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SMAA-TRI: A PARAMETER STABILITY ANALYSIS METHOD FOR ELECTRE TRI

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SMAA-TRI: A PARAMETER STABILITY ANALYSIS METHOD FOR ELECTRE TRI

Abstract

ELECTRE TRI is a multiple criteria decision aiding sorting method with a history of successful real-life applications. In ELECTRE TRI, values for certain parameters, such as criteria weights, thresholds, category profiles, and lambda cutting level, have to be provided. We propose a new method, SMAA-TRI, that is based on Stochastic Multicriteria Acceptability Analysis (SMAA), for analyzing the stability of such parameters. The stability analysis can be used for deriving robust conclusions. SMAA-TRI allows ELECTRE TRI to be used with imprecise, arbitrarily distributed values for weights and the lambda cutting level. The method consists of analyzing through Monte Carlo simulation finite spaces of arbitrarily distributed parameter values in order to describe for each action, the share of parameter values that have it assigned to different categories. We provide algorithms for the method, and demonstrate the real-life applicability by re-analyzing a case study in the field of risk assessment.

Keywords: *SMAA-TRI; Multiple Criteria Decision Aiding (MCDA); ELECTRE TRI; Stochastic Multicriteria Acceptability Analysis (SMAA); Stability Analysis; Robust Conclusions.*

1 Introduction

Partitioning a set of objects into groups (clusters, classes, or categories) is among the most researched areas in various disciplines. The first contributions on grouping techniques came from statistical and econometric fields in form of discriminant, logit and probit analyses (Berkson, 1944; Bliss, 1934; Fisher, 1936; Smith, 1947). During the last three decades, the development of grouping techniques has been based on operations research and artificial intelligence. These methodologies can be divided into two families: the non-parametric techniques such as neural networks, machine learning, rough sets and fuzzy sets (Dembczynski et al., 2002; Greco et al., 1999, 2001, 2002; Pawlak, 1982; Pawlak and Słowiński, 1994), and the ones based on outranking relations (Belacel, 2000; Belacel and Boulassel, 2004; Perny, 1998; Roy and Bouyssou, 1993; Yu, 1992a) and particular cases of multiple attribute utility theory (Zopounidis and Doumpos, 1999, 2000).

The groups can be defined *a priori* or *a posteriori* and be ordered or not. In the case of *a priori* defined ordered groups the problem is called an *ordinal classification* or *sorting* problem, and the objects are assigned to *categories* based on upper and lower profiles, central objects or other norms (Doumpos and Zopounidis, 2002).

In the late seventies a trichotomic procedure based on the outranking approach for sorting problems was proposed by Moscarola and Roy (see Moscarola, 1977; Moscarola and Roy, 1977; Roy, 1981). Several years later, in order to help decision making in a large banking company faced with a problem of accepting or refusing credit requests, a new method with a name of ELECTRE A was developed and applied in 10 sectors of activity (Figueira et al., 2005b). Based on these earlier works, in 1992 a method called ELECTRE TRI (Yu, 1992a,b) emerged. It is one of the most successful and applied methods for MCDA sorting problems (see e.g. Damart et al., 2002; Dimitras et al., 2001; Gabrel, 1994; Georgopoulou et al., 2003; Mavrotas et al., 2003; Merad et al., 2004; Raju et al., 2000).

ELECTRE TRI requires an input of numerous parameters. These parameters can be divided into *preference parameters* (relative importance coefficients of criteria or weights, thresholds, and profiles) and the *technical parameter* (lambda cutting level). The weight elicitation process in general is one of the the most difficult problems in MCDA, because MCDA methods are supported by mathematical models and therefore the preferences need to be expressed in mathematical terms. There are numerous weight elicitation techniques proposed for ELECTRE methods, see e.g. Figueira and Roy (2002); Hokkanen and Salminen (1997); Mousseau (1995); Mousseau et al. (2001); Rogers and Bruen (1998). All these techniques produce different values for weights, and therefore it is advisable to perform some kind of robustness analysis when they are applied (Roy, 2002, 2005).

Important research has been made about parameter inference and robustness analysis for ELECTRE TRI. The first advance on this topic was made by Mousseau and Słowiński (1998), who presented a method for inferring indifference and preference thresholds, profiles and weights, from assignment examples through non-linear optimization. A user-friendly software for this method was presented by Mousseau et al. (2000). A linear programming method for inferring the weights from assignment examples was introduced by Mousseau et al. (2001). Dias and Clímaco (1999, 2000) presented a method for deriving robust conclusions with imprecise parameter values

that were defined with linear constraints. These works were combined into a unified framework by Dias et al. (2002), allowing to infer the parameters and to derive robust conclusions based on assignment examples. Nevertheless, inconsistent judgements can appear when these methods are applied. Algorithms for solving interactively the inconsistencies in the inferred parameter values were proposed by Mousseau et al. (2004, 2003). An approach for inferring category limits was introduced by Ngo The and Mousseau (2002). The methodology was complemented by Dias and Mousseau (2004), who introduced a partial inference procedure for inferring the veto-related parameters.

In this paper we introduce the SMAA-TRI method that can be used for analyzing the robustness of ELECTRE TRI results based on parameter stability analysis. A *parameter stability analysis* consists of analyzing a space of feasible parameters for possible changes in the output of the method. Stability analysis allows the model to include non-deterministic parameters and provides the DMs with more output than parameter inference. SMAA-TRI is based on Stochastic Multicriteria Acceptability Analysis (SMAA) (Lahdelma et al., 1998; Lahdelma and Salminen, 2001), that is a family of decision support methods to aid decision makers (DMs) in discrete decision making problems. The SMAA methods for the ranking problem statement (see Durbach, 2005; Lahdelma et al., 1998, 2003, 2005; Lahdelma and Salminen, 2001, 2005b; Tervonen et al., 2004) are based on inverse weight space analysis that produces descriptive values characterizing the decision making problem. They have been applied in numerous real-life situations (see e.g. Hokkanen et al., 1998, 1999, 2000; Kangas et al., 2003; Lahdelma and Salminen, 2005a; Lahdelma et al., 2002, 2001). SMAA-TRI is the first SMAA method for the sorting problem statement.

We demonstrate the application of SMAA-TRI by re-analyzing a case study in the field of risk assessment. To make the method readily applicable, we provide complete algorithms for the method in the appendices.

The rest of the paper is organized as follows: a comprehensive description of ELECTRE TRI is presented in Section 2. SMAA-TRI is introduced in Section 3. Section 4 contains the re-analysis of a case-study. We end the paper with conclusions and avenues for future research in Section 5.

2 ELECTRE TRI

ELECTRE TRI was designed to assign a set of alternatives, objects or items (actions in general) to pre-defined and ordered categories. Each category is characterized by a lower and an upper profile. The assignment of an action to a certain category results from the comparison of the action with the profiles. The comparison is based on the credibility of the assertions “the action outranks the category profile and *vice-versa*”. In what follows, we will assume, without any loss of generality, that the scales of the criteria are ascending (therefore, all the criteria are to be maximized).

In this paper, we will use the following notation:

- $F = \{g_1, \dots, g_j, \dots, g_n\}$ is the set or family of *criteria*. Let \mathcal{J} denote the set of criterion indices.
- $A = \{a_1, \dots, a_i, \dots, a_m\}$ is the set of *actions*. Let I denote the set of action indices.
- $C = \{C_1, \dots, C_h, \dots, C_k\}$ is the set of *categories* in ascending preference order (C_1 is the “worst” category). Let \mathcal{C} denote the set of category indices.
- $B = \{b_1, \dots, b_h, \dots, b_{k-1}\}$ is the set of *profiles*. The profile b_h is the upper limit of category C_h and the lower limit of category C_{h+1} , for all $h \in \mathcal{B}$, where \mathcal{B} is the set of profile indices.
- $w = (w_1, \dots, w_j, \dots, w_n)$ is the *weight vector* modelling the preferences of DMs. For the sake of simplicity, let us assume that $\sum_{j \in \mathcal{J}} w_j = 1$ (normalized weights).
- $g_j(a_i)$ is the *evaluation* of action a_i on criterion g_j for all $i \in I$ and $j \in \mathcal{J}$.
- M is the evaluation matrix composed of $g_j(a_i)$ for all $i \in I$ and $j \in \mathcal{J}$.

The following comprehensive binary relations are used that allow to compare a_i and b_h :

- P is the *strict preference* relation, that is $a_i P b_h$ denotes the relation “ a_i is strictly preferred over b_h ”.
- I is the *indifference* relation, that is $a_i I b_h$ denotes the relation “ a_i is indifferent to b_h ”.
- Q is the *weak preference* relation, that is $a_i Q b_h$ denotes the relation “ a_i is weakly preferred over b_h ”, which means hesitation between indifference and strict preference.
- R is the *incomparability* relation, that is $a_i R b_h$ denotes that action a_i and b_h are incomparable.
- S is the *outranking* relation, that is $a_i S b_h$ denotes that “ a_i is at least as good as b_h ”.
- \succ is the *preference* (weak and strict) relation.

When the relational operator is subscripted (for example, S_j) it denotes that the relation holds with respect to the criterion indexed by the subscript.

The thresholds are denoted as follows:

- q_j is the *indifference threshold* for the criterion g_j . $q = (q_1, \dots, q_n)$ is the vector of indifference thresholds.
- p_j is the *preference threshold* for the criterion g_j . $p = (p_1, \dots, p_n)$ is the vector of preference thresholds.
- v_j is the *veto threshold* for the criterion g_j . $v = (v_1, \dots, v_n)$ is the vector of veto thresholds.

These thresholds can also vary along the scale of each criterion, and in ELECTRE TRI they are always defined on profiles and they can be interpreted as locally constant (see Roy and Bouyssou, 1993). In what follows we will consider variable thresholds, i.e. $q_j(g_j(b_h))$, $p_j(g_j(b_h))$, and $v_j(g_j(b_h))$.

The comparisons in ELECTRE TRI are based on the pseudo-criterion model. A pseudo-criterion is a function g_j associated with two threshold functions $q(g_j(\cdot))$ and $p(g_j(\cdot))$ satisfying the following conditions, for all $a, b \in A$ (Roy, 1996):

$$q(g_j(b)) - q(g_j(a)) \geq g_j(a) - g_j(b) \quad (1)$$

$$p(g_j(b)) - p(g_j(a)) \geq g_j(a) - g_j(b), \quad (2)$$

and such that, for all $a, b \in A$ with $g_j(a) \geq g_j(b)$:

$$aI_jb \Leftrightarrow g_j(a) \leq g_j(b) + q(g_j(b)) \quad (3)$$

$$aQ_jb \Leftrightarrow g_j(b) + q(g_j(b)) < g_j(a) \leq p(g_j(b)) \quad (4)$$

$$aP_jb \Leftrightarrow g_j(b) + p(g_j(b)) < g_j(a). \quad (5)$$

In Figure 1, an example of a profile scheme is presented graphically. The scheme consists of a set of four criteria $\{g_1, g_2, g_3, g_4\}$ with ascending scales, and a set of three categories $\{C_1, C_2, C_3\}$. The profile b_2 is the upper bound for C_2 and the lower bound for C_3 . The dashed lines represent two different actions. The evaluations of action a_1 on the left does not require any kind of preference information since it fits perfectly on the worst category, C_1 , and therefore it should be assigned to it. But with a_2 the situation is different: the assignment of a_2 depends on the definition of parameter values of ELECTRE TRI.

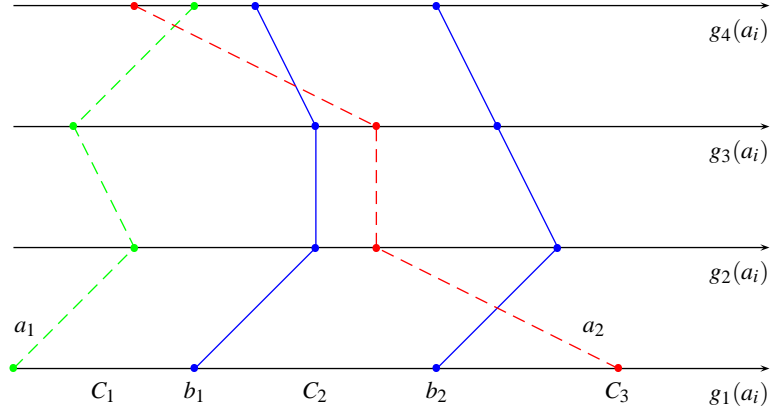


Figure 1: An example of a profile scheme.

2.1 The construction of an outranking relation

The construction of an outranking relation requires the definition of *credibility indices* for the outranking relations $a_i S b_h$ and $b_h S a_i$. Let $\rho(a_i, b_h)$ denote the credibility index of the assertion $a_i S b_h$. It is defined by using both a comprehensive concordance index, $c(a_i, b_h)$, and a discordance index for each criterion $g_j \in F$, that is, $d_j(a_i, b_h)$, for all $j \in J$. The definition of $\rho(b_h, a_i)$ is similar, with the exception that the thresholds in ELECTRE TRI are always computed based on the criterion value of the profile b_h . In what follows we only exemplify the computation for the relation $a_i S b_h$.

2.1.1 The comprehensive concordance index

The concordance index is computed by considering individually for each criterion g_j the support it provides for the assertion $a_i S_j b_h$. The partial concordance index is a fuzzy index measuring whether action a_i is at least as good as profile b_h on criterion g_j . The partial concordance indices are computed as follows, for all $j \in J$, $i \in I$, and $h \in \mathcal{B}$:

$$c_j(a_i, b_h) = \begin{cases} 1, & \text{if } g_j(a_i) \geq g_j(b_h) - q_j(g_j(b_h)), \\ 0, & \text{if } g_j(a_i) < g_j(b_h) - p_j(g_j(b_h)), \\ \frac{g_j(a_i) + p_j(g_j(b_h)) - g_j(b_h)}{p_j(g_j(b_h)) - q_j(g_j(b_h))}, & \text{otherwise.} \end{cases} \quad (6)$$

Therefore $c_j(a_i, b_h)$ increases linearly from 0 to 1, when $g_j(a_i)$ increases in the range

$$[g_j(b_h) - p_j(g_j(b_h)), g_j(b_h) - q_j(g_j(b_h))].$$

The definition of the partial concordance index is illustrated graphically in Figure 2.

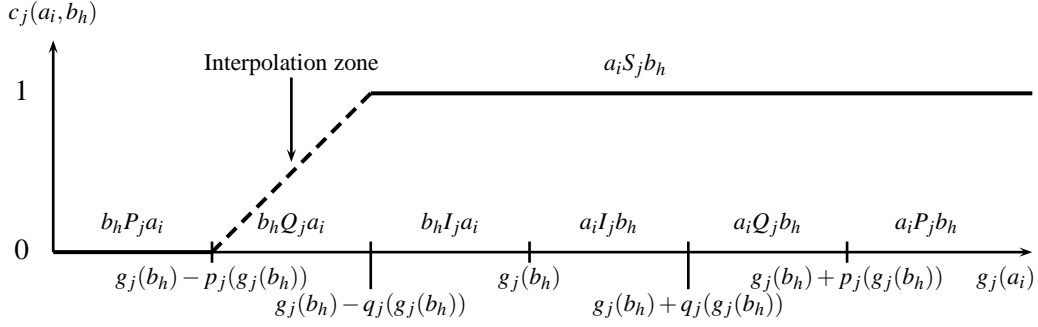


Figure 2: The partial concordance index $c_j(a_i, b_h)$.

After computing the partial concordance indices, the comprehensive concordance index is calculated as follows,

$$c(a_i, b_h) = \sum_{j \in \mathcal{J}} w_j c_j(a_i, b_h). \quad (7)$$

2.1.2 The partial discordance indices

The discordance of a criterion g_j describes the veto effect that the criterion provides against the assertion $a_i S_j b_h$. The discordance indices are computed separately for all criteria. A discordance index is also a fuzzy index, and it reaches the maximal value when criterion g_j puts its veto against the outranking relation. It is minimal when the criterion g_j is not discordant with that relation. To define the value of the discordance index on the intermediate zone a linear interpolation is used. The partial discordance indices are computed as follows, for all $j \in \mathcal{J}$, $i \in I$, and $h \in \mathcal{B}$:

$$d_j(a_i, b_h) = \begin{cases} 1, & \text{if } g_j(a_i) < g_j(b_h) - v_j(g_j(b_h)) \\ 0, & \text{if } g_j(a_i) \geq g_j(b_h) - p_j(g_j(b_h)) \\ \frac{g_j(b_h) - g_j(a_i) - p_j(g_j(b_h))}{v_j(g_j(b_h)) - p_j(g_j(b_h))}, & \text{otherwise.} \end{cases} \quad (8)$$

Therefore $d_j(a_i, b_h)$ decreases linearly from 1 to 0, when $g_j(a_i)$ increases in the range

$$]g_j(b_h) - v_j(g_j(b_h)), g_j(b_h) - p_j(g_j(b_h))].$$

The definition of the discordance index is illustrated graphically in Figure 3.

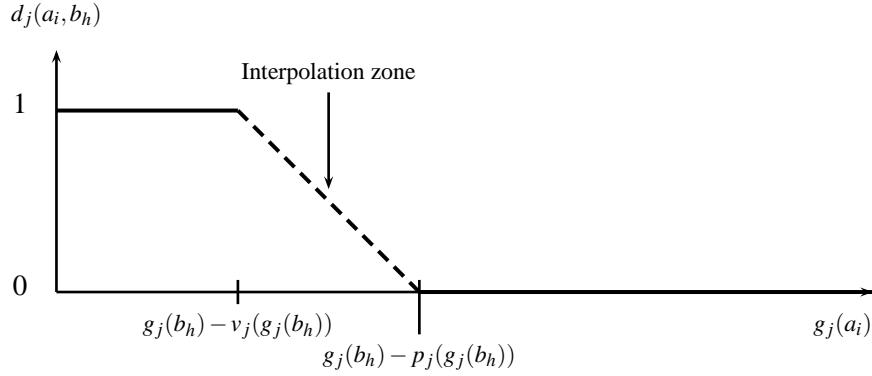


Figure 3: The partial discordance index $d_j(a_i, b_h)$.

2.1.3 The credibility index: a fuzzy outranking relation

The outranking relation is constructed by defining the credibility of the assertion $a_i S b_h$ as follows

$$\rho(a_i, b_h) = \begin{cases} c(a_i, b_h) \prod_{j \in V} \frac{1 - d_j(a_i, b_h)}{1 - c(a_i, b_h)}, & \text{if } V \neq \emptyset, \\ c(a_i, b_h), & \text{otherwise,} \end{cases} \quad (9)$$

with

$$V = \{j \in \mathcal{J} : d_j(a_i, b_h) > c(a_i, b_h)\}. \quad (10)$$

Notice that when $d_j(a_i, b_h) = 1$ for any $j \in \mathcal{J}$, this implies that $\rho(a_i, b_h) = 0$.

2.1.4 Converting a fuzzy relation into a crisp one

After determining the credibility index, the λ -cutting level has to be defined. The cutting level is used to transform the fuzzy outranking relation into a crisp one. It is defined as the smallest credibility index value compatible with the assertion $a_i S b_h$:

$$\begin{aligned} \rho(a_i, b_h) \geq \lambda &\Rightarrow a_i S b_h \\ \rho(a_i, b_h) < \lambda &\Rightarrow \neg a_i S b_h \\ \rho(b_h, a_i) \geq \lambda &\Rightarrow b_h S a_i \\ \rho(b_h, a_i) < \lambda &\Rightarrow \neg b_h S a_i \end{aligned} \quad (11)$$

The λ should be in the range $[0.5, 1]$, and it describes the summation of the weights of the coalition of criteria that must support the assertion $a_i S b_h$.

2.2 The exploitation procedure

The objective of the exploitation procedure is to exploit the binary relations introduced in the previous sections in order to assign actions to categories.

2.2.1 Comparing actions with profiles

The action a_i and the profile b_h can be compared by using the obtained relations. Based on different combinations, an action a_i can be preferred to a profile b_h (\succ) or *vice-versa*, they can be indifferent (I), or they can be incomparable (R). The fuzzy outranking relation can be decomposed into these crisp relations as follows:

1. $a_i I b_h \Leftrightarrow a_i S b_h \wedge b_h S a_i$
2. $a_i \succ b_h \Leftrightarrow a_i S b_h \wedge \neg b_h S a_i$
3. $b_h \succ a_i \Leftrightarrow \neg a_i S b_h \wedge b_h S a_i$
4. $a_i R b_h \Leftrightarrow \neg a_i S b_h \wedge \neg b_h S a_i$

The different possible relations are illustrated in Figure 4.

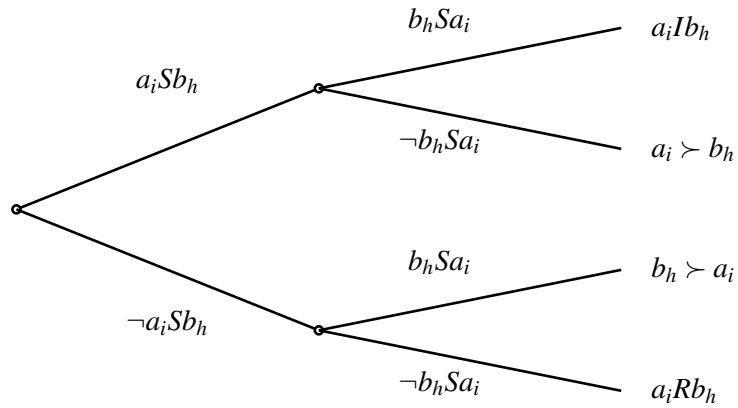


Figure 4: Definition of \succ , I , and R based on the outranking relation S .

2.2.2 Two rules for assigning actions to categories

The sorting procedure extends two well-known procedures: the conjunctive and the disjunctive (Figueira et al., 2005b). Based on these logics, there are two possible exploitation rules: the pessimistic and the optimistic. Let us consider two more “profiles”, b_0 (which every action is preferred to) and b_k (which is preferred over all actions), and let Δ denote the *dominance* relation. The profiles must be connected with the dominance relation as follows:

$$b_k \Delta b_{k-1} \Delta \dots \Delta b_h \Delta \dots \Delta b_1 \Delta b_0. \quad (12)$$

These are two possible rules for assigning actions to categories:

The pessimistic rule: In the pessimistic rule, an action a_i is successively compared with b_k, b_{k-1}, \dots , until $a_i S b_{k-1}$. Then a_i is assigned to the best category C_h such that $a_i S b_{h-1}$.

The optimistic rule: In the optimistic rule, an action a_i is successively compared with b_0, b_1, \dots , until $b_h \succ a_i$. Then a_i is assigned to the worst category C_h such that $b_h \succ a_i$.

3 SMAA-TRI

The fundamental idea of SMAA is to use Monte Carlo simulation for exploring the weight space in order to provide DMs with values characterizing the problem. The SMAA methodology has been developed for discrete stochastic MCDA problems with multiple DMs. The SMAA-2 method (Lahdelma and Salminen, 2001) applies inverse weight space analysis to describe for each action what kind of preferences make it the most preferred one, or place it on any particular rank. In SMAA, the criteria evaluations can be generated based on arbitrary distributions, or they can be sampled from an external source.

SMAA-TRI is developed for parameter stability analysis of ELECTRE TRI, and consists of analyzing finite spaces of arbitrarily distributed parameter values in order to describe for each action the share of parameter values that assign it to different categories. We analyze the stability of weights and the cutting level, and consider the remaining parameters to have deterministic values for easier comprehensibility. The method can easily be extended to consider non-deterministic values for thresholds.

For analyzing ELECTRE TRI, we will denote the input for ELECTRE TRI in SMAA-TRI as follows:

1. The lambda cutting level is presented by a stochastic variable Λ with a density function $f_L(\Lambda)$ defined within the valid range $[0.5, 1]$.

2. The weights are represented by a weight distribution with a joint density function $f_W(w)$ in the feasible weight space W . Total lack of preference information is represented in “Bayesian” spirit by a uniform weight distribution in W , that is, $f_W(w) = 1/\text{vol}(W)$. The weights are non-negative and normalized: the weight space is an $n - 1$ dimensional simplex in n dimensional space:

$$W = \left\{ w \in \mathbb{R}^n : w \geq 0 \text{ and } \sum_{j=1}^n w_j = 1 \right\}. \quad (13)$$

3. The data and the other parameters of ELECTRE TRI are represented by the set $T = \{M, B, q, p, v\}$. Recall that M is the criteria evaluation matrix and B is the set of profiles. These components are considered to have deterministic values for the sake of simplicity.

SMAA-TRI produces category acceptability indices for all pairs of actions and categories. The category acceptability index π_i^h describes the share of possible parameter values that have an action a_i assigned to category C_h , and it is most conveniently expressed percentage-wise. It is a generalization of the rank acceptability index of SMAA-2 (Lahdelma and Salminen, 2001). Let us define a *categorization function* that evaluates as the category index h to which an action a_i is assigned by ELECTRE TRI:

$$h = K(i, \Lambda, w, T), \quad (14)$$

and a category membership function

$$m_i^h(\lambda, w, T) = \begin{cases} 1, & \text{if } K(i, \Lambda, w, T) = h, \\ 0, & \text{otherwise,} \end{cases} \quad (15)$$

which is applied in computing the category acceptability index numerically as a multidimensional integral over the finite parameter spaces as

$$\pi_i^h = \int_0^1 f_L(\Lambda) \int_{w \in W} f_W(w) m_i^h(\Lambda, w, T) dw d\Lambda. \quad (16)$$

The category acceptability index measures the stability of the assignment, and it can be interpreted as a fuzzy measure or a probability for membership in the category. Evidently, the category acceptability indices are within the range $[0,1]$, where 0 indicates that the action will never be assigned to the category, and 1 indicates that it will be assigned to the category with any combination of feasible parameter values. For each action, the acceptabilities for different categories sum to unity. If the parameters are stable, the category acceptability indices for each action should be 1 for one category, and 0 for the others. In this case the assignments are said here to be robust with respect to the imprecise parameters.

The category acceptability indices provide a measure of uncertainty for the results of the sensitivity and robustness analyses as they were considered in ELECTRE TRI. While traditional way to

perform sensitivity analysis in ELECTRE TRI is to consider the extremes of what can be considered possible values for the imprecise parameters (Merad et al., 2004), the category acceptability indices consider the whole space which can be determined with arbitrary joint probability distributions. Therefore, while robustness analysis for ELECTRE TRI (Dias et al., 2002) provides a result such as “depending on the parameter values, the action is assigned either to category 2 or 3”, the SMAA-TRI provides the result as “the action is assigned to category 2 with 5% of the feasible parameter values, and to category 3 with 95% of the feasible parameter values”.

There are three advantages gained with the additional information:

1. The cognitive effort required in determining the extremes of the parameters considered in the sensitivity analysis is reduced, because the space can be determined to be, for example, uniformly distributed and thus small changes in the interval do not change the results dramatically.
2. Quantifying the amount of parameter values that result in “unstable” assignment determines the risk related with imprecise parameters. This will later be demonstrated in the re-analysis of the case study.
3. Weight elicitation techniques provide different weight values, and thus it seems more relevant to elicit the weights as imprecise values rather than deterministic ones (see Tervonen et al. (2004)).

In addition to providing parameter stability analysis, SMAA-TRI also allows ELECTRE TRI to be applied when multiple DMs with conflicting preferences participate in the decision making process. The method allows arbitrarily distributed weights, and therefore they can be defined, for example, as intervals containing the preferences of all DMs (Lahdelma and Salminen, 2001). In this case the results of the analysis (the category acceptability indices) can be used to find assignments accepted by majority of the DMs. Also the extremes of parameter combinations that assign actions to certain categories can be computed simultaneously with the parameter stability analysis.

The category acceptability indices are computed through Monte Carlo simulation, quite similarly as in SMAA-2. The algorithms for SMAA-2 together with analyses of complexity and running times have been presented by Lahdelma and Tervonen (2004). For more information on weight generation technique and handling of preference information, we advise the reader to consult Lahdelma and Tervonen (2004).

The ELECTRE TRI procedure is illustrated in Figure 5, and the SMAA-TRI simulation scheme in Figure 6. The algorithms for ELECTRE TRI are provided in Appendix A, and the algorithms for SMAA-TRI in Appendix B.

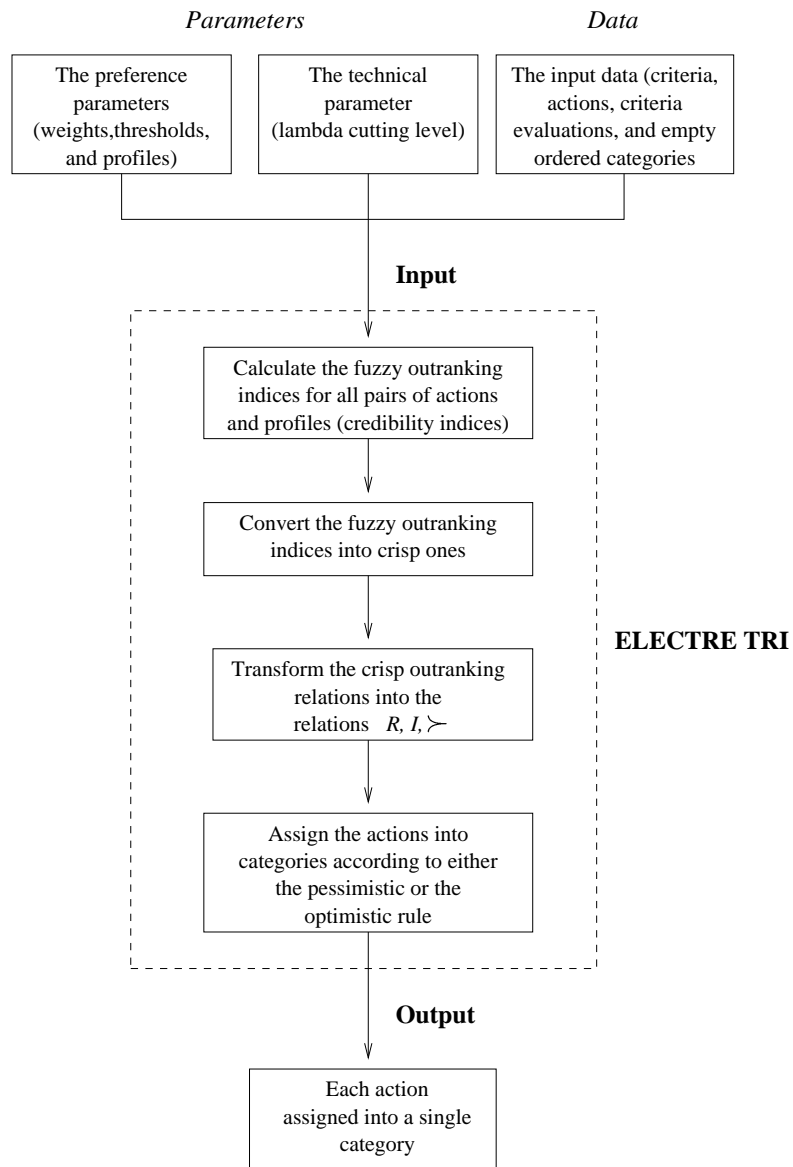


Figure 5: The ELECTRE TRI procedure.

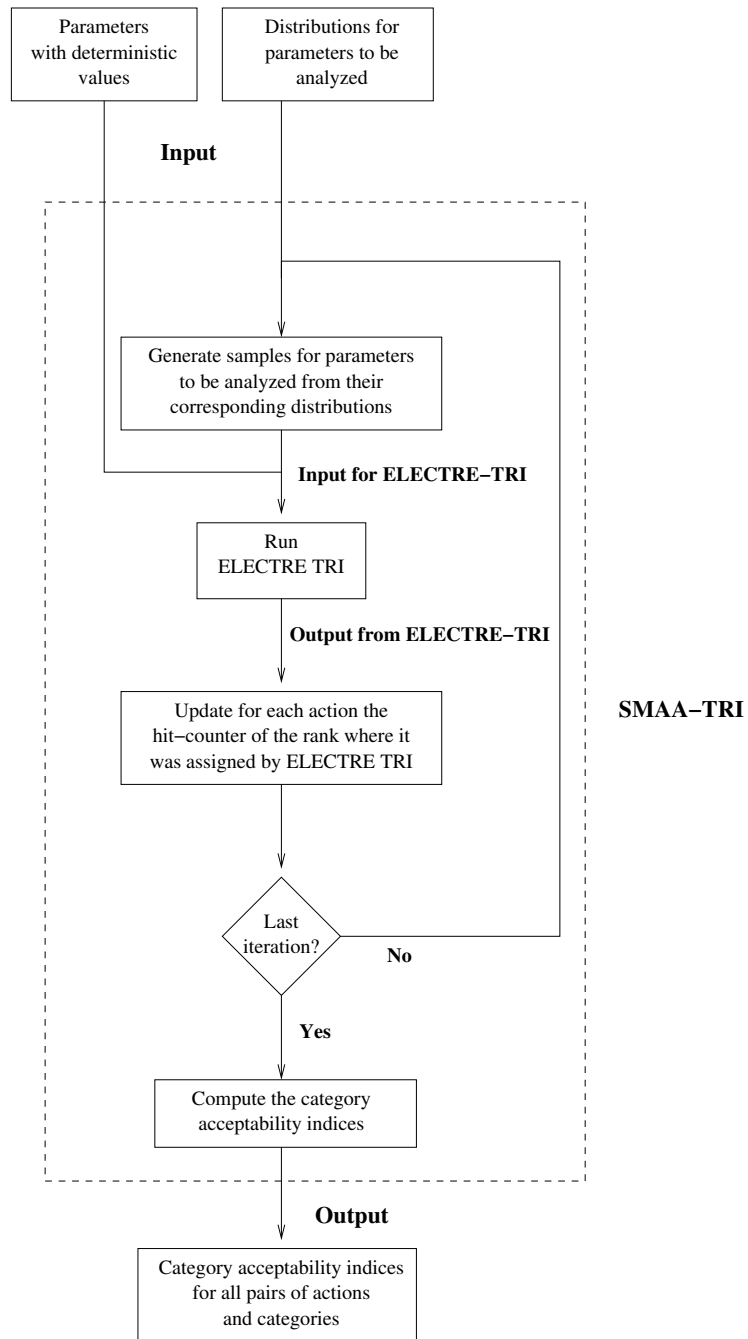


Figure 6: The SMAA-TRI simulation scheme.

4 Case study: experiments and results

In this case study we re-analyze the recent real-world application of ELECTRE TRI in the field of risk analysis. The original analysis is presented by Merad et al. (2004). The study concentrates on France's Lorraine region, where iron has been mined for more than a century. The underground mining tunnels have caused land subsidence, which has caused buildings to collapse. The object of this study was to partition land into zones and assign these zones into predefined risk categories in order to decide which zones need constant surveillance. We will re-analyze the assignment procedure by using the data provided in the case study.

The assignment phase consists of 10 homogenous zones (actions), a_1, \dots, a_{10} , that are evaluated in terms of 10 criteria, g_1, \dots, g_{10} . The criteria are presented in Table 1 (adapted from Merad et al., 2004). There are 4 risk categories where the zones are to be assigned, Category 1 is for zones with highest risk and Category 4 for lowest. The risk categories are separated by the three profiles b_1, b_2, b_3 . Performances of the zones together with profiles and thresholds are presented in Table 3 (adapted from Merad et al., 2004). The authors used the Revised Simos' procedure by Figueira and Roy (2002) to elicit the criteria weights. These weights are presented in Table 2.

The authors of the original case study used lambda cutting level of 0.65, but also analyzed the sensitivity of the results by altering the lambda to 0.7, 0.75, 0.8, and 0.85. In the sensitivity analysis also different profiles were applied, but the authors did not provide them in the paper. The results including the sensitivity analysis are presented in Table 4.

We performed stability analysis to this case study with SMAA-TRI. We chose cutting level to be represented by a stochastic variable uniformly distributed in the range [0.65,0.85]. The feasible weight space was defined with constraints provided in Table 5. These constraints are not probably the best constraints possible, as quantifying the imprecision should have been done along with the original case study.

Table 1: Criteria of the case study.

No.	Criterion	Nature	Units	Encoding (scale)	Direction risk increase
G1. Susceptibility of the mine to collapse					
g_1	Corrected mean stress applied on pillars	Quanti.	MPa	$k \times \frac{0.25 \times H}{1-\tau}$	+
g_2	Existence of fault	Quali.	(code)	0: no; 10: yes.	+
g_3	Superimposition of pillars	Quali.	(code)	0: only one mined layer; 10: two well superimposed layers or thick intermediate layer (≥ 7 m); 40: two bad superimposed layers or thin intermediate layer (≥ 7 m).	+
g_4	Size and regularity of pillars	Quali.	(code)	0: large pillars; 10: small regular pillars; 20: small irregular pillats.	+
g_5	Sensitivity of rock to flooding (depending on the rock type)	Quali.	(code)	0: no sensitivity; 10: sensitive; 20: very sensitive; 30: highly sensitive.	+
G2. Surface sensitivity					
g_6	Depth of the top mined layer	Quanti.	m	Given on maps. Called H.	-
g_7	Maximum expected subsidence	Quanti.	m	Deduced from subsidence models. Called A_m .	+
g_8	Expected surface deformation (deduced from subsidence models)	Quanti.	mm/m	$\epsilon_{\max} = 1.5 \times \frac{A_m}{H}$	+
g_9	Zone extent	Quanti.	km ²	Given on maps.	+
g_{10}	Vulnerability of building	Quali.	(code)	5: commercial zones; 10: isolated zones; 20: grouped houses; 30: long buildings; 40: urban road;	+

Table 2: Weights of the case study.

Weight	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
Non-normalized	5	1	1	1	5	1	1	20	1	10
Normalized	0.109	0.022	0.022	0.022	0.109	0.022	0.022	0.435	0.022	0.217

SMAA-TRI was executed with 10000 Monte Carlo iterations. The resulting category acceptability indices are presented in Table 6. Visualization of the results is important in SMAA methods, especially if there is a large amount of actions and/or criteria. Because the categories are ordered and therefore upwards inclusive, they are visualized with stacked columns in Figure 7.

The results of the re-analysis show the usefulness of SMAA-TRI. Although the stability analysis results are quite different from the ones by Merad et al. (2004), SMAA-TRI provides more information. For example, compare the sensitivity analysis results for Zone 5 in Table 4 and the

Table 3: Criteria performances, profiles, and thresholds.

Zone	Criterion									
	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
a_1	5.8	10	0	20	0	35	2.37	6.8	3.6	20
a_2	4.8	0	40	0	0	70	1.28	1.83	0.2	10
a_3	9.7	10	10	0	30	200	1.67	0.84	7.4	30
a_4	10.4	10	10	10	30	203	1.68	0.83	9.0	20
a_5	9.7	0	10	0	10	222	1.2	0.54	1.8	20
a_6	9.8	10	0	20	0	50	1.27	2.54	6.7	20
a_7	12.3	0	0	0	30	155	0.96	0.61	14.1	10
a_8	11.2	10	0	0	30	180	0.71	0.39	6.4	20
a_9	11.3	0	40	20	0	115	2.18	1.89	2.5	10
a_{10}	11.0	10	0	10	30	180	0.31	0.18	2.6	20
Profile										
b_1	14	10	40	20	20	110	1.8	1	35	30
b_2	10	10	10	10	10	150	1.4	0.82	20	20
b_3	8	0	10	10	10	190	1	0.63	6	20
Threshold										
q	0.05	0	0	0	0	10	0.1	0.05	0.5	0
p	0.1	0	0	0	0	20	0.2	0.09	1	0

Table 4: Original results of the case study and sensitivity analysis.

Zone	Result	Sensitivity Analysis
a_1	Category 1	Categories 1 and 2
a_2	Category 1	Categories 1 and 2
a_3	Category 2	Stable
a_4	Category 2	Stable
a_5	Category 4	Categories 3 and 4
a_6	Category 1	Categories 1 and 2
a_7	Category 3	Categories 3 and 4
a_8	Category 4	Categories 3 and 4
a_9	Category 1	Categories 1 and 2
a_{10}	Category 4	Stable

category acceptability indices for the same zone in Table 6. The original sensitivity analysis gives information that Zone 5 can be assigned to risk categories 3 or 4, and with this information the DMs (especially if they are risk-aware) should treat the zone as it would be assigned to risk category 3, which is of higher risk than category 4. But with the information provided by the category acceptability indices more informed decision can be made: regarding our imprecise and uncertain information about the parameters, we can quite safely (98% acceptability) place the zone in risk category 4.

Table 5: Weight constraints for the re-analysis.

Weight	Lower bound	Upper bound
w_1	3	7
w_2	0	2
w_3	0	2
w_4	0	2
w_5	3	7
w_6	0	2
w_7	0	2
w_8	15	25
w_9	0	2
w_{10}	7	13

Table 6: Category acceptability indices.

Zone	Category 1	Category 2	Category 3	Category 4
a_1	100	0	0	0
a_2	100	0	0	0
a_3	0	100	0	0
a_4	0	100	0	0
a_5	0	0	2	98
a_6	100	0	0	0
a_7	0	34	54	12
a_8	0	34	10	56
a_9	100	0	0	0
a_{10}	0	34	21	45

In this re-analysis using imprecise weights provides some interesting results. The original sensitivity analysis considered the assignment of Zone 10 stable, but by considering the weights imprecise ($\pm 30\%$), the assignment of the zone is quite unstable. With only 45% of the feasible parameter values the zone is placed in risk category 4, and a quite large share of the feasible values (34%) places the zone in risk category 2. If the original case study would have been performed with imprecise weight values, the actions chosen based on the assignment would probably have been quite different.

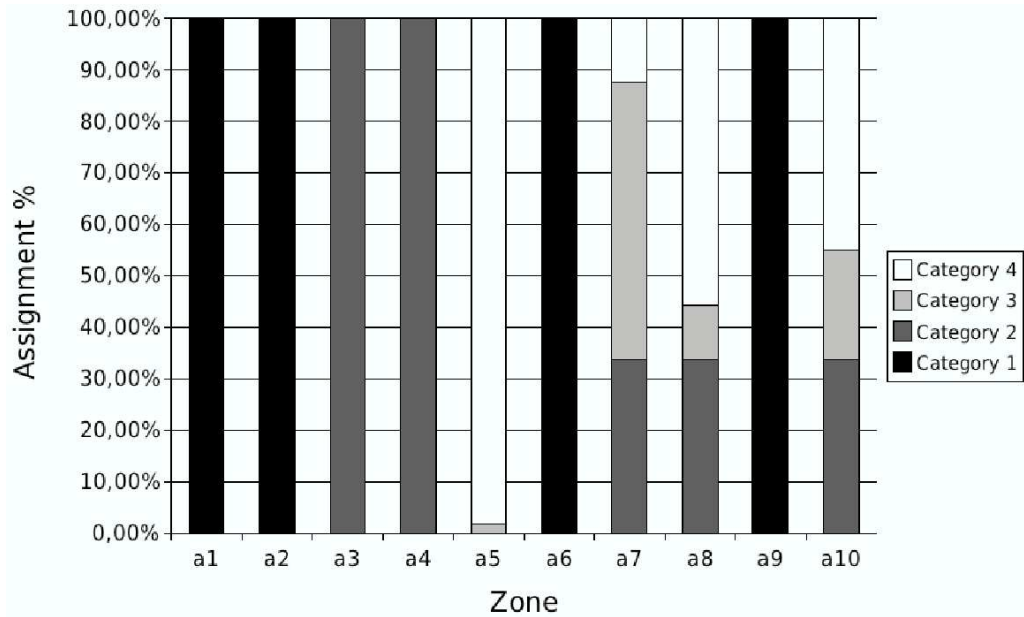


Figure 7: The category acceptability indices.

5 Conclusions and avenues for future research

Defining parameter values for ELECTRE TRI model is not an easy task. Moreover, if there are multiple DMs with conflicting preferences, it might even be impossible to reach consensus about weight values. With our approach the possibility to define the model by using stochastic variables “solves” these problems: the lambda cutting level can be defined with imprecise value, and the weights can be defined as intervals containing the preferences of all DMs.

In this paper we presented the SMAA-TRI method that allows ELECTRE TRI to be applied with stochastic values for lambda cutting level and weights. The SMAA-TRI analysis results in category acceptability indices for all pairs of actions and categories, and these can be used to analyze the stability of the parameters. The indices can be used also to derive robust conclusions, or if not possible, to quantify the “amount of instability” in the results induced by the imprecise parameter values.

We presented a re-analysis of the case study in which the usefulness of SMAA-TRI was demonstrated. By visualizing the category acceptability indices with stacked columns the uncertainty related with each assignment decision can be presented to the DMs in a comprehensible way. We provide the algorithms for the method as appendices, and hope that SMAA-TRI will be applied in future by decision analysts for deriving robust conclusions when ELECTRE TRI is chosen as the sorting method.

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Appendices

A The ELECTRE TRI algorithm

ELECTRE TRI algorithm is presented in four sequential phases:

- Phase 1 consists of calculating the fuzzy outranking indices for all pairs (a_i, b_h) and (b_h, a_i) , for all $i \in I$ and $h \in \mathcal{B}$. This procedure is presented in Algorithm 1.
- In Phase 2, the fuzzy outranking relation is converted into a crisp one as defined in Section 2.1.4. This procedure is presented in Algorithm 2.
- After this, in Phase 3, the crisp outranking relations are converted into relations R, I , and \succ . This procedure is similar to the scheme presented in Figure 4.
- In Phase 4 the actions are assigned into categories according to the pessimistic or the optimistic rule. The pessimistic rule is presented in Algorithm 3, and the optimistic in Algorithm 4.

The input of the algorithm consists of the basic data:

- A , the set of actions,
- F , the set of criteria,
- $g_j(a_i), \forall i \in I, j \in \mathcal{J}$, the criteria evaluations,
- C , the set of categories, $C_h = \emptyset, \forall h \in C$,

the preference parameters:

- B , the set of category profiles,
- w , the weight vector,
- $q_j(g_j(b_h)) \forall j \in \mathcal{J}, \forall h \in B$, the set of indifference thresholds,
- $p_j(g_j(b_h)) \forall j \in \mathcal{J}, \forall h \in B$, the set of preference thresholds,
- $v_j(g_j(b_h)) \forall j \in \mathcal{J}, \forall h \in B$, the set of veto thresholds,

and the technical parameter λ (the cutting level), that is defined in general by the decision analyst. The procedure assigns all actions into the pre-defined categories. The procedure leads to a partition (output) of the set of actions A in subsets or categories C_1, \dots, C_k :

$$\bigcup_{h \in C} C_h = A, \text{ and}$$
$$C_h \cap C_\ell = \emptyset, \forall h, \ell \in C, h \neq \ell.$$

```

1: for  $i \in I$  do
2:   for  $h \in \mathcal{B}$  do
3:     for  $j \in \mathcal{J}$  do
4:       if  $g_j(a_i) \geq g_j(b_h) - q_j(g_j(b_h))$  then
5:          $c_j(a_i, b_h) \leftarrow 1$ 
6:       else if  $g_j(a_i) < g_j(b_h) - p_j(g_j(b_h))$  then
7:          $c_j(a_i, b_h) \leftarrow 0$ 
8:       else
9:          $c_j(a_i, b_h) \leftarrow \frac{p_j(g_j(b_h)) + g_j(a_i) - g_j(b_h)}{p_j(g_j(b_h)) - q_j(g_j(b_h))}$ 
10:      end if
11:     if  $g_j(a_i) < g_j(b_h) - v_j(g_j(b_h))$  then
12:        $d_j(a_i, b_h) \leftarrow 1$ 
13:     else if  $g_j(a_i) \geq g_j(b_h) - p_j(g_j(b_h))$  then
14:        $d_j(a_i, b_h) \leftarrow 0$ 
15:     else
16:        $d_j(a_i, b_h) \leftarrow \frac{g_j(b_h) - g_j(a_i) - p_j(g_j(b_h))}{v_j(g_j(b_h)) - p_j(g_j(b_h))}$ 
17:     end if
18:   end for
19:    $c(a_i, b_h) \leftarrow \sum_{j \in \mathcal{J}} w_j c_j(a_i, b_h)$ 
20:    $V \leftarrow \{j \in \mathcal{J} : d_j(a_i, b_h) > c(a_i, b_h)\}$ 
21:   if  $V \neq \emptyset$  then
22:      $\rho(a_i, b_h) \leftarrow c(a_i, b_h) \prod_{j \in V} \frac{1 - d_j(a_i, b_h)}{1 - c(a_i, b_h)}$ 
23:   else
24:      $\rho(a_i, b_h) \leftarrow c(a_i, b_h)$ 
25:   end if
26: end for
27: end for

```

Algorithm 1: ELECTRE TRI, Phase 1: Calculating the fuzzy outranking indices.

```

1: for  $i \in I$  do
2:   for  $h \in \mathcal{B}$  do
3:     if  $\rho(a, b_h) \geq \lambda$  then
4:        $a_i S b_h$ 
5:     else
6:        $\neg a_i S b_h$ 
7:     end if
8:     if  $\rho(b_h, a_i) \geq \lambda$  then
9:        $b_h S a_i$ 
10:    else
11:       $\neg b_h S a_i$ 
12:    end if
13:   end for
14: end for

```

Algorithm 2: ELECTRE TRI, Phase 2: Converting the fuzzy outranking relation into a crisp one.

```

1: for  $i \in I$  do
2:    $h \leftarrow k$ 
3:   repeat
4:      $h \leftarrow h - 1$ 
5:   until  $a_i S b_h$ 
6:    $C_{h+1} \leftarrow C_{h+1} \cup \{a_i\}$ 
7: end for

```

Algorithm 3: ELECTRE TRI, Phase 4, the pessimistic rule: Assigns all actions into categories.

```

1: for  $i \in I$  do
2:    $h \leftarrow 0$ 
3:   repeat
4:      $h \leftarrow h + 1$ 
5:   until  $b_h \succ a_i$ 
6:    $C_h \leftarrow C_h \cup \{a_i\}$ 
7: end for

```

Algorithm 4: ELECTRE TRI, Phase 4, the optimistic rule: Assigns all actions into categories.

B The SMAA-TRI algorithm

The following symbols are used in the SMAA-TRI algorithm:

c A vector of category indices.

K The number of Monte Carlo simulations.

And the following functions and subroutines:

$RAND_W()$ Function returning a random weight vector from weight distribution f_W .

$RAND_\lambda()$ Function returning a value from lambda cutting level distribution f_λ .

$ELECTRE_TRI(w, \lambda, T)$ Execution of ELECTRE TRI. Returns a vector, where in position i is the category index in which action a_i is assigned with the given parameter values. Parameters are the following: w is the weight vector, λ is the cutting level, and T is the vector of parameters that have deterministic values through the simulation (see Section 2).

The procedure is presented in Algorithm 5.

```
1: for  $k \leftarrow 1$  to  $K$  do
2:    $w \leftarrow RAND_W()$ 
3:    $\lambda \leftarrow RAND_\lambda()$ 
4:    $c \leftarrow ELECTRE\_TRI(w, \lambda, T)$ 
5:   for  $i \in I$  do
6:      $\pi_i^{c_i} \leftarrow \pi_i^{c_i} + 1$ 
7:   end for
8: end for
9: for  $i \in I$  do
10:  for  $h \in C$  do
11:     $\pi_i^h \leftarrow \pi_i^h / K$ 
12:  end for
13: end for
```

Algorithm 5: The SMAA-TRI algorithm: Monte Carlo simulation to compute the category acceptability indices π_i^h .