AN AUTOMATIC REFERENCE POINT-LIKE APPROACH FOR DEALING WITH A MULTICRITERIA TELETRAFFIC ROUTING MODEL

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1 Motivation

Nowadays the exponential and convergent development of informatics and telecommunications makes that, in many communication networks, important decisions of technical nature have to be made in short time periods or even in real-time. In these circumstances problems of "automatic decision" become increasingly important. In these cases the main function of automatic decision is not just to replace humans in the execution of repetitive decisions (see Bouyssou [1]) but also to enable a rapid adaptation of the systems to diverse working conditions that are known from an automatic monitoring of the systems.

This work is dedicated to a case where, in the context of a multicriteria shortest path model, on the one hand it is necessary to be prepared to replace a route (i.e., a loopless path to be used by a certain origin-destination communication flow) by another route, motivated by failure or congestion; and on the other hand, it is necessary to up-date periodically the conditions in which the choices are made, taking into account the state of the network. The latter function will not be object of study in this work and could be implemented by a rule-based decision system capable of executing an adequate periodic up-date of the parameters of the mathematical model used for selecting the implemented paths and the candidate substitute paths, indicating the corresponding priority order. Obviously such rule-based decision system should be dedicated to the case study that we seek to treat.

In this paper we will present a mathematical model for the automatic ordering and selection of paths in telecommunication networks using multicriteria routing models and a method of analysis dedicated to this type of models based on a reference point approach. Although the proposed model is not of universal use it can be applied in studies concerning diverse routing systems.

In section 2 of this work we will present the main features of the addressed routing model. The aim of the developed method is the automatic determination (for each node to node traffic flow), ordering and selection of K loopless paths between the originating and terminating nodes, which are solutions to this problem, taking into account preference thresholds for the objective functions. The use of preference thresholds establishes sub-regions of the objective function space with different priority order, concerning the choice of solutions. The appropriate selection of reference points and a tuning of the Chebyshev metric allows us to search exhaustively the different sub-regions taking into account their priority order. Section 3 will present the problem addressed in relation to the routing model and describes the method used for ranking and selecting the solutions. This includes the description of its main procedures. Section 4 typifies the case study where the model will be applied. This case study is dedicated to a traffic routing problem in a broadband network of ATM type, formulated as a bicriteria shortest path problem with several constraints. In this problem the two metrics to be optimised are the number of arcs of the path and the path cost expressed in terms of the available bandwidth in its arcs and the constraints correspond to available bandwidth, delay and jitter bounds. The final objective of the routing method associated with the formulation of the routing problem is to order and select possible paths which may be used by each origin-destination traffic flow, characterised by certain technical QoS (Quality of Service) requirements. In the tested examples the routing of video traffic flows will be considered in randomly generated networks with a few thousands of nodes, assuming different network working conditions.

Finally, section 5 presents and analyses the computational results and section 6 discusses the results obtained with the proposed approach and outlines conclusions and topics for further development of this work.

2 Features of the routing model

In the routing model that we are interested in a communication network is represented by an undirected graph $(\mathcal{N}, \mathcal{A})$, with a set $\mathcal{N}$ of n nodes, that may represent switches, routers or servers, and a set $\mathcal{A}$ of m arcs (or links) which correspond to transmission facilities with a certain capacity expressed in terms of bandwidth. Let $\mathcal{P}$ denote the set of loopless paths from a source node s to a terminal node t in $(\mathcal{N}, \mathcal{A})$, and associate with each arc (i, j): the cost $c_{ij} > 0$, the available bandwidth $b_{ij} > 0$, and the delay $d_{ij} > 0$. Given a path p we also define the functions: $c(p) = \sum_{(i,j)\in p} c_{ij}$, $b(p) = \min_{(i,j)\in p}\{b_{ij}\}$ (usually designated as "bottleneck" bandwidth), $d(p) = \sum_{(i,j)\in p} d_{ij}$ (which corresponds to the total delay experienced when p is used), and the number of arcs in p (denoted by $h(p)$). The model considers two objective functions: the cost c and the number of arcs h.

In the formulation considered for a specific traffic routing problem in an ATM type network the cost is a function of the available bandwidth on the arc, $c_{ij} = 1/b_{ij}$, and constraints on the permitted delay jitter (Δ_{jitter}), required bandwidth ($\Delta_{\text{bandwidth}}$) and maximum delay ($\Delta_{\text{delay}}$) have to be introduced. For certain queueing disciplines in the nodes the jitter constraint can be transformed into a constraint in the number of arcs, namely: $\Delta_{\text{jitter}} = m_p(s,t) + \Delta_{\text{arcs}}$ where $m_p(s,t)$ is the minimum number of arcs of a feasible path from s to t, and Δ_{arcs} is an integer typically equal to 2 or 4 (see Pornavalai *et al.* [8]).

Once an origin-destination pair is fixed, the routing problem here considered is formulated as:

$$
\begin{array}{ll}
\min & \{c(p) : p \in \mathcal{P}\} \\
\min & \{h(p) : p \in \mathcal{P}\} \\
\text{s.t.} & b(p) \geq \Delta_{\text{bandwidth}} \\
& d(p) \leq \Delta_{\text{delay}} \\
& h(p) \leq \Delta_{\text{jitter}}
\end{array}
\tag{1}
$$

This is a bicriteria shortest path problem with additional constraints. In [2] an algorithm was presented to compute the whole set of non-dominated feasible solutions. Later in [3] two procedures for selecting paths to be used by the calls of each node to node traffic flow in an automatic manner were developed, based on the utilization of weights for the 2 objective functions and on the definition of preference thresholds on the solution space.

In the following a different method for calculating, ordering and selecting solutions to this problem will be presented, which is based on reference point approach. For this purpose it will be presented a procedure for determining automatically a number K of solutions in a search region, by non-decreasing order of a distance function between each solution and a reference point. This technique is then combined with the definition of tighter search regions obtained by defining preference thresholds on the objective function space.

3 Problem definition

Let $(\mathcal{N}, \mathcal{A})$ be a network where each $(x, y) \in \mathcal{A}$ is associated with two costs, $c^i_{xy} \in \mathbb{R}$, with $i = 1, 2$. Assume that

$$c_i(p) = \sum_{(x,y)\in p} c^i_{xy}, \; i = 1, 2,$$

are two objective functions, defined for any path between a pair of nodes in a network, which we want to minimise. The aim is the automatic determination (for each given pair of nodes (s, t)), ordering and selection of K loopless paths from s to t which are solutions to this bicriteria shortest path problem, taking into account preference thresholds defined in the objective function space. An underlying assumption to this ordering and selection process is that the first selected path may be, in certain situations, unavailable in the network (typically as a result of congestion, excessive delay or failure in some of its components), in which case the second path in the selected path list will be chosen, and so on. Note that in these conditions we seek not only non-dominated solutions since in some applications there may be cases in which it is justified to select as second (third, ...) choice, solutions which may be dominated. This means that it is justified to use a path dominated by another previously obtained by the procedure whenever the latter is unavailable in the actual network. An example of such application is the routing model described in [6]. The introduced approach is based on listing a given number K of loopless paths according to a Chebyshev distance to a reference point. One can show that we can tune the parameters in the Chebyshev metric in order to sweep a specified region of the solution set. The use of preference thresholds establishes sub-regions of the objective function space with different priority order, concerning the choice of solutions. The appropriate selection of reference points and a tuning of the Chebyshev metric allows us to search exhaustively the different sub-regions taking into account their priority order. The proposed search procedure, that will be applied to the model described above, combines this enumeration technique with the definition of priority regions obtained from preference thresholds concerning acceptable and requested values for each objective function.

Figure 1 illustrates the scheme of the ranking procedure in the region within the solid lines, where the star represents the reference point, and the bullets represent the solutions. Solutions are found along the dashed lines, from the smaller rectangle to the largest rectangle, all rectangles having vertex in the reference point. The number attached to each bullet is the order in which the corresponding solution is found.

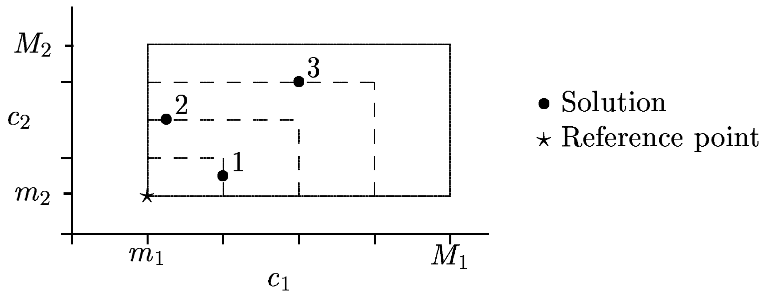

Figure 1: Solutions ranked with the new procedure, in the area within solid lines

We will use a reference point type approach for selecting solutions in an ordered way, where the Chebyshev distance to a given point, usually considered as the ideal one, that is, (c_1^*, c_2^*) such that $c_i^* = \min\{c_1(p) : p \in \mathcal{P}\}$, $i = 1, 2$, is minimised. In the procedure, once the reference point $(\bar{c}_1, \bar{c}_2)$ is established, we will solve the problem

$$\min_{p \in \mathcal{P}} \max_{i=1,2} \{|c_i(p) - \bar{c}_i|\}.$$

However, as mentioned above, we want to find several alternatives to the optimal solution, near $(\bar{c}_1, \bar{c}_2)$, by ranking loopless paths in non-decreasing order of $\max_{i=1,2}\{|c_i(p) - \bar{c}_i|\}$.

As a ranking strategy, and taking into account the existence of constraints on the loopless paths we are looking for, one can combine the methods proposed by Murthy and Her [7] and by Martins, Pascoal and Santos [5]. Murthy and Her introduced a labelling algorithm to solve a min-max type path problem, where the labels of non-dominated nodes can be eliminated. On the other hand, Martins *et al.* described ranking shortest path algorithms, where a label is associated with each k-th shortest path determined from s until a certain node, $k \leq K$. In the procedure developed here several paths from s to other nodes in the network are constructed, noticing that, for each $x \in \mathcal{N}$, it is sufficient to keep the labels $(\pi_x^{1,1}, \pi_x^{1,2})$, ..., $(\pi_x^{K,1}, \pi_x^{K,2})$, corresponding to the K-th best paths from s to x relatively to the objective function $\max_{i=1,2}\{|c_i(p) - \bar{c}_i|\}$, and that the remaining labels (and paths) can be ignored. As in [7], also some other labels can be avoided. Assume that the shortest trees from any node to t in terms of c_1 and c_2 (which can be easily computed by Dijkstra's algorithm [4]) are known. Let those trees be $\mathcal{T}_t^1$ and $\mathcal{T}_t^2$, respectively, and let $\mathcal{T}_t^i(x)$ denote the path from x to t in $\mathcal{T}_t^i$, $i = 1, 2$. Then the labels obtained for $y \in \mathcal{N}$, such that $(x, y) \in \mathcal{A}$, with the form $(\pi_y^{j,1}, \pi_y^{j,2}) + (c_{xy}^1, c_{xy}^2)$, for some $1 \leq j < K$, which satisfy:

$$\max\left\{|\pi_y^{j,1} + c_{xy}^1 + c_1(\mathcal{T}_t^1(y)) - \bar{c}_1|, |\pi_y^{j,2} + c_{xy}^2 + c_2(\mathcal{T}_t^2(y)) - \bar{c}_2|\right\} \geq \max\left\{|\pi_x^{K,1} - \bar{c}_1|, |\pi_x^{K,2} - \bar{c}_2|\right\}$$

could never lead us to a path closer to $(\bar{c}_1, \bar{c}_2)$ than label $(\pi_x^{K,1}, \pi_x^{K,2})$, therefore they can be ignored.

The procedure used for ranking loopless paths according to the Chebyshev metric is now summarised.

Procedure 1:

1. Compute $\mathcal{T}_t^i$, $i = 1, 2$
2. `For` $(i \in \mathcal{N} - \{s\})$ `Do:` $cont_i \leftarrow 1$
3. $cont_s \leftarrow -1$

4. $total \leftarrow 1$; $\pi^i_{total} \leftarrow 0$, $i = 1, 2$
5. $paths \leftarrow 0$; $MaxNorm \leftarrow 0$
6. $X \leftarrow \{total\}$
7. `While` ($paths < K$ and $X \neq \emptyset$) `Do:`
   (a) $k \leftarrow$ element in X with the lexicografic lowest label; Remove k from X
   (b) $x \leftarrow$ node of $\mathcal{N}$ corresponding to k
   (c) $cont_x \leftarrow cont_x + 1$
   (d) `If` ($x = t$) `Then`
      i. `If` ($paths < K$) `Then`
         A. $paths \leftarrow paths + 1$; $\pi_t^{paths,i} \leftarrow \pi^i_k$
         B. Store the path that corresponds to the loopless path from 1 to k
         C. $MaxNorm \leftarrow \max\{MaxNorm, \max_{i=1,2}\{|\pi^i_k - \bar{c}_i|\}\}$
      ii. `Else If` ($\max_{i=1,2}\left\{|\pi^i_k - \bar{c}_i|\right\} < MaxNorm$) `Then`
         A. Replace the K-th best optimal path with the path in $\mathcal{P}$ corresponding to the loopless path from 1 to k
         B. $(\pi_t^{K,1}, \pi_t^{K,2}) \leftarrow$ label of the K-th best optimal path
         C. Update $MaxNorm$
   (e) `For` ($(x, y) \in \mathcal{A}$) `Do:`
      i. `If` ($(|\pi^i_k + c^i_{xy} + c_i(\mathcal{T}^i_t(y)) - \bar{c}_i| \leq \max_{i=1,2}\left\{|\pi_t^{K,i} - \bar{c}_i|\right\}$, $i = 1, 2$, and y doesn't form a loop) `Then`
         A. $total \leftarrow total + 1$; $\pi^i_{total} \leftarrow \pi^i_k + c^i_{xy}$, $i = 1, 2$
         B. Insert $total$ in X
8. Select the K-best optimal paths

Taking into account the nature of the additional constraints in problem (1) it is simple to insert instructions in this procedure which allow the skipping of more labels.

After the process of ranking solutions according to the Chebyshev metric has been established it is important to notice that one can assure that the search is made within a specific solution subset. In fact, to find solutions $p \in \mathcal{P}$ such that $m_i \leq c_i(p) \leq M_i$, for $i = 1, 2$, with bounds m_i, M_i previously fixed, it suffices to consider the "left bottom corner" of the rectangle we want to sweep, that is, using (m_1, m_2) as the reference point, and a weighted Chebyshev metric in proportion to sizes of that rectangle. Let us consider a search region bounded by m_i, M_i, where $m_i < M_i$, for $i = 1, 2$, and the parameters

$$w_1 = M_2 - m_2,\ w_2 = M_1 - m_1. \tag{2}$$

Then, the reference point with coordinates $\bar{c}_i = m_i$, $i = 1, 2$, is used with the weighted Chebyshev metric

$$\max_{i=1,2}\{w_i|c_i(p) - \bar{c}_i|\}. \tag{3}$$

Figure 1 represents the search that results from the application of the above procedure with this metric.

Ranking paths relatively to a reference point can be combined with the establishment of preference thresholds, in a simple manner by letting m_i and M_i vary with the regions defined by those constraints.

Assume that the user specifies aspiration and reservation levels, by means of a requested value, c^i_{req}, and an acceptable value, c^i_{acc}, for each objective function, $i = 1, 2$. Solutions are searched in each region bounded by the requested and acceptable values, following the priorities chosen by the user. Once those parameters are imposed, the solutions which satisfy the requested values (which define the first priority region) are listed – zone A in figure 2. The second priority region is formed by solutions which satisfy only one of the requested values, the value of the other objective function being just an acceptable value – zones B_1 and B_2. Finally acceptable solutions with respect to both the objectives, are searched – zone C. Notice that the priorities used in this search are defined by the user, so they can be different from this description, and that the search in each zone corresponds to the scheme illustrated in figure 1.

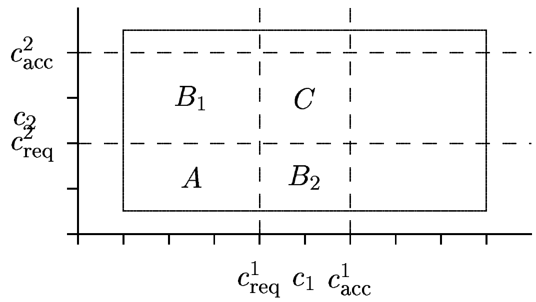

Figure 2: Priority regions

Finally, it is remarked that the proposed strategy can be easily extended to problems with $r > 2$ objectives. In that case we may divide the solution space in 2^r regions with different priorities[1] and generalise the metric (3) by doing

$$w_i = \prod_{j=1, j\neq i}^{r} (M_j - m_j).$$

We can now summarise the essential procedure of the method developed for ordering and selecting up to K paths, by using the procedure presented above (enabling the enumeration of loopless paths using metric (3)).

Procedure 2:

1. Define M_i, m_i, for $i = 1, \ldots, r$, and K
2. $k = 0$
3. For each region bounded by m_i and M_i, $i = 1, \ldots, r$, Do:
   (a) If $k \geq K$ Then Stop
   (b) $\bar{c}_i = m_i$, for $i = 1, \ldots, r$
   (c) $w_i = \prod_{j=1, j\neq i}^{r} (M_j - m_j)$, for $i = 1, \ldots, r$
   (d) $k = k + 1$
   (e) Determine p_k, the k-th loopless path nearest to $(\bar{c}_1, \ldots, \bar{c}_r)$ relatively to $\max_{i=1,\ldots,r} \{w_i |c_i(p) - \bar{c}_i|\}$

[1]If $r = 3$ we have 2^3 3-dimensional search zones, instead of the 2^2 rectangles defined when $r = 2$.

4 Application model

The model presented in section 2 was applied to a video traffic routing problem in an ATM type network by considering two sets of test networks. In these networks we considered $B_{ij} = 155.52$ Mb/s as the capacity of any arc (i, j), and the bounds of the routing model, $\Delta_{\text{bandwidth}} = 1.5$ Mb/s, $\Delta_{\text{arcs}} \in \{2, 4, 5, 6\}$, $\Delta_{\text{delay}} \in \{10, 15, \ldots, 60\}$ ms. The values of the available bandwidths in the arcs b_{ij} were randomly generated in the interval $[0.52, 150.52]$ Mb/s, according to different statistical distributions. The networks in the first set were randomly generated in a rectangular grid with 400×240 points where the mesh space unit corresponds to 10 Km. The second set of networks was obtained from 1088 US cities coordinates. In both sets topological requirements in terms of connectivity and average node degree (equal to 4 in the first set and equal to 8 in the second set) were imposed. This application test bed was used, with minor adaptations, in the present work.

The required and acceptable values considered where those already used in [3]. For the number of arcs in a path the required values, h_{req}, and acceptable values, h_{acc}, are expressed as:

$$\begin{aligned} h_{\text{req}} &= \text{int}(\overline{m}_p) + 1, \\ h_{\text{acc}} &= \text{int}(\overline{m}_p) + \Delta_{\text{arcs}} - 1 \quad (\text{with } \Delta_{\text{arcs}} > 2), \end{aligned} \tag{4}$$

where $\text{int}(x)$ is the smallest integer greater than or equal to x and $\overline{m}_p$ denotes the average value of the length of the shortest paths for all the node pairs considered in a given test network.

Concerning the path costs the aspiration level, c_{req}, and the reservation level, c_{acc}, are defined by:

$$\begin{aligned} c_{\text{req}} &= \frac{\bar{c}_{\min} + c_m}{2}, \\ c_{\text{acc}} &= \frac{\bar{c}_{\max} + c_m}{2}, \end{aligned} \tag{5}$$

where $\bar{c}_{\min}$ and $\bar{c}_{\max}$ denote the average values of the minimal and maximal cost paths obtained for all node pairs in each computational experience, and $c_m = (\bar{c}_{\min} + \bar{c}_{\max})/2$.

5 Some computational results

Some computational tests with the procedures described above were run for the set of randomly generated networks constructed on a grid with 400×240 points, with an average node degree equal to 4. The statistical distribution used for the available bandwidths in the links is given by:

I_0	I_1	I_2	I_3	I_4
5%	10%	15%	20%	50%

where I_n are sub-intervals of values in [0.52,150.52] Mb/s, defined by:

$$\begin{aligned} I_i &= \{0.52 + 2k : k = 15i, \ldots, 15(i+1) - 1\}, \ i = 0, 1, 2, 3, \\ I_4 &= \{0.52 + 2k : k = 60, \ldots, 75\}, \end{aligned}$$

and the bounds imposed over the paths on the experiments where $\Delta_{\text{bandwidth}} = 1.5$ Mb/s, $\Delta_{\text{delay}} = 60$ s, and $\Delta_{\text{jitter}} = m_p(s, t) + \Delta_{\text{arcs}}$, where $m_p(s, t)$ is the minimum number of arcs of a feasible path from s to t and $\Delta_{\text{arcs}} = 6$.

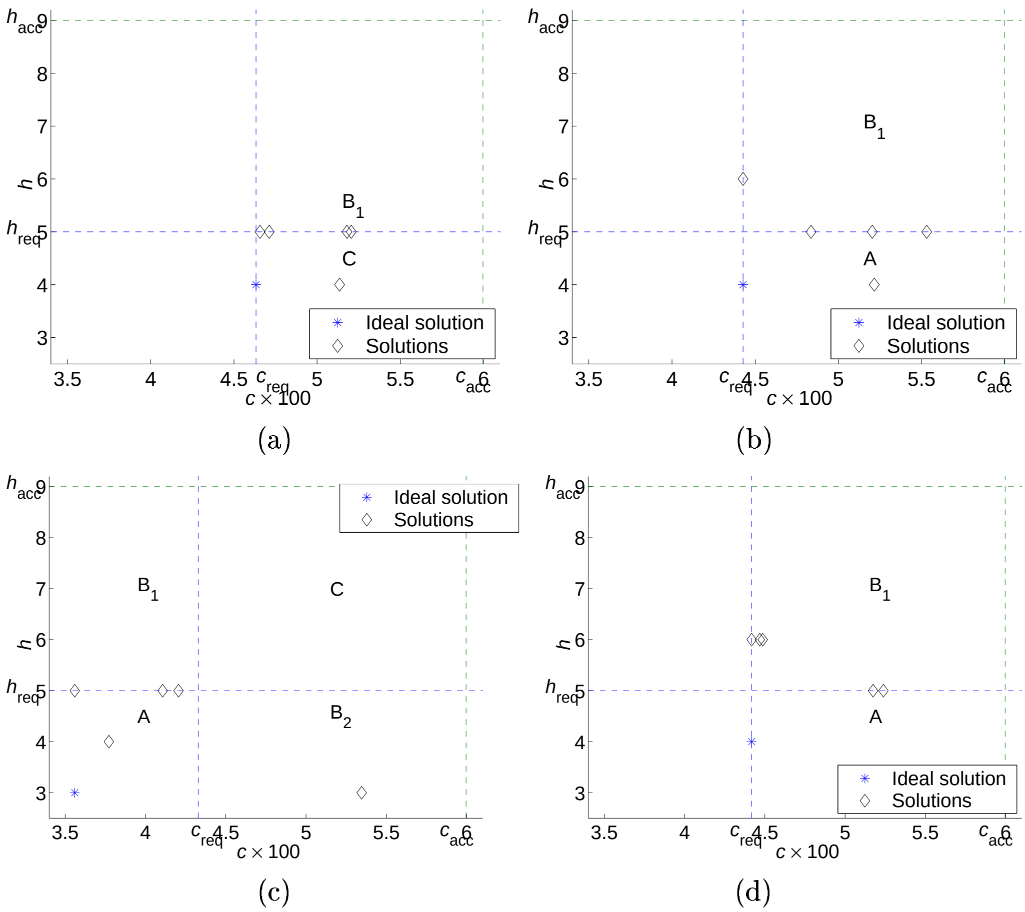

Figure 3: Solutions obtained by the method for $n = 1000$, $m = 4n$

Figures 3 and 4 illustrate the solutions obtained when $K = 5$ for two of these test networks (with 1000 and 2000 nodes, respectively), for 4 different node pairs, using procedure 2 of path ranking described above. In the figures the points designated as 'ideal solutions' (in general not feasible) were obtained by signalling in the objective function space the minimal values (c^*, h^*) of the objective functions corresponding to solutions which satisfy the constraints of the problem (1). Tables 1 and 2 show the values obtained for the objective functions and the left hand-side of the constraints in (1), for the selected solutions. The computational experiments showed that it was possible to obtain the required number of paths in a short time according to the prescribed preference thresholds. Note that in typical applications in communication networks it would be required to obtain at most two or three paths for each end to end connection. Finally, note that when weakly non-dominated solutions appear in the list of selected paths, if required, they can be reordered according to the metric which distinguishes those solutions.

6 Conclusions

A mathematical model for the automatic ordering and selection of paths of telecommunication networks using multicriteria routing models and a method of analysis dedicated to this type of model,

	$c \times 10^2$	h	b	d
1	5.1352	4	38.52	38.1749
2	5.1794	5	50.52	50.1275
3	5.2051	5	50.52	39.6911
4	4.6565	5	74.52	49.6385
5	4.7121	5	74.52	28.5718

(a)

	$c \times 10^2$	h	b	d
1	5.2154	4	48.52	32.9897
2	4.8354	5	66.52	48.0795
3	5.2034	5	48.52	41.7915
4	5.5306	5	48.52	32.5078
5	4.4273	6	126.52	55.0892

(b)

	$c \times 10^2$	h	b	d
1	3.7721	4	68.52	23.7974
2	4.1067	5	92.52	47.1189
3	4.2058	5	92.52	27.8429
4	3.5591	5	130.52	43.9038
5	5.3461	3	36.52	36.3470

(c)

	$c \times 10^2$	h	b	d
1	5.2382	5	68.52	52.2382
2	5.1743	5	50.52	44.7202
3	4.4179	6	118.52	42.2418
4	4.4691	6	120.52	52.6953
5	4.4872	6	122.52	31.9524

(d)

Table 1: Cost values of the solutions in Figure 3

based on a reference point approach, were presented.

Although the methodology introduced here was presented having in mind multicriteria routing problems in multimedia networks, it seems that these ideas can also be useful in other problems where automatic decisions are necessary.

The routine proposed here for ranking paths according to the Chebyshev metric consists of a labeling algorithm, which has the advantage of enabling a simple incorporation of the verification of additional constraints on the solutions. The method for ordering and selecting solutions uses preference thresholds for the objective functions which enable the definition of sub-regions of the objective function space with different priority order, concerning the choice of solutions. A key feature of the method is the appropriate selection of reference points and a tuning of the Chebyshev metric which allows an exhaustive search in the different sub-regions taking into account their priority order.

As a future task one may think of developing procedures for updating the preference thresholds according to changes in the network status. In the present case that can be reduced simply to measuring the status periodically and repeating the application of the formulas used, although other rules could be studied for different problems.

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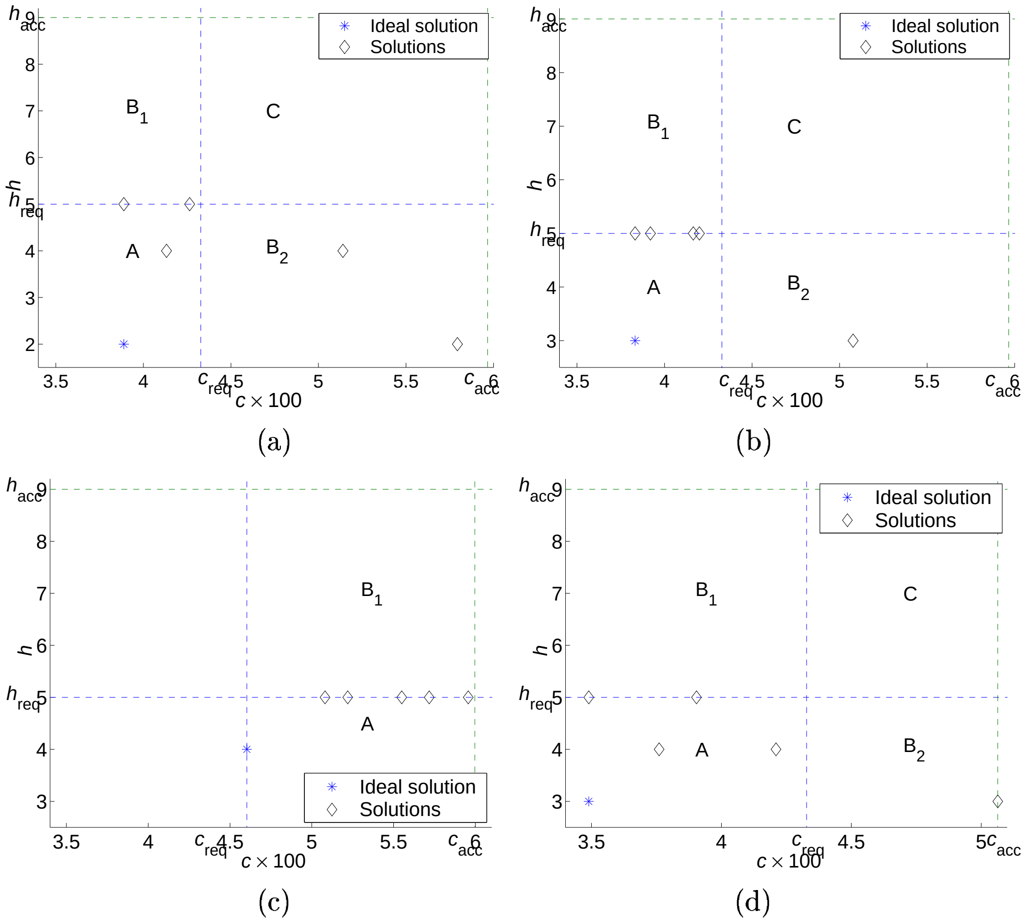

Figure 4: Solutions obtained by the method for $n = 2000$, $m = 4n$

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	$c \times 10^2$	h	b	d
1	4.1320	4	58.52	36.0601
2	3.8885	5	120.52	51.1521
3	4.2642	5	94.52	20.4429
4	5.1395	4	62.52	34.4545
5	5.7937	2	34.52	28.3891

(a)

	$c \times 10^2$	h	b	d
1	3.8319	5	110.52	38.2714
2	3.9196	5	104.52	43.2535
3	4.1993	5	104.52	47.9667
4	4.1643	5	100.52	41.1944
5	5.0771	3	28.52	21.0270

(b)

	$c \times 10^2$	h	b	d
1	5.9564	5	40.52	27.7652
2	5.5501	5	54.52	33.1852
3	5.0810	5	68.52	38.6802
4	5.7178	5	72.52	55.9505
5	5.2195	5	72.52	42.5627

(c)

	$c \times 10^2$	h	b	d
1	3.7603	4	72.52	30.2497
2	4.2101	4	68.52	39.1189
3	3.4894	5	140.52	29.0226
4	3.9042	5	116.52	41.7276
5	5.0639	3	36.52	21.0052

(d)

Table 2: Cost values of the solutions in Figure 4