execution of the heuristic and the routing method would be intractable. Note that the accuracy of the results of the traffic model parameters is very important just in relative terms, rather than in terms of absolute values. In fact, the aim of the routing 'optimisation' procedure is the comparison of different routing solutions in terms of various objective functions obtained with the same traffic model, rather than to obtain very accurate results for those functions.

Comparing the analytical results obtained with the HMOR-S2 and the dynamic routing model simulation results for the objective functions of the first level W_Q and $B_{Mm|Q}$, for $\alpha = 0.0; 0.5$ the 95% confidence intervals either include the corresponding analytical value or are better than the corresponding analytical value, which clearly is a good result. In fact, on the one hand the analytical model, as explained earlier, tends to overestimate the objective functions to be maximised and on the other hand, its results correspond to a static routing solution obtained in perfectly stationary conditions. So, in these cases, the improvement in routing solutions obtained by the dynamic model enables the out-

Table 5.4: Average objective function values, and 95% confidence intervals, for simulation of the static (considering a warm-up time of 8h) and the dynamic routing model (considering $t_0 = t_1 = 4$h, updates every 15m and $b = 0.7$), a total simulation time of 48h on the test network M1, for different values of α, when the HMOR-S2 was used to update the routing plan

Objective Functions	Analytical Results	Simulation Results: Static Model	Simulation Results: Dynamic Model
		$\alpha = 0.0$	
W_Q	64731.51	64642.53±64.17(0.10%)	**64728.91 ± 70.22**(0.11%)
$B_{Mm\|Q}$	0.0898	*0.0887±0.00336*(3.79%)	***0.0796±0.00347***(4.35%)
$B_{m1\|Q}$	0.0898	*0.0887±0.00336*(3.79%)	***0.0796±0.00347***(4.35%)
$B_{m2\|Q}$	0.0199	0.0246±0.000647(2.63%)	0.0246±0.00101(4.11%)
$B_{m3\|Q}$	0.00216	0.00226±0.0000663(2.93%)	***0.00207±0.0000652***(3.15%)
$B_{M1\|Q}$	0.691	***0.684±0.00802***(1.17%)	*0.688±0.0190*(2.77%)
$B_{M2\|Q}$	0.0723	**0.0843 ± 0.00242**(2.87%)	0.0983±0.00744(7.56%)
$B_{M3\|Q}$	0.0287	**0.0291 ± 0.000206**(0.71%)	0.0294±0.000528(1.80%)
W_B	17007.15	16982.33±37.02(0.22%)	***17172.13±43.11***(0.25%)
		$\alpha = 0.5$	
W_Q	60569.09	60491.22±50.79(0.08%)	***60637.17±60.26***(0.10%)
$B_{Mm\|Q}$	0.0424	0.0460±0.00163(3.54%)	***0.0328±0.00162***(4.92%)
$B_{m1\|Q}$	0.0424	0.0460±0.00163(3.54%)	***0.0328±0.00162***(4.92%)
$B_{m2\|Q}$	0.00534	0.00809±0.000328(4.06%)	**0.00586 ± 0.000590**(10.07%)
$B_{m3\|Q}$	0.00119	0.00126±0.0000403(3.20%)	***0.000904±0.0000397***(4.39%)
$B_{M1\|Q}$	0.628	0.631±0.0151(2.40%)	***0.621±0.0255***(4.10%)
$B_{M2\|Q}$	0.0432	0.0503±0.00266(5.29%)	***0.0190±0.00151***(7.92%)
$B_{M3\|Q}$	0.0243	0.0245±0.000196(0.80%)	***0.0239±0.000160***(0.67%)
W_B	16904.99	**16899.02 ± 38.69**(0.23%)	16848.00±66.68(0.40%)
		$\alpha = 1.0$	
W_Q	56100.60	56027.72±46.92(0.08%)	**56027.98 ± 62.17**(0.11%)
$B_{Mm\|Q}$	0.0263	**0.0281 ± 0.00126**(4.48%)	0.0282±0.00187(6.65%)
$B_{m1\|Q}$	0.0263	**0.0281 ± 0.00126**(4.48%)	0.0282±0.00187(6.65%)
$B_{m2\|Q}$	0.00515	**0.00832 ± 0.000685**(8.23%)	0.00839±0.000942(11.22%)
$B_{m3\|Q}$	0.000560	0.000637±0.0000154(2.42%)	**0.000587 ± 0.0000429**(7.31%)
$B_{M1\|Q}$	0.544	0.547±0.0281(5.13%)	***0.456±0.0717***(15.74%)
$B_{M2\|Q}$	0.0185	0.0325±0.00353(10.88%)	0.0325±0.00406(12.49%)
$B_{M3\|Q}$	0.0193	0.0195±0.000167(0.86%)	***0.0155±0.00273***(17.55%)
W_B	16479.60	16453.09±17.05(0.10%)	***16481.27±50.90***(0.31%)

performance of the (static) upper bounds of the objective function values provided by the analytical model. However, for $\alpha = 1.0$ the 95% confidence intervals have values slightly worse than the corresponding analytical value. In this case the results of the static and dynamic routing models are quite similar. This means that in this region of lower blocking probabilities the dynamic routing model is not capable of attaining the performance values corresponding to the analytic upper bounds for the static solution. The explanation for this relies on the fact that the errors inherent to the analytic/numerical model tend to be higher for lower traffic loads (i.e. for higher values of α in our routing problem application example) because in this situation, there is an increased tendency to underestimate the blocking probabilities and overestimate the objective function values to be maximised. Furthermore the statistical uncertainty in the simulation results tends to increase in this situation of lower blocking probabilities, because even for hundreds of calls of certain flows, the blocking of a call may be a rare event. Remember that the simulation results for the dynamic routing model are average values of performance in a great number of routing update intervals, while the analytical results are obtained in ideal steady state traffic conditions.

Comparing the static and the dynamic routing model results for the objective functions of the first level W_Q and $B_{Mm|Q}$, for $\alpha = 0.0; 0.5$ the average results are always better with the dynamic routing model. For $\alpha = 1.0$, the average results of the static and the dynamic routing model are almost the same, as noted above. A final remark on the confidence intervals for each objective function: their length is of the same order for both the static and the dynamic routing model.

In global terms and as expected, the results obtained with the dynamic routing model are better (or approximately the same in the worst case) than those obtained with the static routing model, and in situations of higher traffic loads the dynamic model is even capable of outperforming the results anticipated by the analytical model. This shows that the dynamic model is well calibrated for these networks, in terms of the choice of the initial routing solutions to be used by the heuristic and the choice of the routing updating period. In the dynamic routing model, the routing plan is adjusted throughout the simulation run, in accordance with the traffic random fluctuations around the average values corresponding to the nominal traffic matrix $\overline{A}$ that was defined in steady state conditions.

6 Variants of the Heuristic

The study of the heuristic approach HMOR-S2 was completed with a sensitivity analysis, which led to the consideration of variants of this heuristic. In this section, the sensitivity tests and the obtained results are described, followed by the description of the variants to the HMOR-S2. In one of these variants, HMOR-S2$_{\mathrm{R}}$, a floating relaxation is imposed on one of the first level objective function values, either the QoS traffic revenue in the HMOR-S2$_{\mathrm{R}-\Delta W}$ variant or the blocking probability $B_{Mm|Q}$ in the HMOR-S2$_{\mathrm{R}-\Delta B}$ variant. In the other variant, HMOR-S2$_{\mathrm{L}}$, a floating barrier is imposed on one of the first level objective function values, giving the HMOR-S2$_{\mathrm{L}-\Delta W}$ variant or the HMOR-S2$_{\mathrm{L}-\Delta B}$ variant.

6.1 Sensitivity Analysis

The purpose of the sensitivity tests applied to the HMOR-S2 heuristic is to check whether the heuristic is treating the lower level objective functions in a balanced way (that is, to check whether better values of the second level objective functions can be obtained without worsening the values of the first level objective functions) and to check whether the value of an upper level objective function can be improved at the cost of worsening the value of the other upper level objective function.

In the first set of tests, either an upper bound was imposed on one of the blocking probability functions B_{ms} or B_{Ms}, $s \in \mathcal{S}_Q$, or a lower bound was imposed on the BE traffic revenue W_B, $s \in \mathcal{S}_B$. These bounds constitute barriers, in the sense that they are more demanding than the corresponding values obtained at the end of the HMOR-S2 run.

In the second set of tests (relaxation tests), the focus is on the first level objective functions. In one of the tests, the blocking function $B_{Mm|Q}$ is no longer treated as an objective function and an upper bound on its value is imposed. This upper bound is less demanding than the corresponding value $[B_{Mm|Q}]_{\text{basis}}$ obtained at the end of the HMOR-S2 run. The purpose of this test is to check whether the QoS traffic revenue can be improved by relaxing the value of the other main objective function. In the other test, the QoS traffic revenue W_Q is no longer treated as an objective function and a lower bound on its value is imposed. This lower bound is less demanding than the corresponding value $[W_Q]_{\text{basis}}$ obtained at the end of the HMOR-S2 run. The purpose of this test is to check whether the blocking function $B_{Mm|Q}$ can be improved when the value of the other objective function is relaxed.

As mentioned before, the numerical calculations performed in the HMOR-S2 heuristic are quite complex and because of the simplifications assumed in the traffic model underlying the heuristic and numerical errors associated with the resolution of large systems of non-linear equations, significant inaccuracies are expected. Therefore, in the sensitivity analysis that was carried out, different revenue values are considered the same if they differ less than 0.5%. So, as long as a revenue value W_Q is such that $\left|\frac{W_Q-[W_Q]_{\text{basis}}}{[W_Q]_{\text{basis}}}\right| \leq 0.5\%$ then the QoS traffic revenue is considered to be the same as the one obtained when the basic heuristic (i.e. the HMOR-S2 heuristic without any sensitivity analysis) was run. Likewise, for the blocking probabilities, different values are those who differ by more than 5%. Note that ususally the revenue values vary less (in terms of percentage) than the blocking probability values.

Generally speaking, the results of the sensitivity tests for the HMOR-S2 heuristic were as expected, allowing us to assume that the heuristic is balanced in the treatment of the different objective functions. Nonetheless, there are a few results that are worth mentioning.

In the first set of tests, one or both of the upper level objective function values were worse when a barrier (i.e. a stricter value) was imposed on one of the lower level blocking probability functions or BE traffic revenue. That is, when the improvement of one of the lower level functions is imposed, the upper level objective function values tend to be worse (at least for one of those functions). There was however one situation where one of the first level objective function improved and the other worsened. This result is not unexpected, as the two first level objective functions are conflicting in nature, but showed that there was one non-dominated solution that the basic heuristic was not able to detect so far.

In the second set of tests, in one of the sensitivity tests where the upper level objective function $B_{Mm|Q}$ ceased to be treated in the heuristic as an objective function and a relaxed upper bound was imposed on its value, a final solution with slightly better values for both $B_{Mm|Q}$ and W_Q was obtained. Therefore, in spite of allowing the value of $B_{Mm|Q}$ to increase beyond the value obtained when the basic heuristic was run, it actually diminished, and there was a slight improvement of the QoS traffic revenue. This result suggests that, in some rare cases, the heuristic is not capable of finding a solution that slightly dominates the current selected solution.

6.2 Technique of the Floating Relaxation

The apparently odd result obtained in one of the second set of sensitivity tests led to the implementation of HMOR-S2$_{\mathrm{R}}$, a variant of the basic heuristic HMOR-S2, that tries to explore the feature of the heuristic that allows the value of one of the upper level functions to worsen beyond the value obtained when the basic heuristic was run, and yet both upper level functions end up having better values. In this heuristic, one of the upper level functions ceases to be an objective function and a floating bound is imposed on its value. This technique is one of relaxation, because the imposed bound allows the function to worsen its value. The imposed bound is a floating one because it is associated with a percentage of the value of the function in the most adequate solution found up to a specific instant in the heuristic run, i.e. $\max\{W_Q\}$ or $\min\{B_{Mm|Q}\}$.

Two sets of experiences were conducted, each one taking into account a specific first level objective function, either the QoS traffic revenue or the $B_{Mm|Q}$ blocking probability. In this variant, either the QoS traffic revenue criterion or the $B_{Mm|Q}$ criterion for the acceptance of a solution is less stringent than in the basic heuristic HMOR-S2.

In the heuristic HMOR-S2$_{\mathrm{R}-\Delta W}$, a percentage $0 < \Delta W < 1$ of the current best value of W_Q, $\max\{W_Q\}$, was considered and in the points VII.1(b)iF and VII.1(b)iG of the heuristic formalised in the Appendix, the condition $W_Q > \max\{W_Q\}$ is replaced by $W_Q > (1 - \Delta W)\max\{W_Q\}$. For instance, if $\Delta W = 0.5\%$, this implementation will allow the QoS traffic revenue value to worsen down to $1 - \Delta W = 99.5\%$ of its value in the most adequate solution found so far in the heuristic run ($\max\{W_Q\}$), whereas in the basic heuristic the QoS traffic revenue W_Q had to be better than $\max\{W_Q\}$ for the current solution to be considered as the most adequate. The other conditions for the current solution to be considered as the most adequate remain the same, that is, the other upper level objective function value $B_{Mm|Q}$ and the lower level objective function values for the service being analysed, B_{ms} and B_{Ms} if $s \in \mathcal{S}_Q$ or W_B if $s \in \mathcal{S}_B$, have to improve.

In the heuristic HMOR-S2$_{\mathrm{R}-\Delta B}$, a percentage $0 < \Delta B < 1$ of the current $\min\{B_{Mm|Q}\}$ was considered and in the points VII.1(b)iF and VII.1(b)iG of the heuristic formalised in the Appendix, the condition $B_{Mm|Q} < \min\{B_{Mm|Q}\}$ is replaced by $B_{Mm|Q} < (1 + \Delta B)\min\{B_{Mm|Q}\}$. For instance, if $\Delta B = 2.0\%$, this implementation will allow the $B_{Mm|Q}$ value to worsen up to $1 + \Delta B = 102.0\%$ of its value in the most adequate solution found so far in the heuristic run ($\min\{B_{Mm|Q}\}$), whereas in the basic heuristic the $B_{Mm|Q}$ value had to be better than $\min\{B_{Mm|Q}\}$ for the current solution to be considered as the most adequate. Similarly to HMOR-S2$_{\mathrm{R}-\Delta W}$, the other conditions for the current solution to be considered as the most adequate remain the same, that is, the other upper level objective function value and the lower level objective function values for the service being analysed have to improve.

Computational results were obtained through the heuristic HMOR-S2$_{\mathrm{R}}$ in the same network case study that was used for the experiments of HMOR-S2. Only analytical experiments were conducted. As the final analytical results were never better than the results after the HMOR-S2 run, no need was felt to conduct simulation experiments for this variant HMOR-S2$_{\mathrm{R}}$. In the analytical study, the heuristic was run only once, starting with the same initial solution used in the basic heuristic runs, which is typical of Internet routing conventional algorithms. The experiences were conducted with different values of ΔW and ΔB, and the final results were quite similar regardless of the chosen value. An example of the results is displayed in table 6.1. The results with the basic heuristic and each of the variants are compared, with the best value for each objective function presented in bold. For instance, for $\alpha = 0.0$, both variants present better results for $B_{M1|Q}$ than the basic heuristic. These results were obtained for HMOR-S2$_{\mathrm{R}\text{–}\Delta W=0.5\%}$ and HMOR-S2$_{\mathrm{R}\text{–}\Delta B=2.0\%}$.

As the results displayed in the table show, for $\alpha = 0.5; 1.0$ the objective function values of the final solution were the same as the ones obtained with the basic heuristic. The same situation occured for all the considered values of ΔW and ΔB. For $\alpha = 0.0$, the results for the first level objective functions were worse than the ones obtained with the HMOR-S2. No further experiments were conducted with this variant, because it did not manage to find solutions "better" than the ones obtained with the basic heuristic.

6.3 Technique of the Floating Barrier

Another variant, HMOR-S2$_{\mathrm{L}}$, was considered but this time, a bound (or barrier) is imposed on one of the upper level objective functions that is more demanding than the one corresponding to the current most adequate solution. Again the imposed bound is a floating one because it is associated with a percentage of the value of the function in the most adequate solution found up to a specific instant in the heuristic run, i.e. $\max\{W_Q\}$ or $\min\{B_{Mm|Q}\}$. Two sets of experiments were conducted, each one focusing on a specific first level objective function, either the QoS traffic revenue or the $B_{Mm|Q}$ blocking probability. In this variant, either the criterion on the QoS traffic revenue or the criterion on $B_{Mm|Q}$ for the acceptance of a solution is more stringent than in the basic heuristic HMOR-S2.

In the heuristic HMOR-S2$_{\mathrm{L}\text{–}\Delta W}$, a percentage $0 < \Delta W < 1$ on the value $\max\{W_Q\}$ was considered and in the points VII.1(b)iF and VII.1(b)iG of the heuristic formalised in the Appendix, the condition $W_Q > \max\{W_Q\}$ is replaced by $W_Q > (1 + \Delta W)\max\{W_Q\}$. For instance, if $\Delta W = 0.5\%$, this implementation will demand the QoS traffic revenue value to be at least $1 + \Delta W = 100.5\%$ of the value for the most adequate solution found so far in the heuristic run ($\max\{W_Q\}$). The other conditions for the current solution to be considered as the most adequate remain the same.

In the heuristic HMOR-S2$_{\mathrm{L}\text{–}\Delta B}$, a percentage $0 < \Delta B < 1$ of the current $\min\{B_{Mm|Q}\}$ was considered and in the points VII.1(b)iF and VII.1(b)iG of the heuristic formalised in the Appendix, the condition $B_{Mm|Q} < \min\{B_{Mm|Q}\}$ is replaced by $B_{Mm|Q} < (1 - \Delta B)\min\{B_{Mm|Q}\}$. For instance, if $\Delta B = 2.0\%$, this implementation will require the value of $B_{Mm|Q}$ to improve and be at most $1 - \Delta B = 98.0\%$ of the value in the most adequate solution found so far in the heuristic run ($\min\{B_{Mm|Q}\}$). Similarly to HMOR-S2$_{\mathrm{L}\text{–}\Delta W}$, the other conditions for the current solution to be considered as the most adequate remain the same.

In this variant, the same solution is kept throughout the algorithm unless there is a significant improvement in one of the upper level functions. Note that the values for ΔW and ΔB should not be too high, or the algorithm may become too stringent and prevent

Table 6.1: Objective function values for the final solution for HMOR-S2, HMOR-S2$_{\mathrm{R}-\Delta W=0.5\%}$ and HMOR-S2$_{\mathrm{R}-\Delta B=2.0\%}$, on the test network M1, for different values of α

Objective Functions	$\alpha = 0.0$ HMOR-S2 (Basis)	HMOR-S2$_{\mathrm{R}}$ $\Delta W = 0.5\%$	HMOR-S2$_{\mathrm{R}}$ $\Delta B = 2.0\%$
W_Q	**64731.51***	64461.71$\diamond$	64574.12$\star$
$B_{Mm\|Q}$	**0.0898**	0.108	0.101
$B_{m1\|Q}$	**0.0898**	0.108	0.101
$B_{m2\|Q}$	**0.0199**	0.0245	0.0221
$B_{m3\|Q}$	**0.00216**	0.00240	0.00253
$B_{M1\|Q}$	0.691	**0.661**	**0.690**
$B_{M2\|Q}$	**0.0723**	0.102	0.0728
$B_{M3\|Q}$	0.0287	**0.0265**	0.0287
W_B	17007.15	**17982.66**	16986.87

Objective Functions	$\alpha = 0.5$ HMOR-S2 (Basis)	HMOR-S2$_{\mathrm{R}}$ $\Delta W = 0.5\%$	HMOR-S2$_{\mathrm{R}}$ $\Delta B = 2.0\%$
W_Q	60569.09†	60569.09$\bullet$	60569.09$\odot$
$B_{Mm\|Q}$	0.0424	0.0424	0.0424
$B_{m1\|Q}$	0.0424	0.0424	0.0424
$B_{m2\|Q}$	0.00534	0.00534	0.00534
$B_{m3\|Q}$	0.00119	0.00119	0.00119
$B_{M1\|Q}$	0.628	0.628	0.628
$B_{M2\|Q}$	0.0432	0.0432	0.0432
$B_{M3\|Q}$	0.0243	0.0243	0.0243
W_B	16904.99	16904.99	16904.99

Objective Functions	$\alpha = 1.0$ HMOR-S2 (Basis)	HMOR-S2$_{\mathrm{R}}$ $\Delta W = 0.5\%$	HMOR-S2$_{\mathrm{R}}$ $\Delta B = 2.0\%$
W_Q	56100.60‡	56100.60$\square$	56100.60$\otimes$
$B_{Mm\|Q}$	0.0263	0.0263	0.0263
$B_{m1\|Q}$	0.0263	0.0263	0.0263
$B_{m2\|Q}$	0.00515	0.00515	0.00515
$B_{m3\|Q}$	0.000560	0.000560	0.000560
$B_{M1\|Q}$	0.544	0.544	0.544
$B_{M2\|Q}$	0.0185	0.0185	0.0185
$B_{M3\|Q}$	0.0193	0.0193	0.0193
W_B	16479.60	16479.60	16479.60

HMOR-S2: *)99.35%; †)99.57%; ‡)99.58% of the optimal revenue $W_Q^{\max}$ in [37];
HMOR-S2$_{\mathrm{R}-\Delta W=0.5\%}$: $\diamond$)98.93%; $\bullet$)99.57%; $\square$)99.58% of the optimal revenue $W_Q^{\max}$ in [37];
HMOR-S2$_{\mathrm{R}-\Delta B=2.0\%}$: $\star$)99.11%; $\odot$)99.57%; $\otimes$)99.58% of the optimal revenue $W_Q^{\max}$ in [37]

Table 6.2: Objective function values for the final solution for HMOR-S2 and HMOR-S2$_{\mathrm{L}-\Delta B=2.0\%}$, on the test network M1, for different values of α

Objective Functions	$\alpha = 0.0$ HMOR-S2 (Basis)	HMOR-S2$_\mathrm{L}$ $\Delta B = 2.0\%$	$\alpha = 0.5$ HMOR-S2 (Basis)	HMOR-S2$_\mathrm{L}$ $\Delta B = 2.0\%$	$\alpha = 1.0$ HMOR-S2 (Basis)	HMOR-S2$_\mathrm{L}$ $\Delta B = 2.0\%$
W_Q	64731.51*	64731.51$\diamond$	**60569.09**†	60561.38$\bullet$	56100.60‡	**56108.98**$\square$
$B_{Mm\|Q}$	0.0898	0.0898	**0.0424**	0.0435	0.0263	**0.0253**
$B_{m1\|Q}$	0.0898	0.0898	**0.0424**	0.0435	0.0263	**0.0253**
$B_{m2\|Q}$	0.0199	0.0199	0.00534	**0.00531**	0.00515	**0.00497**
$B_{m3\|Q}$	0.00216	0.00216	**0.00119**	0.00121	0.000560	**0.000556**
$B_{M1\|Q}$	0.691	0.691	0.628	**0.623**	**0.544**	0.556
$B_{M2\|Q}$	0.0723	0.0723	0.0432	**0.0422**	0.0185	**0.0178**
$B_{M3\|Q}$	0.0287	0.0287	0.0243	**0.0240**	**0.0193**	0.0200
W_B	17007.15	17007.15	**16904.99**	16894.85	**16479.60**	16466.04

HMOR-S2: *)99.35%; †)99.57%; ‡)99.58% of the optimal revenue $W_Q^{\max}$ in [37];
HMOR-S2$_{\mathrm{L}-\Delta B=2.0\%}$: $\diamond$)99.35%; $\bullet$)99.56%; $\square$)99.59% of the optimal revenue $W_Q^{\max}$ in [37]

us from finding solutions better than the initial one.

Computational results were obtained through the HMOR-S2$_\mathrm{L}$ heuristic in the same network case study that was used for the experiments of HMOR-S2. Analytical and simulation experiments were conducted. In the analytical study, the heuristic was run only once, starting with the same initial solution used in the basic heuristic runs, which is typical of Internet routing conventional algorithms. The experiences were conducted with different values of ΔW and ΔB. The HMOR-S2$_{\mathrm{L}-\Delta W}$ presented good analytical results, so simulation experiments were also conducted on this variant.

An example of the analytical results is displayed in tables 6.2 and 6.3. The results with the basic heuristic and each of the variants are compared, with the best value for each objective function in bold. These results were obtained for HMOR-S2$_{\mathrm{L}-\Delta B=2.0\%}$ and for HMOR-S2$_{\mathrm{L}-\Delta W=0.5\%}$ and HMOR-S2$_{\mathrm{L}-\Delta W=0.3\%}$.

As the results displayed in the table 6.2 show, for the heuristic HMOR-S2$_{\mathrm{L}-\Delta B=2.0\%}$ the objective function values of the final solution were the same as the ones obtained with the basic heuristic for $\alpha = 0.0$. For $\alpha = 0.5$, the results for the upper level objective functions were slightly better with the basic heuristic, and for $\alpha = 1.0$, the best results for W_Q and $B_{Mm|Q}$ were obtained with the variant. There is little difference between the results obtained with HMOR-S2 and HMOR-S2$_{\mathrm{L}-\Delta B=2.0\%}$, so this variant does not bring any extra value to our study.

As for the analytical results displayed in the table 6.3, for $\Delta W = 0.5\%$ the upper level objective functions values improved for $\alpha = 0.0$ and $\alpha = 1.0$. In particular, the QoS traffic revenue values improved 0.114% and 0.230%, respectively. However, the values of W_Q and $B_{Mm|Q}$ did not improve for $\alpha = 0.5$ and $\Delta W = 0.5\%$. Only for lower values of ΔW did those values improve. For $\Delta W = 0.3\%$, the QoS traffic revenue improved 0.150% for $\alpha = 0.5$ and 0.250% for $\alpha = 1.0$. Note that when the HMOR-S2$_{\mathrm{L}-\Delta W}$ allowed for an improvement in the QoS traffic revenue, the value of the blocking probability function $B_{Mm|Q}$ also improved and there was a trend of improvement for the lower level objective

Table 6.3: Objective function values for the final solution for HMOR-S2 and HMOR-S2$_{\mathrm{L}\text{–}\Delta W}$ with $\Delta W = 0.5\%$ and $\Delta W = 0.3\%$, on the test network M1, for different values of α

Objective Functions	HMOR-S2 (Basis)	HMOR-S2$_{\mathrm{L}\text{–}\Delta W}$ $\Delta W = 0.5\%$	HMOR-S2$_{\mathrm{L}\text{–}\Delta W}$ $\Delta W = 0.3\%$
		$\alpha = 0.0$	
W_Q	64731.51*	**64805.18**$\diamond$	64667.80$\star$
$B_{Mm\|Q}$	0.0898	**0.0831**	0.0937
$B_{m1\|Q}$	0.0898	**0.0831**	0.0937
$B_{m2\|Q}$	0.0199	**0.0193**	0.0208
$B_{m3\|Q}$	0.00216	**0.00213**	0.00246
$B_{M1\|Q}$	0.691	**0.689**	**0.678**
$B_{M2\|Q}$	**0.0723**	0.0987	0.100
$B_{M3\|Q}$	0.0287	**0.0286**	**0.0277**
W_B	17007.15	**17403.22**	**17575.26**
		$\alpha = 0.5$	
W_Q	60569.09†	60118.00$\bullet$	**60659.66**$\odot$
$B_{Mm\|Q}$	0.0424	0.0954	**0.0343**
$B_{m1\|Q}$	0.0424	0.0954	**0.0343**
$B_{m2\|Q}$	0.00534	0.00728	**0.00389**
$B_{m3\|Q}$	0.00119	0.00267	**0.000987**
$B_{M1\|Q}$	0.628	0.643	**0.623**
$B_{M2\|Q}$	0.0432	**0.0268**	**0.0275**
$B_{M3\|Q}$	0.0243	0.0251	**0.0240**
W_B	16904.99	**16992.69**	16864.90
		$\alpha = 1.0$	
W_Q	56100.60‡	**56229.80**$\square$	**56240.59**$\otimes$
$B_{Mm\|Q}$	0.0263	**0.0123**	**0.0158**
$B_{m1\|Q}$	0.0263	**0.0123**	**0.0158**
$B_{m2\|Q}$	0.00515	**0.000607**	**0.0000433**
$B_{m3\|Q}$	0.000560	0.00119	0.000626
$B_{M1\|Q}$	0.544	**0.388**	0.562
$B_{M2\|Q}$	0.0185	**0.00340**	**0.00474**
$B_{M3\|Q}$	0.0193	**0.0121**	0.0203
W_B	**16479.60**	15748.50	15300.08

HMOR-S2: *)99.35%; †)99.57%; ‡)99.58% of the optimal revenue $W_Q^{\max}$ in [37];
HMOR-S2$_{\mathrm{L}\text{–}\Delta W=0.5\%}$: $\diamond$)99.46%; $\bullet$)98.83%; $\square$)99.81% of the optimal revenue $W_Q^{\max}$ in [37];
HMOR-S2$_{\mathrm{L}\text{–}\Delta W=0.3\%}$: $\star$)99.25%; $\odot$)99.72%; $\otimes$)99.83% of the optimal revenue $W_Q^{\max}$ in [37]

functions. Therefore, the solutions with the variant HMOR-S2$_{\text{L}-\Delta W}$ tend to be globally "better" than the solutions with the basic heuristic.

This variant is more stringent than the basic heuristic in terms of the QoS traffic revenue values and nevertheless the final results for both the upper level functions are often better. This situation may be related to the way in which the heuristic works, as a specific solution is always compared to the most adequate solution found up to the current instant in the heuristic run. There may be a situation where due to the requirement that $W_Q > (1+\Delta W)\max\{W_Q\}$ with $\Delta W > 0$ a specific solution is rejected (whereas it would have been accepted if the requirement was only $W_Q > \max\{W_Q\}$) and after that rejection the algorithm ends up leading to a slightly better solution in terms of first and second level objective functions (for the service under scrutiny). The basic heuristics do not have any mechanism that allows the detection of the situations where a solution is better than the most adequate solution up to the current instant and yet it would be advantageous to reject it in order to achieve even better results with other solutions. That is, the final potential consequence of accepting or rejecting a solution can not be evaluated in these heuristics.

Simulation experiments were conducted for the HMOR-S2$_{\text{L}-\Delta W}$ variant, with $\Delta W = 0.5\%$ and $\Delta W = 0.3\%$. The experiments, both with the static routing model and with the dynamic routing model, were performed as described in sections 5.3.1 and 5.3.2. In the static routing model simulation, the routing plan calculated by the HMOR-S2$_{\text{L}-\Delta W}$ variant is considered and it is never changed throughout the simulation. In the periodic type state-dependent dynamic routing model simulation, that routing plan is the initial solution at the start of the simulation and the routing plans are updated throughout the simulation experiment by the HMOR-S2$_{\text{L}-\Delta W}$ variant.

The simulation results (average value $\pm$ half length of the 95% confidence interval) and the analytical results for HMOR-S2$_{\text{L}-\Delta W=0.5\%}$ and HMOR-S2$_{\text{L}-\Delta W=0.3\%}$ are presented in tables 6.4 and 6.5, respectively. The information is displayed as in table 5.4, that is, the best simulation results obtained for each function are indicated in bold and if some simulation result is better than the corresponding analytical value, this is indicated in italic.

The results in tables 6.4 and 6.5 show that for $\Delta W = 0.5\%$ and $\alpha = 0.0; 1.0$, and also for $\Delta W = 0.3\%$ for all the considered values of α, the objective function values are the same for the simulations of the static and dynamic routing model. Therefore, in these situations, the dynamic routing model is not advantageous with respect to the static routing model. Despite the periodic update of the routing plan, in accordance with the traffic random fluctuations around the average values corresponding to the nominal traffic matrix, in these situations the routing plan was never changed, i.e. the HMOR-S2$_{\text{L}-\Delta W}$ variant did not manage to find a routing plan better than the one used at the start of the simulation. As explained earlier, this variant is more stringent than the basic heuristic. Therefore, it is not entirely surprising that the solution stays the same throughout the simulation run.

For $\Delta W = 0.5\%$ and $\alpha = 0.5$, the average results are better with the dynamic routing model than with the static routing model for all the objective functions but $B_{m2|Q}$ and $B_{M2|Q}$. Note that in this case the routing plan obtained in the analytical test is not a good solution, as the analytical results are not very good: the QoS traffic revenue has a low value and the blocking probability function value $B_{Mm|Q}$ is high. As this routing plan is never changed throughout the static routing model simulation, the simulation results are not good either. As for the dynamic routing model simulation, the update of the routing

Table 6.4: Average objective function values, and 95% confidence intervals, for simulation of the static (considering a warm-up time of 8h) and the dynamic routing model (considering $t_0 = t_1 =$ 4h, updates every 15m and $b = 0.7$), a total simulation time of 48h on the test network M1, for different values of α, when the HMOR-S2$_{\text{L}-\Delta W=0.5\%}$ variant was used to update the routing plan

		$\alpha = 0.0$	
Objective Functions	Analytical Results	Simulation Results: Static Model	Simulation Results: Dynamic Model
W_Q	64805.18	64697.18±55.99(0.09%)	64697.18±55.99(0.09%)
$B_{Mm\|Q}$	0.0831	0.0840±0.00247(2.94%)	0.0840±0.00247(2.94%)
$B_{m1\|Q}$	0.0831	0.0840±0.00247(2.94%)	0.0840±0.00247(2.94%)
$B_{m2\|Q}$	0.0193	0.0241±0.000687(2.86%)	0.0241±0.000687(2.86%)
$B_{m3\|Q}$	0.00213	0.00223±0.0000488(2.19%)	0.00223±0.0000488(2.19%)
$B_{M1\|Q}$	0.689	*0.679±0.0152*(2.23%)	*0.679±0.0152*(2.23%)
$B_{M2\|Q}$	0.0987	0.109±0.00405(3.70%)	0.109±0.00405(3.70%)
$B_{M3\|Q}$	0.0286	0.0289±0.000366(1.27%)	0.0289±0.000366(1.27%)
W_B	17403.22	*17444.00±40.13*(0.23%)	*17444.00±40.13*(0.23%)

		$\alpha = 0.5$	
Objective Functions	Analytical Results	Simulation Results: Static Model	Simulation Results: Dynamic Model
W_Q	60118.00	60031.57±43.87(0.07%)	***60398.09±209.82***(0.35%)
$B_{Mm\|Q}$	0.0954	0.0979±0.00143(1.46%)	***0.0488±0.0129***(26.35%)
$B_{m1\|Q}$	0.0954	0.0979±0.00143(1.46%)	***0.0488±0.0129***(26.35%)
$B_{m2\|Q}$	0.00728	**0.0109 ± 0.000494**(4.54%)	0.0121±0.00581(47.94%)
$B_{m3\|Q}$	0.00267	0.00274±0.0000412(1.50%)	***0.00112±0.000199***(17.71%)
$B_{M1\|Q}$	0.643	*0.627±0.00738*(1.18%)	***0.555±0.0725***(13.07%)
$B_{M2\|Q}$	0.0268	**0.0389 ± 0.00211**(5.41%)	0.0598±0.0271(45.30%)
$B_{M3\|Q}$	0.0251	*0.0249±0.000466*(1.87%)	***0.0201±0.00312***(15.51%)
W_B	16992.69	*17043.82±25.42*(0.15%)	***17123.67±319.71***(1.87%)

		$\alpha = 1.0$	
Objective Functions	Analytical Results	Simulation Results: Static Model	Simulation Results: Dynamic Model
W_Q	56229.80	56193.05±52.91(0.09%)	56193.05±52.91(0.09%)
$B_{Mm\|Q}$	0.0123	0.0144±0.000915(6.37%)	0.0144±0.000915(6.37%)
$B_{m1\|Q}$	0.0123	0.0144±0.000915(6.37%)	0.0144±0.000915(6.37%)
$B_{m2\|Q}$	0.000607	0.00167±0.000172(10.32%)	0.00167±0.000172(10.32%)
$B_{m3\|Q}$	0.00119	0.00124±0.0000158(1.27%)	0.00124±0.0000158(1.27%)
$B_{M1\|Q}$	0.388	0.394±0.0287(7.30%)	0.394±0.0287(7.30%)
$B_{M2\|Q}$	0.00340	0.00743±0.00133(17.91%)	0.00743±0.00133(17.91%)
$B_{M3\|Q}$	0.0121	0.0124±0.000387(3.13%)	0.0124±0.000387(3.13%)
W_B	15748.50	*15753.26±17.98*(0.11%)	*15753.26±17.98*(0.11%)

Table 6.5: Average objective function values, and 95% confidence intervals, for simulation of the static (considering a warm-up time of 8h) and the dynamic routing model (considering $t_0 = t_1 =$ 4h, updates every 15m and $b = 0.7$), a total simulation time of 48h on the test network M1, for different values of α, when the HMOR-S2$_{\mathrm{L}-\Delta W=0.3\%}$ variant was used to update the routing plan

	$\alpha = 0.0$		
		Simulation Results	
Objective Functions	Analytical Results	Static Model	Dynamic Model
W_Q	64667.80	64563.55±60.49(0.09%)	64563.55±60.49(0.09%)
$B_{Mm\|Q}$	0.0937	*0.0928±0.00314*(3.39%)	*0.0928±0.00314*(3.39%)
$B_{m1\|Q}$	0.0937	*0.0928±0.00314*(3.39%)	*0.0928±0.00314*(3.39%)
$B_{m2\|Q}$	0.0208	0.0262±0.000508(1.94%)	0.0262±0.000508(1.94%)
$B_{m3\|Q}$	0.00246	0.00251±0.0000540(2.15%)	0.00251±0.0000540(2.15%)
$B_{M1\|Q}$	0.678	*0.673±0.0119*(1.76%)	*0.673±0.0119*(1.76%)
$B_{M2\|Q}$	0.100	0.112±0.00244(2.19%)	0.112±0.00244(2.19%)
$B_{M3\|Q}$	0.0277	0.0281±0.000232(0.82%)	0.0281±0.000232(0.82%)
W_B	17575.26	17554.38±51.05(0.29%)	17554.38±51.05(0.29%)

	$\alpha = 0.5$		
		Simulation Results	
Objective Functions	Analytical Results	Static Model	Dynamic Model
W_Q	60659.66	60579.06±57.01(0.09%)	60579.06±57.01(0.09%)
$B_{Mm\|Q}$	0.0343	0.0384±0.00127(3.31%)	0.0384±0.00127(3.31%)
$B_{m1\|Q}$	0.0343	0.0384±0.00127(3.31%)	0.0384±0.00127(3.31%)
$B_{m2\|Q}$	0.00389	0.00662±0.000330(4.99%)	0.00662±0.000330(4.99%)
$B_{m3\|Q}$	0.000987	0.00106±0.0000315(2.97%)	0.00106±0.0000315(2.97%)
$B_{M1\|Q}$	0.623	0.629±0.0152(2.41%)	0.629±0.0152(2.41%)
$B_{M2\|Q}$	0.0275	0.0336±0.000921(2.74%)	0.0336±0.000921(2.74%)
$B_{M3\|Q}$	0.0240	0.0242±0.000323(1.33%)	0.0242±0.000323(1.33%)
W_B	16864.90	16862.04±41.38(0.25%)	16862.04±41.38(0.25%)

	$\alpha = 1.0$		
		Simulation Results	
Objective Functions	Analytical Results	Static Model	Dynamic Model
W_Q	56240.59	56231.31±55.75(0.10%)	56231.31±55.75(0.10%)
$B_{Mm\|Q}$	0.0158	0.0162±0.000661(4.08%)	0.0162±0.000661(4.08%)
$B_{m1\|Q}$	0.0158	0.0162±0.000661(4.08%)	0.0162±0.000661(4.08%)
$B_{m2\|Q}$	0.0000433	0.000102±0.0000311(30.49%)	0.000102±0.0000311(30.49%)
$B_{m3\|Q}$	0.000626	*0.000624±0.00000660*(1.06%)	*0.000624±0.00000660*(1.06%)
$B_{M1\|Q}$	0.562	0.569±0.0277(4.87%)	0.569±0.0277(4.87%)
$B_{M2\|Q}$	0.00474	0.00796±0.00364(45.72%)	0.00796±0.00364(45.72%)
$B_{M3\|Q}$	0.0203	0.0203±0.000314(1.55%)	0.0203±0.000314(1.55%)
W_B	15300.08	*15300.61±16.41*(0.11%)	*15300.61±16.41*(0.11%)

plan throughout the simulation allows for better simulation results. A final remark on the confidence intervals for each objective function: their length is higher for the dynamic routing model.

In most situations, the analytical results obtained with the HMOR-S2$_{\mathrm{L}-\Delta W}$ are not inside the 95% confidence interval, but they are of the same order of magnitude. In particular, for the QoS traffic revenue, only for $\alpha = 1.0$ with $\Delta W = 0.5\%$ and $\Delta W = 0.3\%$ was the analytical value of W_Q inside the corresponding confidence interval. As for the other upper level objective function, $B_{Mm|Q}$, for $\alpha = 0.0$ with $\Delta W = 0.5\%$ and $\Delta W = 0.3\%$, and for $\alpha = 1.0$ with $\Delta W = 0.3\%$, the 95% confidence intervals include the analytical results. The differences between the analytical and the simulation results are mainly due to the inaccuracies intrinsic to the analytic/numerical resolution, namely those associated to the simplifications of the traffic model, and the associated error propagation as analysed in detail in section 5.3.3.

Two other sets of good results can be found in tables 6.4 and 6.5 for $\Delta W = 0.5\%$ and $\Delta W = 0.3\%$, respectively. For $\Delta W = 0.3\%$ and $\alpha = 1.0$, the analytical results of all the functions except $B_{m2|Q}$ and $B_{M2|Q}$ are quite close to the ones obtained with the simulations and they are inside the respective 95% confidence interval. The simulation results of the dynamic model for $\Delta W = 0.5\%$ and $\alpha = 0.5$ are better than the corresponding analytical results for all the functions except $B_{m2|Q}$ and $B_{M2|Q}$, and the 95% confidence intervals include the corresponding analytical value or are even better than that analytical value. As mentioned earlier, the analytical results are not very good in this situation so the fact that the dynamic model simulation results are better is not surprising.

As expected, the dynamic model simulation results are better (or the same in the worst case) than those obtained with the static model. In the dynamic routing model the routing plan is updated throughout the simulation in accordance with the traffic random fluctuations around the average values corresponding to the nominal traffic matrix $\overline{A}$. However, in these simulation experiments only for $\Delta W = 0.5\%$ and $\alpha = 0.5$ did the routing plan actually change.

7 Conclusions and Further Work

This report began by reviewing a hierarchical bi-level multiobjective routing model in MPLS networks with alternative routing formulated within the general modelling framework developed by the authors in [8], with two classes of services (and different types of traffic flows in each class), in particular QoS and BE services, with different priorities in the optimisation model.

A new specialised heuristic strategy for finding "good" compromise solutions to this very complex routing optimisation problem, adapted from the one described in [9], was put forward. This new heuristic has the same methodological framework of the previous heuristic resolution, but it is more effective. It was applied to a test network used in a benchmarking case study that uses a lexicographic optimisation routing approach, including admission control for BE traffic, based on a deterministic traffic representation, with the expected revenues associated with QoS and BE traffic as objective functions. The analytical results obtained with the new heuristic (HMOR-S2) were compared with the optimal values in the benchmarking study [37] and with the values obtained with the previous heuristic (MOR-S2) and seem quite encouraging concerning the potential performance of a multicriteria routing model of this nature. The final solution in the HMOR-S2 heuristic has significantly better values for all the objective functions when compared to the values obtained with the initial solution (typical of Internet classical routing algorithms). Comparing the results obtained with MOR-S2, the present heuristic clearly produces final solutions with better values for the first level objective functions, which makes it better suited to the resolution of the very complex bi-level multiobjective routing problem formulated for networks with two service classes.

A dynamic version of the routing method of periodic state-dependent type was formulated and its performance was analysed in the same case study network. A discrete event simulation platform was developed, which allowed for a more exact evaluation of the results of the heuristic in a stochastic environment closer to real network working conditions. In most cases, the analytical results obtained with the HMOR-S2 are not inside the 95% confidence interval of the static routing model simulation results, although they are of similar magnitudes, due to the inaccuracies intrinsic to the analytic/numerical resolution, namely those associated with the simplifications of the traffic model, and the associated error propagation. As expected, in most cases, better results were obtained with the dynamic routing model because the routing plan is adjusted throughout the simulation run, in response to relevant traffic variations, evaluated periodically through appropriate dynamic estimates of the traffic offered by every network node-to-node flow.

A sensitivity analysis was performed by introducing relaxation and floating barrier techniques in the heuristic, in order to assess the balance in the treatment of the different objective functions. Generally speaking, the results of the sensitivity tests showed that the objective functions are treated in a balanced way. However, some unexpected results occurred and they led to the formulation of variants of the basic heuristic. In both variants, one of the upper level functions ceases to be an objective function and a floating bound (associated with a variation of a percentage of the value of the function) is imposed on its value. In the first variant, a technique of relaxation is used, where the imposed bound allows the function to worsen its value. In the second variant, a stricter barrier is imposed on the value of the function. Analytical results for these variants were presented. As the variant with a floating barrier imposed on the QoS traffic revenue W_Q showed promising analytical results (with an improvement in the values of the upper level objective functions in the final solution), this variant was studied experimentally, in the same way as the HMOR-S2 heuristic. The simulation results were in line with those obtained for HMOR-S2, except for the fact that in many cases the simulation results are the same for the static and for the dynamic routing model.

The inherent great complexity and the associated computational burden of this type of model are its major limitations and they constitute the reverse of its 'ambitious' features, namely, network-wide optimisation, multiobjective nature with two levels of objective functions, use of alternative routing and a stochastic representation of traffic flows of multiple service types. Therefore, its potential practical application at present is restrained to networks with a limited number of nodes, such as the core and intermediate (metro-core) level networks of low dimension.

Further work on this model will focus on the search for improvements in the heuristic procedure, through the possible development of metaheuristics for this very complex network routing problem. Also the extension of the model to broader routing principles such as probabilistic load sharing or traffic splitting might be studied and tested.

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Appendix – Formalisation of the HMOR-S2 Heuristic

I. $\overline{R}_a \leftarrow \overline{R}_o$
II. Compute $\overline{B}$ and $W_Q, B_{Mm|Q}$ for $\overline{R}_a$
III. $W_Q^o \leftarrow W_Q, B_{Mm|Q}^o \leftarrow B_{Mm|Q}$
IV. $\overline{R}_* \leftarrow \overline{R}_a$
V. Compute $\overline{B}$ for $\overline{R}_a$
Compute $W_Q, B_{Mm|Q}, B_{ms|Q}, B_{Ms|Q}(\forall s \in \mathcal{S}_Q), W_B$ for $\overline{R}_a$
VI. $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
$\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}(\forall s \in \mathcal{S}_Q)$ and $\max\{W_B\} \leftarrow W_B$
VII. For $nPaths = |\mathcal{F}|$ to $nPaths = 1$

1. For $ape = 0$ to $ape = 1$

   (a) If $ape = 0$, $z_{APR} \leftarrow 1.0$
   Else, $z_{APR} \leftarrow 0.01 \cdot nPaths$

   (b) For $s = 1$ to $s = |\mathcal{S}|$

   i. For $nCycles = 1$ to $nCycles = 0$

   A. Compute $\overline{B}$ and $\overline{c}^Q, s \in \mathcal{S}_Q$ or $\overline{c}^B, s \in \mathcal{S}_B$ for $\overline{R}_a$
   B. Compute and order the values of the function $\xi(f_s)$, with $\xi(f_s) = F_L(f_s)$ if $nCycles = 1$ and $\xi(f_s) = F_C^{Q(B)}(f_s)$ if $nCycles = 0$
   C. Find the $nPaths$ flows with lower value of $\xi(f_s)$
   D. Compute with MMRA-S2 new candidate paths for the corresponding O-D pairs and define a new set of first and second choice paths for the service s, $\overline{R}_a(s)$, according to the rules established for each service
   E. Compute $\overline{B}$ for $\overline{R}_a$
   Compute $B_{ms|Q}, B_{Ms|Q}$ if $s \in \mathcal{S}_Q$ or W_B if $s \in \mathcal{S}_B$ for $\overline{R}_a$

   F. If $s \in \mathcal{S}_Q$ then
   - If $(B_{ms|Q} < \min\{B_{ms|Q}\}$ and $B_{Ms|Q} < \min\{B_{Ms|Q}\})$ then
     - Compute $W_Q, B_{Mm|Q}$
     - If $(W_Q > \max\{W_Q\}$ and $B_{Mm|Q} < \min\{B_{Mm|Q}\})$ then
       * $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}$
       * $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
       * $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$

   G. Else ($s \in \mathcal{S}_B$)
   - If $(W_B > \max\{W_B\})$ then
     - Compute $W_Q, B_{Mm|Q}$
     - If $(W_Q > \max\{W_Q\}$ and $B_{Mm|Q} < \min\{B_{Mm|Q}\})$ then
       * $\max\{W_B\} \leftarrow W_B$
       * $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
       * $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$

   H. $\overline{R}_a(s) \leftarrow \overline{R}_*(s)$

   End of the cycle For ($nCycles$)

   End of the cycle For (s)

End of the cycle For (ape)

End of the cycle For ($nPaths$)
VIII. If $W_Q^o > \max\{W_Q\}$ or $B_{Mm|Q}^o < \min\{B_{Mm|Q}\}$ then

1. The best solution is $\overline{R}_o$

IX. Otherwise, the best solution is $\overline{R}_*$
X. Compute the objective function values for the best solution