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Forecasting intraday prices in financial markets using metaheuristics

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Abstract

Forecasting the intraday prices that stocks or financial indexes will achieve in a trading session is of great importance for defining the investors' position in the market. Three problems are studied in this paper: 1) forecasting a given time series using a nonlinear transformation of a modified $AR(p)$ model with the parameters being estimated by a metaheuristic procedure; 2) forecasting a bandwidth for the range of daily prices; 3) forecasting upper and lower bounds for the maximum and for the minimum daily prices. Extensive computational experiments are performed in order to define, for each problem, the best models for the NASDAQ, DAX, CAC40 and S&P financial series.

Key-Words: Forecasting, nonlinear models, financial markets, technical analysis, metaheuristics

1 Introduction

Many studies exist for forecasting financial time series, particularly the closing prices of stocks or financial indexes. The approaches consists mainly in time series methods like traditional processes AR, ARMA, ARIMA, GARCH, ARCH,... Simple methods such as weighted averages are also very popular in the technical analysis domain. Due to their restrictive assumptions such as linear or restricted nonlinear specifications, approximate methods are of increasing interest among researchers in financial market studies since its flexibility enable the use of more complex and more adequate representation models, without requiring any particularly constrained assumption necessary for estimation purposes. Therefore, less restrictive methods like genetic algorithms, scatter search, neural networks and fuzzy logic are being successfully applied in this field (Malliaris and Salchenberger, 1996, Weddig and Cios, 1996, Kim and Kim, 1997, Lendasse et al., 2000, Hwarng, 2001, Sheta and Jong, 2001, Zhang, 2003, Chen et al., 2005, Hwarng and Hui-Kuang, 2006, Gómez et al., 2007, Singh, 2007, Valenzuelaa et al., 2008, Zang and Kline, 2007, Gomes da Silva, 2008).

This paper is about the study of the behavior of intraday prices in financial markets. The daily behavior of the stock prices is a complex mix of several different forces induced by the expectations of the investors, reflecting their attitudes to the market. The action and reaction of the investors is heavily influenced by their time horizon: some are short term investors, other assume intermediate or long time interests. A complex combination of these horizons defines the market prices. The classical perspective is of the efficiency of the markets and as a consequence the behavior of prices is random in nature and cannot be predicted. Some other authors argue that there are at least some predictable periods of time in the prices pattern. The prediction field in the financial markets, and the daily process in particular, is a challenging task with practical interest since it can enhance the daily financial exposure of investors (McDonald, 2002; Mendelsohn, 2000). In this paper it is considered three estimation problems for the intraday maximum and minimum prices. Problem 1 is about the search for the best model for the maximum prices time series (M_t) and for the minimum prices time series (m_t) using an autoregressive process added by one different explanatory variable. Problem 2 is about the generation of a bandwidth for the daily interval prices, i.e., a bandwidth for the maximum and minimum prices. Problem 3 aims at finding the tightest lower and upper bound for the maximum and for the minimum time series with an additional constraint that imposes that the forecasted prices are always not above the maximum prices, for the first time series, and that the forecasted prices are always not below the minimum prices, for the second time series. This problem revealed to be much more difficult to solve, where the exact time series functions is a constrained barrier.

The models in each of the above problems consists of a nonlinear transformation of a mix of a linear autoregressive process with an additional explanatory variable. With a nonlinear specification, the best set of parameters for the models are difficult to find, justifying the application of a metaheuristic process for estimating the best set of parameters.

The paper is organized as follows. Section 2 presents the estimation process used to derive the best model for each time series in each problem. Section 3, 4 and 5 are devoted to problems 1, 2 and 3, respectively. Section 5 concludes the paper.

2 The estimation process

The estimation process is an optimization problem where a given error criteria is to be minimized. Here it is used the mean squared errors, MSE . Let X represent the set of parameters to be estimate, and X_k its k th component.

In Gomes da Silva (2008) it was proposed the application of the scatter search metaheuristic for deriving the set of parameters for a nonlinear autoregressive moving average process in the next section. This approach is modified and adapted for estimating the above different nonlinear models (5) - (6) .

The usual scatter search metaheuristic repeats the following five components (Glover, 1999; Laguna and Martí, 2003): 1) Diversification, 2) Improvement, 3) Reference set update, 4) Subset generation, and 5) Solution combination.

The five components of scatter search metaheuristic are summarized below:

1) Diversification: The diversification component seeks to generate a diversified population of solutions, P , that is representative of the search space.

2) Improvement: The improvement component starts by searching for a new parameter vector that reduces the current evaluation criterion, usually by conducting a local search around a good solution. The local search procedures are problem dependent.

3) Reference set update: The reference set, R , is composed of a solution subset subjected to an evolutionary process in an attempt to determine the optimal solution. One option is to define the reference set as composed of a small set of the best solutions found. This is an elitist reference set in that only the best solutions are maintained.

4) Subset generation: The subset generation step works to combine solutions from R to create new subsets, S^1, S^2, \dots

5) Solution combination: The combination step encourages the combination of the solutions contained in each subset in order to further explore their structure. Although, in order to discover other regions of the search space, it is also necessary to “escape” from the original solutions.

The methods Diversification, Reference set update, Subset generation and Solution combination are defined as in Gomes da Silva (2008). The Improvement method, was changed in order to explore more rapidly other search directions. The improvement method progressively searches a new value in a small neighborhood of X_k , $N_r(X_k)$. As in the original proposal, the objective is not to destroy completely the underlying solution, X , but rather to use it as a guide to obtain a better solution. This procedure is repeated until all the elements have been analyzed. A search direction, d , is built using the results concerning the MSE. An improvement loop moves along a search direction and it is repeated until a maximum number of iterations has been achieved. During the loop a new parameter vector is defined and if the new vector does not produce a better value for the MSE, then a reverse search direction is defined. This was not present in Gomes da Silva (2008). Expressed formally, the improvement procedure is:

```
Procedure Improve ( $X$ )
Begin
   $k \leftarrow 1$ ;
  Repeat
     $W \leftarrow X$ ;
```

```


$$Y_k \leftarrow N_{r_k}(X_k);$$

If  $\text{MSE}(W) < \text{MSE}(X)$  Then  $d_k \leftarrow W_k - X_k; X_k \leftarrow W_k$  else  $d_k \leftarrow X_k - W_k;$ 
 $k \leftarrow k + 1;$ 
Until  $(k > |X|)$ //find a better solution in a small neighborhood
 $k \leftarrow 1;$ 
Repeat
 $W \leftarrow X + \theta \times d;$ //search along direction  $d$  with  $\theta > 0$ 
If  $\text{MSE}(W) < \text{MSE}(X)$  Then
Begin
 $d \leftarrow W - X;$ 
 $X \leftarrow W;$ 
 $\theta \leftarrow \theta + \xi;$ //update the step-size
End
else  $d \leftarrow X - W; \theta \leftarrow \theta - \epsilon;$ //revert the search direction  $d$  using a smaller step-size
 $k \leftarrow k + 1;$ 
Until  $(k > \text{maximproves})$ 
End

```

where $N_{r_k}(X_k) \equiv \{\widetilde{X}_k : \widetilde{X}_k = \text{rand}(X_k(1 \pm r_k))\}$.

The overall structure of the proposed forecasting procedure is presented below. Beginning with a randomly-generated population, the algorithm evaluates each element of the population using the MSE, and a subset of the best members is used to define the reference set, R . Each element of R is then improved. An evolutionary phase is repeated until a maximum number of iterations has been achieved (stopping criterion). This loop contains the improvement, subset generation, and combination steps. After the evolutionary phase, X^* is the best set of parameters found for the original model.

Procedure Forecast NGAR⁺

```

Begin
 $P \leftarrow$ randomly generated vectors  $X : X^1, X^2, \dots, X^p;$ 
 $R \leftarrow$ The  $|R|$  vectors with the smallest MSE;
 $R \leftarrow \{ \text{Improve}(X^k) : X^k \in R \};$ 
Repeat
Generate subsets from  $R : S^1, \dots, S^v;$ //  $v$  subsets
 $k \leftarrow 1;$ 
 $list \leftarrow \emptyset;$ 
Repeat
Combine( $S^j$ );//define Candidates( $S^k$ )
 $list \leftarrow list \cup_{X^j \in \text{Candidates}(S_k)} \text{Improve}(X^j);$ 
 $k \leftarrow k + 1;$ 
Until  $(k > v)$ //all the subsets were analyzed
Update R with  $list;$ 
Until (stopping criterion is achieved)
 $X^* \leftarrow \text{argmin}\{\text{MSE}(X^j) : X^j \in R\}$ //the best set of parameters
End

```

In the experiments that follow the following running conditions were kept constant:
Population: 100 solutions; Reference Set: 20 solutions; Iterations: 100.

The initial solutions were randomly generated as follows: $\delta \in [-1, 1]$; $\alpha \in [-1, 1]$; $v \in [-1, 1]$, $r \in [0.25, 1]$.

Parameters of the improvement method: $\theta = 1.05$; $\xi = 1.05$; $\epsilon = 0.6$.

3 Problem 1: Modelling the maximum and the minimum prices

Two models are built for forecasting intraday maximum and minimum prices of stocks and financial indexes values using the opening value (o_t), the maximum value (M_t), the minimum value (m_t) and the closing value (c_t) information.

In the proposed models, the dependent variables maximum and minimum values of a stock or index are a function of their past values and of an additional explanatory variable that is the strength of the opening direction, i.e., $\phi_t = \frac{o_t - c_{t-1}}{c_{t-1}}$.

The past values are commonly sources of explanation, particularly in time series analysis. Moving averages, very popular in technical analysis studies, are also based on these sources of information. Although, it is also known that the past behavior in financial markets does not explain everything. The inclusion of ϕ_t is justified to mitigate this gap of explanation, trying to capture the information and expectations of investors concerning the stock, following the end of the previous session. It is assumed that ϕ_t is the beginning of the dynamic source that influences the daily behavior of the stock (indexes values), and can be seen as a proxy of other explanatory models, proposed by the fundamental school.

For the maximum value, M_t , the proposed model is:

$$M_t = F_M(M_{t-1}, \dots, M_{t-p_M}; \phi_t) + u_t^M \quad (1)$$

where u_t^M is a stochastic error term and p_M is the number of autoregressive terms of M_t .

A similar model is proposed for the minimum prices (values) time series:

$$m_t = F_m(m_{t-1}, \dots, m_{t-p_m}; \phi_t) + u_t^m \quad (2)$$

where u_t^m is a stochastic error term and p_m is the number of autoregressive terms of m_t .

Using linear functions to express $F_M(\bullet)$ and $F_m(\bullet)$, the above models become:

$$M_t = \alpha_o^M + \alpha_1^M M_{t-1} + \dots + \alpha_{t-p_M}^M M_{t-p_M} + \gamma^M \phi_t + u_t^M \quad (3)$$

$$m_t = \alpha_o^m + \alpha_1^m m_{t-1} + \dots + \alpha_{t-p_m}^m m_{t-p_m} + \gamma^m \phi_t + u_t^m \quad (4)$$

Without the variable ϕ_t the above models become the usual autoregressive models, namely

$AR(p_M)$ and $AR(p_m)$.

These are simple models and easy to deal with. Nevertheless, these linear models are not frequently suffice to correctly describe the complex pattern of economic and financial variables, with nonlinear models being more adequate (Clements et al., 2004). When using these kind of models, several difficult questions must be addressed. Indeed, as referred in Clement et al.

(2004), when using nonlinear models, much remain to be done in the areas of specification, estimation and testing.

One of the difficulties is that the parameters estimation is much harder, and exact alternative methods assume theoretical assumptions, which condition the set of nonlinear models to be used. Evolutionary procedures, which do not require specific assumptions, may be very interesting in this task.

In this paper it is only evaluated the ability of a linear combination of Gaussian functions are used to represent maximum (M_t) and minimum (m_t) prices time series. The models became:

$$M_t = \delta_0^M + \sum_{j=1}^{f_M} \delta_j^M \exp \left(- \left(\frac{(\alpha_o^M + \alpha_1^M M_{t-1} + \dots + \alpha_{t-p_M}^M M_{t-p_M} + \gamma^M \phi_t) - v_j^M}{r_j^M} \right)^2 \right) + \tilde{u}_t^M \quad (5)$$

$$m_t = \delta_0^m + \sum_{j=1}^{f_m} \delta_j^m \exp \left(- \left(\frac{(\alpha_o^m + \alpha_1^m m_{t-1} + \dots + \alpha_{t-p_m}^m m_{t-p_m} + \gamma^m \phi_t) - v_j^m}{r_j^m} \right)^2 \right) + \tilde{u}_t^m \quad (6)$$

where v_j^M, v_j^m are the centers of the Gaussian functions and r_j^M, r_j^m the radius of these functions.

Let these nonlinear models be noted by $\text{NGAR}_{(p_M, f_M)}^+$ and $\text{NGAR}_{(p_m, f_m)}^+$, respectively.

For simplification reasons in the text, let Y_t be the observation t ($t = 1, \dots, n$) of the dependent variable Y (M_t and m_t) and \hat{Y}_t its estimated value.

3.1 Empirical testing

The proposed models (5)-(6) and estimation procedure presented in Section 2 will be tested on the financial indexes Nasdaq, Dax, CAC40 and S&P. Several model specifications were tested: $\text{NGAR}_{(1,1)}^+$, $\text{NGAR}_{(1,2)}^+$, $\text{NGAR}_{(1,3)}^+$, $\text{NGAR}_{(2,1)}^+$, $\text{NGAR}_{(2,2)}^+$ and $\text{NGAR}_{(2,3)}^+$, and for each of them a set of 15 runs were performed. Thus, for each time series, 90 simulations were computed.

The data concerning each time series is divided into two subsets. One subset of data is used for training the models. The other is used for testing them.

The used daily data of each index, the total number of observations and the separation between training and testing sets are presented below.

Index	Period	Nº Obs.	Training	Testing
NASDAQ	16/11/2006-14/11/2008	549	499	50
DAX	05/03/2006-08/07/2008	366	316	50
CAC40	16/11/2006-14/11/2008	547	497	50
S&P	05/03/2007-08/07/2008	366	316	50

The original data was standardized by dividing all the transaction values of each series by the highest value of it. Figures 1 to 4 shows the behavior of the above referred financial indexes, for the period of time presented above, and where the vertical line separates the training from the test data.

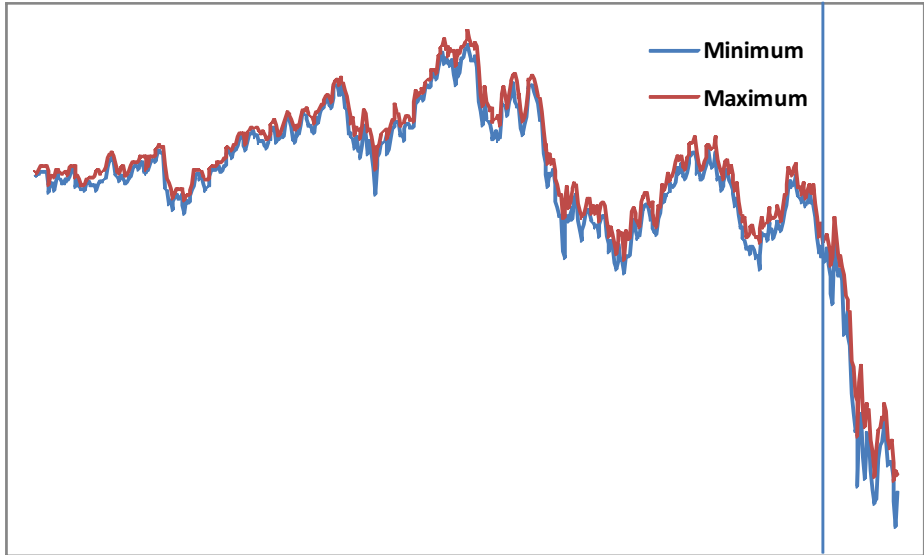


Figure 1: NASDAQ

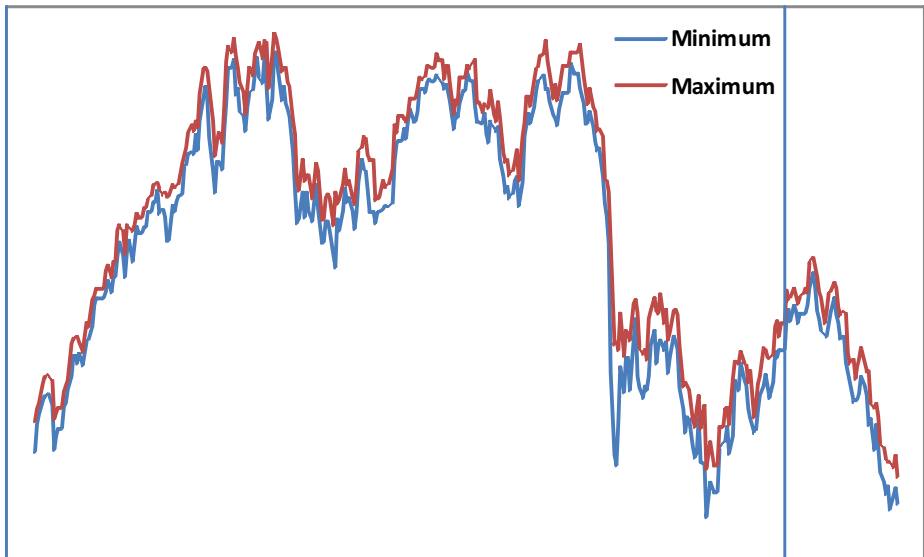


Figure 2: DAX

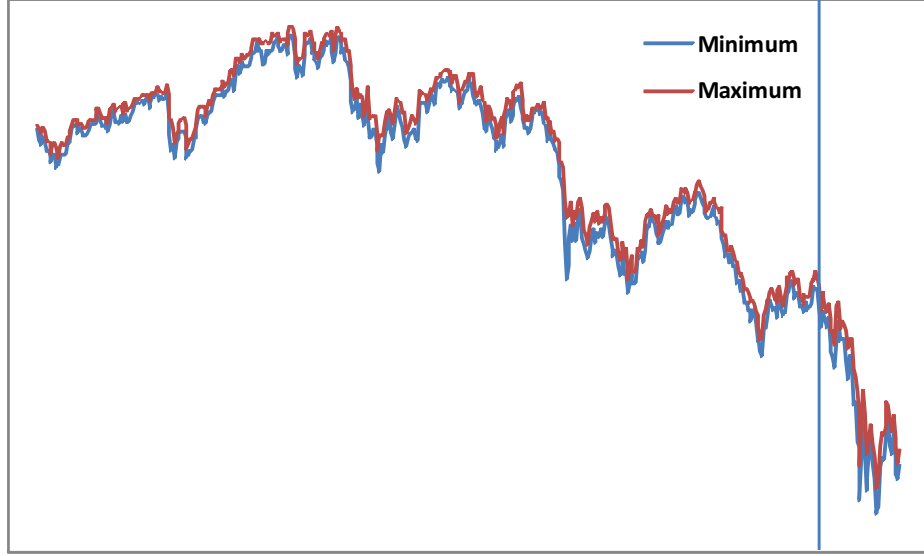


Figure 3: CAC40

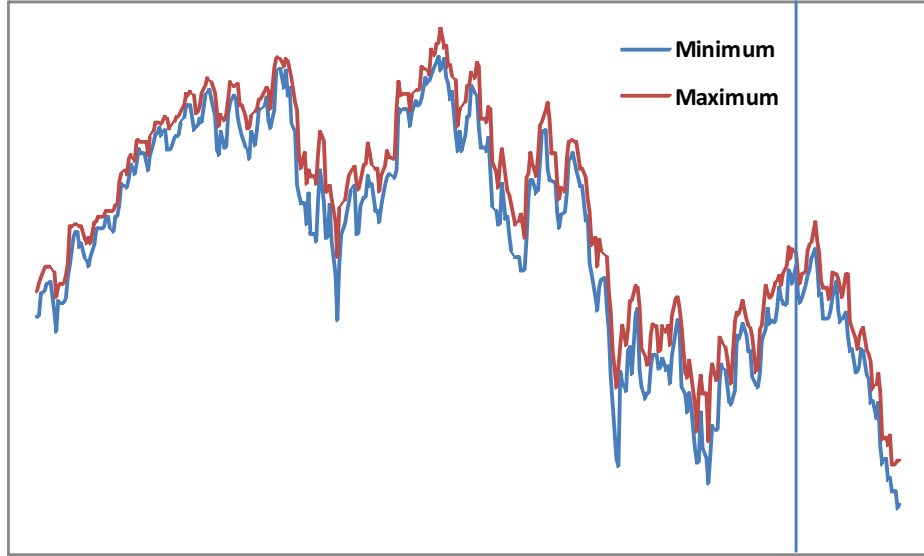


Figure 4: S&P

The optimization criterion is the MSE (or RMSE), but several other error measures are computed and presented such as the MAE, MaxAPE and MaxAPE. All the measures are defined as follows:

$$MSE = \frac{\sum_{t=p_k+1}^n (Y_t - \hat{Y}_t)^2}{n-p_k}; \quad RMSE = \sqrt{\frac{\sum_{t=p_k+1}^n (Y_t - \hat{Y}_t)^2}{n-p_k}}; \quad MAE = \frac{\sum_{t=p_k+1}^n |Y_t - \hat{Y}_t|}{n-p_k}; \quad MaxAE = \max_{t=p_k+1, \dots, n} \left\{ |Y_t - \hat{Y}_t| \right\}; \quad MaxAPE = \max_{t=p_k+1, \dots, n} \left\{ \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \right\} \times 100.$$

In the estimation process presented in Section 2, $X = (\delta, \alpha, v, r)$, with $\delta = (\delta_0^k, \delta_1^k, \dots, \delta_{f_k}^k)$, $\alpha = (\alpha_0^k, \alpha_1^k, \dots, \alpha_{t-p_k}^k, \gamma^k)$, $v = (v_1^k, \dots, v_{f_k}^k)$, $r = (r_1^k, \dots, r_{f_k}^k)$ where $k = M, m$.

<i>RMSE</i>	$\text{NGAR}_{(1,1)}^+$	$\text{NGAR}_{(1,2)}^+$	$\text{NGAR}_{(1,3)}^+$	$\text{NGAR}_{(2,1)}^+$	$\text{NGAR}_{(2,2)}^+$	$\text{NGAR}_{(2,3)}^+$
Nasdaq						
<i>Average</i>	0,0074366	0,0078698	*0,0074025	0,0080168	0,0086996	0,0087946
<i>Maximum</i>	0,0086681	0,0083100	*0,0081308	0,0127615	0,0110850	0,0136370
<i>Minimum</i>	0,0068975	0,0069003	0,0068960	0,0068796	*0,0068498	0,0069216
DAX						
<i>Average</i>	*0,0084704	0,0086477	0,0105175	0,0101871	0,0086294	0,0104611
<i>Maximum</i>	0,0094664	*0,0094092	0,0149959	0,0143516	0,0094562	0,0144731
<i>Minimum</i>	0,0073990	0,0073938	0,0079213	*0,0073713	0,0073919	0,0077824
CAC40						
<i>Average</i>	0,0080609	*0,0078193	0,0079703	0,0092007	0,0101116	0,0080609
<i>Maximum</i>	0,0090682	0,0088518	*0,0087553	0,0126005	0,0154534	0,0090682
<i>Minimum</i>	0,0067326	*0,0067187	0,0067330	0,0067834	0,0069364	0,0067326
S&P						
<i>Average</i>	*0,0072778	0,0072840	0,0075990	0,0092413	0,0093307	0,0092413
<i>Maximum</i>	*0,0077735	0,0082916	0,0078909	0,0121178	0,0123997	0,0121178
<i>Minimum</i>	0,0070179	0,0070089	0,0070294	0,0070122	*0,0069534	0,0070122

Table 1: Results for the maximum time series (Problem 1)

For the maximum time series, the best average models, not necessarily the absolute best models, are obtained with one autoregressive term and a number of functions varying between 1 and 3 (see Table 1, where the asterisk means the minimum value of the line). These results are compatible with a short time memory in the stock prices behavior.

For the best average model of each time series, the evaluation measures MSE, RMSE, MAE, MaxAE and MaxAPE are displayed in Table 2. The ability of the models to forecast outside the training sample increases just in the case of the DAX index, with the measures having better values for the testing sample. For the other time series the models seems to lose its ability to represent the unknown values. This feature may be attributed to a different pattern of the data, not present in the training sample (see Figures 1-4).

For the minimum time series the best average results are also obtained with simple models (see Table 3): one autoregressive term and no more than two nonlinear functions. The overall minimum is obtained with two autoregressive terms and one nonlinear function. The worst values are achieved with three nonlinear functions and two autoregressive terms, except for the NASDAQ index, where that model is the $\text{NGAR}_{(2,2)}^+$.

Table 4 displays the evaluation measures MSE, RMSE, MAE, MaxAE and MaxAPE for the best average model for each time series. The performance of the models, outside the training sample, increases just for the DAX and S&P indexes.

<i>DataSet</i>	<i>MSE</i>	<i>RMSE</i>	<i>MAE</i>	<i>MaxAE</i>	<i>MaxAPE</i>
NASDAQ					
<i>Serie</i>	0,0002527	0,0158968	0,0082857	0,0899048	16,4619135
<i>Training</i>	0,0000476	0,0068960	0,0051427	0,0226745	2,8666992
<i>Test</i>	0,0022551	0,0474883	0,0395893	0,0899048	16,4619135
DAX					
<i>Serie</i>	0,0000522	0,0072261	0,0053387	0,0278749	3,1471820
<i>Training</i>	0,0000547	0,0073990	0,0055078	0,0278749	3,1471820
<i>Test</i>	0,0000363	0,0060236	0,0042736	0,0186811	2,2972783
CAC40					
<i>Serie</i>	0,0000641	0,0080077	0,0054898	0,0459891	8,4681497
<i>Training</i>	0,0000451	0,0067187	0,0048153	0,0325058	4,0777173
<i>Test</i>	0,0002524	0,0158878	0,0121801	0,0459891	8,4681497
S&P					
<i>Serie</i>	0,0000499	0,0070647	0,0050573	0,0239305	2,7957192
<i>Training</i>	0,0000491	0,0070179	0,0050322	0,0239305	2,6477900
<i>Test</i>	0,0000549	0,0074102	0,0052183	0,0236279	2,7957192

Table 2: Evaluation measures for the maximum times series (Problem 1)

<i>RMSE</i>	$\text{NGAR}_{(1,1)}^+$	$\text{NGAR}_{(1,2)}^+$	$\text{NGAR}_{(1,3)}^+$	$\text{NGAR}_{(2,1)}^+$	$\text{NGAR}_{(2,2)}^+$	$\text{NGAR}_{(2,3)}^+$
NASDAQ						
<i>Average</i>	0,0085713	*0,0084010	0,0084040	0,0096211	0,0102169	0,0103853
<i>Maximum</i>	0,0092548	*0,0092240	0,0092620	0,0133099	0,0148145	0,0142159
<i>Minimum</i>	0,0078261	0,0078393	0,0078372	*0,0077802	0,0079205	0,0083571
DAX						
<i>Average</i>	*0,0094302	0,0094691	0,0095618	0,0106680	0,0113228	0,0121156
<i>Maximum</i>	*0,0103911	0,0107524	0,0104605	0,0158471	0,0152653	0,0160732
<i>Minimum</i>	0,0084156	0,0084158	0,0084158	0,0082771	*0,0083334	0,0086317
CAC40						
<i>Average</i>	0,0084328	*0,0083711	0,0087169	0,0102302	0,0100081	0,0106633
<i>Maximum</i>	0,0098882	*0,0097620	0,0098204	0,0135903	0,0144520	0,0151031
<i>Minimum</i>	0,0071135	0,0071127	0,0071125	*0,0070918	0,0072242	0,0071140
S&P						
<i>Average</i>	0,0087259	*0,0085912	0,0087730	0,0106859	0,0095765	0,0110179
<i>Maximum</i>	*0,0091142	0,0091197	0,0090841	0,0136479	0,0114644	0,0141728
<i>Minimum</i>	0,0081467	0,0081360	0,0081462	*0,0080859	0,0083117	0,0087866

Table 3: Results for the minimum time series (Problem 1)

<i>DataSet</i>	<i>MSE</i>	<i>RMSE</i>	<i>MAE</i>	<i>MaxAE</i>	<i>MaxAPE</i>
NASDAQ					
<i>Serie</i>	0,0002457	0,0156737	0,0089675	0,0959142	19,2125777
<i>Training</i>	0,0000615	0,0078393	0,0060597	0,0365569	4,4966176
<i>Test</i>	0,0020419	0,0451874	0,0379293	0,0959142	19,2125777
DAX					
<i>Serie</i>	0,0000674	0,0082081	0,0058286	0,0620586	7,4802969
<i>Training</i>	0,0000709	0,0084156	0,0059493	0,0620586	7,4802969
<i>Test</i>	0,0000452	0,0067250	0,0050682	0,0186038	2,2313751
CAC40					
<i>Serie</i>	0,0000840	0,0091649	0,0061965	0,0492565	9,4710312
<i>Training</i>	0,0000506	0,0071127	0,0051886	0,0414045	5,4030492
<i>Test</i>	0,0004154	0,0203809	0,0161956	0,0492565	9,4710312
S&P					
<i>Serie</i>	0,0000641	0,0080093	0,0060506	0,0340556	4,0202915
<i>Training</i>	0,0000662	0,0081360	0,0060893	0,0340556	4,0202915
<i>Test</i>	0,0000513	0,0071599	0,0058066	0,0160723	1,9741264

Table 4: Evaluation measures for the minimum times series (Problem 1)

<i>Data Set</i>	<i>Time series</i>	<i>NASDAQ</i>	<i>DAX</i>	<i>CAC40</i>	<i>S&P</i>
<i>Entire data</i>	<i>Maximum</i>	0,9773	0,9868	0,9952	0,9763
	<i>Minimum</i>	0,9797	0,9839	0,9944	0,9728
<i>Test data</i>	<i>Maximum</i>	0,9702	0,9655	0,9608	0,9477
	<i>Minimum</i>	0,9606	0,9635	0,9497	0,9683

Table 5: Coefficients of determination (Problem 1)

In order to assess the quality of the approximation, it was computed the coefficient of determination, R^2 (see Table 5). For all the financial indexes the coefficients are high, revealing a good property for the representation of a time series. The values presented in Table 5 state that for all the financial indexes the explanation ability of the model is at least 94%.

The results show that the models and the approach are adequate to represent the behavior of the financial variables under study.

4 Problem 2: Modelling a bandwidth for the daily interval prices

Providing the investors with an interval (bandwidth) variation of the daily prices is particularly interesting for designing their investments strategies. Despite the importance of this problem, it is a less explored research topic. A suggestion for this problem can be found in Mendelsohn (2000, p. 82).

From the estimation point of view, this is not a trivial problem. Indeed, such an interval can not be simply obtained by joining the separate estimations of each time series (maximum and

minimum time series) since it may lead to an incoherent bandwidth, where the estimation for the maximum values are below than the estimation for the minimum values. This inconsistency is particularly present when the financial variables shows a small daily gap.

Let a bandwidth be defined in a coherent manner, i.e., as an estimated interval with the constraint that the upper bound is never lower than the lower bound. Figure 5 illustrates the meaning of a coherent bandwidth, where the dotted lines correspond to estimated values.

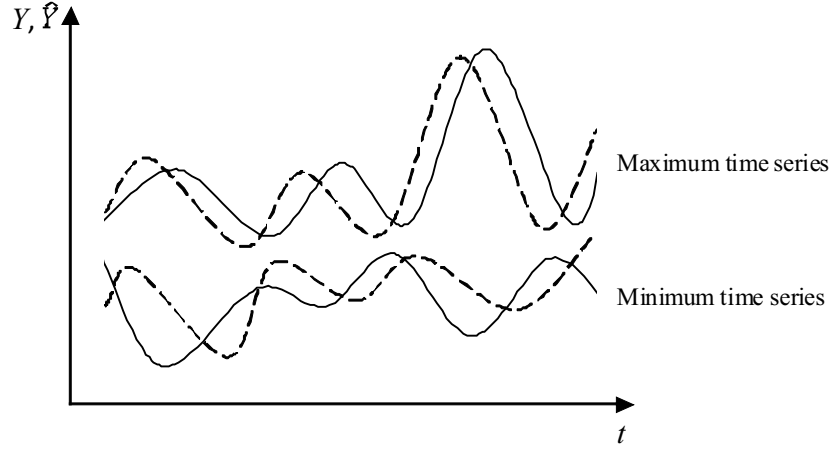


Figure 5: A coherent bandwidth

The upper bound of the bandwidth is an estimation of the time series maximum prices and the lower bound of the bandwidth is an estimation for the time series minimum price. As estimations, the upper bound is allowed to be greater or smaller than the real maximum prices and the lower bound can also be greater or smaller than the minimum price.

The proposed estimation process for the bandwidth is as follows.

Firstly, it is computed a time series that is the average of the maximum and the minimum prices time series of the underlying time series, $\bar{Y}_t = \frac{M_t + m_t}{2}, t = 1, \dots, n$.

Then, it is estimated a model for fitting this new time series. This model is only restricted to be found among the nonlinear $AR(p)$ modified processes, as presented in the previous section.

After having estimated the model, $\widehat{Y}_t, t = p+1, \dots, n$, the bandwidth for each period of time, t , is given by:

$$\widehat{B}_t \equiv \left[\widehat{Y}_t + \widehat{\pi}^M; \widehat{Y}_t - \widehat{\pi}^m \right]$$

where the estimated parameters $\widehat{\pi}^M$ and $\widehat{\pi}^m$ are defined in order to minimize the sum of the MSE, TMSE:

$$TMSE = \frac{\sum_{j=p+1}^n \left(\widehat{Y}_t + \widehat{\pi}^M - M_t \right)^2}{n-p} + \frac{\sum_{j=p+1}^n \left(\widehat{Y}_t - \widehat{\pi}^m - m_t \right)^2}{n-p}$$

It can be easily proved that the optimal parameters, $\widehat{\pi}^M$ and $\widehat{\pi}^m$ are:

<i>TRMSE</i>	NGAR ⁺ _(1,1)	NGAR ⁺ _(1,2)	NGAR ⁺ _(1,3)	NGAR ⁺ _(2,1)	NGAR ⁺ _(2,2)	NGAR ⁺ _(2,3)
NASDAQ						
<i>Average</i>	*0,0181692	0,0182604	0,0201229	0,0211599	0,0199196	0,0248413
<i>Maximum</i>	*0,0206655	0,0232838	0,0370334	0,0427896	0,0313167	0,0731904
<i>Minimum</i>	0,0171315	0,0171386	*0,0171271	0,0171495	0,0171625	0,0171500
DAX						
<i>Average</i>	0,0170228	*0,0170211	0,0170212	0,0172081	0,0171204	0,0171279
<i>Maximum</i>	0,0170624	0,0170327	*0,0170312	0,0187726	0,0171857	0,0171752
<i>Minimum</i>	*0,0170173	0,0170180	0,0170176	0,0170467	0,0170482	0,0170503
CAC40						
<i>Average</i>	*0,0165107	0,0169456	0,0167254	0,0178256	0,0169456	0,0174664
<i>Maximum</i>	*0,0174139	0,0184815	0,0182728	0,0212157	0,0184815	0,0290007
<i>Minimum</i>	0,0160881	0,0161103	0,0160907	0,0160933	0,0161103	*0,0160858
S&P						
<i>Average</i>	0,0168506	0,0168540	0,0168555	*0,0168129	0,0169105	0,0168270
<i>Maximum</i>	0,0168670	0,0168662	0,0169004	*0,0168637	0,0174724	0,0169084
<i>Minimum</i>	0,0168308	0,0168357	0,0168292	*0,0167587	0,0167921	0,0167858

Table 6: Results for the Bandwidth (Problem 2)

$$\widehat{\pi}^M = \frac{\sum_{j=p+1}^n (M_t - \widehat{Y}_t)}{n-p}; \widehat{\pi}^m = \frac{\sum_{j=p+1}^n (\widehat{Y}_t - m_t)}{n-p}$$

The best bandwidth, B^* , is the one that, in the evolutionary process, minimizes the sum of the MSE concerning the maximum and the minimum time series, i.e., with the lowest value of TMSE.

4.1 Empirical testing

Table 6 displays the results about the *TRMSE* (squared root of TMSE) for the different tested nonlinear models and for the four financial indexes: NASDAQ, DAX, CAC40 and S&P. The average, maximum and minimum of the set of experiments (also 15 runs) are shown. A first comment is that the most suitable (on average) models for the bandwidth are different from the most suitable models for modeling each time series (see results for Problem 1). For all the financial indexes, the number of nonlinear functions remains small (no more than 2). The number of autoregressive terms is one, except for the S&P index.

Considering the model associated with the best average results for each financial index, Tables 7 and 8 display the evaluation measures concerning both the maximum and minimum upper bound for the best experiment with that model. For all the financial series, the results concerning the maximum time series are better than for the minimum time series.

<i>DataSet</i>	<i>MSE</i>	<i>RMSE</i>	<i>MAE</i>	<i>MaxAE</i>	<i>MaxAPE</i>
NASDAQ					
<i>Serie</i>	0,000066	0,008096	0,005851	0,038397	6,163727
<i>Training</i>	0,000045	0,006690	0,005084	0,029792	3,674380
<i>Test</i>	0,000273	0,016509	0,013493	0,038397	6,163727
DAX					
<i>Serie</i>	0,000051	0,007173	0,005159	0,039196	7,217335
<i>Training</i>	0,000038	0,006171	0,004597	0,034173	4,298207
<i>Test</i>	0,000184	0,013569	0,010736	0,039196	7,217335
CAC40					
<i>Serie</i>	0,000043	0,006535	0,004984	0,026727	3,235122
<i>Training</i>	0,000045	0,006727	0,005130	0,026727	3,235122
<i>Test</i>	0,000027	0,005168	0,004063	0,012103	1,442489
S&P					
<i>Serie</i>	0,000044	0,006627	0,004943	0,030365	3,574625
<i>Training</i>	0,000044	0,006652	0,004923	0,030365	3,574625
<i>Test</i>	0,000042	0,006473	0,005069	0,019799	2,454405

Table 7: Evaluation measures for the maximum times series (Problem 2)

For the NASDAQ and DAX indexes, the performance of the model decreased in the test data, both in the maximum and minimum time series. This reveals that the model is not so adequate to represent out of sample data. For the CAC40 and S&P indexes, the forecasting ability of the models for out of sample data is present with a better performance of the evaluation measures in the test data.

Observing the maximum absolute percentual error, it can be seen that they are obtained with the minimum time series, except for the S&P index. In the four indexes, this measure varies between 2,5% and 8,5% for the training data and between 1,4% and 10,4% in the test data.

Once again, the MSE and RMSE measures reveal a tight approximation of the model to the data. The coefficient of determination confirm the above statement (see Table 9).

It is interesting to observe that the results for Problem 2, both for the maximum and the minimum time series, are comparable (even better in some statistics) to the ones obtained for Problem 1. This shows that the simultaneous use of intraday extreme prices can provide a good source of information for the forecasting ability of the models.

5 Problem 3: Modelling upper and lower bounds

One of the features of the estimations in Problems 1 and 2 is that the estimated values are placed above and below the real values. Consequently, if the forecasted maximum is above the real one then an investment order of selling at that level was not processed and a possible earning is not fulfilled. In the same way, if the forecasted minimum is below the real one then an investment order of buying at that level was not processed and possible future earnings with the stock are not possible. When the estimates are used for forecasting, the investor has an additional task of assuming the risk of placing its order equal, higher or lower than the forecasted value.

<i>DataSet</i>	<i>MSE</i>	<i>RMSE</i>	<i>MAE</i>	<i>MaxAE</i>	<i>MaxAPE</i>
NASDAQ					
<i>Serie</i>	0,000143	0,011978	0,008101	0,064296	11,298871
<i>Training</i>	0,000065	0,008052	0,006421	0,038471	4,157262
<i>Test</i>	0,000927	0,030443	0,024841	0,064296	11,298871
DAX					
<i>Serie</i>	0,000103	0,010173	0,006874	0,054867	10,64453
<i>Training</i>	0,000057	0,007534	0,005633	0,049074	6,403855
<i>Test</i>	0,000567	0,023815	0,019180	0,054867	10,64453
CAC40					
<i>Serie</i>	0,000078	0,008812	0,006379	0,068517	8,258703
<i>Training</i>	0,000084	0,009152	0,006642	0,068517	8,258703
<i>Test</i>	0,000039	0,006260	0,004718	0,018006	2,127306
S&P					
<i>Serie</i>	0,000060	0,007754	0,006112	0,021846	2,540004
<i>Training</i>	0,000061	0,007781	0,006117	0,021846	2,540004
<i>Test</i>	0,000057	0,007579	0,006077	0,018715	2,336970

Table 8: Evaluation measures for the minimum times series (Problem 2)

<i>Data Set</i>	<i>Time series</i>	<i>Nasdaq</i>	<i>Dax</i>	<i>CAC40</i>	<i>S&P</i>
<i>Entire data</i>	<i>Maximum</i>	0,9900	0,9894	0,9958	0,9795
	<i>Minimum</i>	0,9855	0,9817	0,9938	0,9756
<i>Test data</i>	<i>Maximum</i>	0,9718	0,9765	0,9610	0,9685
	<i>Minimum</i>	0,9569	0,9683	0,9530	0,9626

Table 9: Coefficients of determination (Problem 2)

An additional useful information for this purpose is providing the investor with a reliable lower bound for the maximum values and a reliable upper bound for the minimum. The problem becomes an optimization problem with constraints, where the constraints are: $\hat{Y}_t \leq Y_t$ ($t = 1, \dots, n$) in the case of the upper bound and $\hat{Y}_t \geq Y_t$ ($t = 1, \dots, n$) in the case of the lower bound (see Figure 6). This is a much more demanding problem, as will be seen below. This bound problem is not referred in the literature of financial time series prediction.

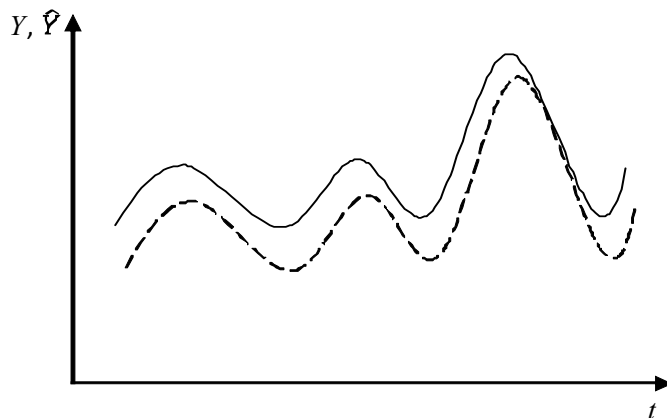


Figure 6: Lower bound problem

In metaheuristic procedures, when constraints are present in the optimization process the most used approach consists of using penalty functions in order to penalize the infeasible solutions in terms of the optimization criterion (Michalewicz, 1996). Other approaches does not allow the existence of infeasible solutions, considering the recovery of feasibility when an infeasible solution is found and when feasible solutions are easy to obtain.

The penalty approach will be used in this paper with a recovery of feasibility at the end of the evolutionary process. This process is also used to enhance the best found solution. Note that recovering feasibility in our bound problem can be easily made by simply adjusting the parameter δ_0^k ($k = M, m$) reducing its value by the maximum amount of violation: $\max_{t=1, \dots, n} \{ \hat{Y}_t - Y_t | \hat{Y}_t - Y_t > 0 \}$ in the case of the upper bound or $\max_{t=1, \dots, n} \{ Y_t - \hat{Y}_t | \hat{Y}_t - Y_t < 0 \}$ in the case of the lower bound.

Thus, in order to produce the required bounds the performance measure of the model is changed, penalizing the errors above the maximum values when the maximum time series is being modeled and the errors below the minimum values, when the minimum time series is being modeled. The tighter the bound the better.

The total penalty is assumed to be proportional to the number of violations of the bounds:

$$(e_t^M)^2 = \begin{cases} (Y_t - \hat{Y}_t)^2 & , \hat{Y}_t \leq Y_t \\ G + (Y_t - \hat{Y}_t)^2 & , \text{otherwise} \end{cases} \quad (7)$$

$$(e_t^m)^2 = \begin{cases} (Y_t - \hat{Y}_t)^2 & , \hat{Y}_t \geq Y_t \\ G + (Y_t - \hat{Y}_t)^2 & , \text{otherwise} \end{cases} \quad (8)$$

with G being a high positive value and $Y = M, m; \hat{Y} = \widehat{M}, \widehat{m}$.

With this penalty function, the number of violations is to be minimized and any infeasible solution is worst than any feasible solution.

The optimization criterion for deriving the parameters is $MSE G^k = \frac{\sum_{t=p_k+1}^n (e_t^k)^2}{n-p_k}$, with $k = M, m$.

At the end of the procedure, if the best solution is infeasible then the parameter δ_0^k ($k = M, m$) is adjusted. Due to the nature of the problem, a similar adjustment in that parameter can be used to enhance the quality of the solution. Indeed, a better feasible solution is obtained if δ_0^M is the minimum possible or δ_0^m is the maximum possible, without violating the feasibility.

With an easy way of recovering feasibility, one strategy of dealing with the bound constraints would be to maintain the procedure stated as in Section 2 and recovering the feasibility at the end. However, previous computational experiments lead to worst results when compared with the mix of penalty and feasibility recovery, which is justified since no effort was made for tuning the other parameters of the model considering the constraints.

5.1 Empirical testing

The estimation of the bounds are made over the same financial series, keeping the same parameters and the data sets used above. The error measures are presented without the penalty factor, although it was considered in the optimization process. The number of observations above the maximum values and below the minimum values are identified for each sample ($> sup$; $< inf$). As the feasibility is always assumed at the end of the procedure, the bounds are always respected for the testing data.

In the experiments, the penalty parameter G was set equal to 100.000. The results obtained with NGAR models are displayed in Tables 10 and 12 and the error measure for the best model of the selected model are presented in Tables 11 and 13.

Taking into account the feasibility of the estimations in the test data, it can be observed that it can be considered effective for all the financial indexes, except for the CAC40 with the maximum time series (see Tables 11 and 13).

Also for this bound problem, good results for the training data do not necessarily mean good results for the test data.

Comparing the NGAR models with the results for the Problem 1, it can be seen that the most adequate models are of different nature. The constraint lead to a significant increase in the value of the average RMSE measure. Although, the worst cases measures (MaxAE and MaxAPE) were less affected. These consequences are valid for both the maximum and minimum time series.

The quest that the estimations must always remain above (in the minimum case) or below (in the maximum case) impose that the shape of the time series must be respected. But, as the results show, the structure of the nonlinear models used, despite its enhancement with the inclusion of several nonlinear functions, revealed to have difficulties to achieve this objective. In order to preserve the bound constraint, most of the time the model generates a worst approximation. Nevertheless, the investors can combine this information with the provided in the sections above to reduce its uncertainty.

It seems that the shape of the original data is difficult to reproduce and the improvement of the estimations is stopped by the points where the estimations are equal to the observed values

<i>RMSE</i>	$\text{NGAR}_{(1,1)}^+$	$\text{NGAR}_{(1,2)}^+$	$\text{NGAR}_{(1,3)}^+$	$\text{NGAR}_{(2,1)}^+$	$\text{NGAR}_{(2,2)}^+$	$\text{NGAR}_{(2,3)}^+$
NASDAQ						
<i>Average</i>	0,0308929	*0,0231592	0,0303952	0,0310225	0,0325747	0,0313863
<i>Maximum</i>	0,1105157	*0,0284956	0,1099754	0,0432835	0,0434197	0,0386467
<i>Minimum</i>	0,0213120	*0,0212525	0,0214608	0,0219277	0,0226680	0,0217273
DAX						
<i>Average</i>	0,0390549	0,0357633	*0,0341354	0,0511766	0,0488907	0,0463657
<i>Maximum</i>	0,0917290	*0,0510874	0,0526895	0,0711300	0,0828292	0,0884224
<i>Minimum</i>	*0,0238117	0,0241414	0,0239500	0,0283964	0,0240542	0,0241986
CAC40						
<i>Average</i>	0,0450469	0,0433436	0,0332839	*0,0332753	0,0345993	0,0373369
<i>Maximum</i>	0,2233670	0,2226818	0,0533427	*0,0396901	0,0571969	0,0536397
<i>Minimum</i>	0,0236934	0,0239857	0,0265402	*0,0231854	0,0244089	0,0249238
S&P						
<i>Average</i>	0,0250449	0,0307095	*0,0242597	0,0268050	0,0322100	0,0373369
<i>Maximum</i>	0,0295940	0,1148026	*0,0249385	0,0363226	0,1176104	0,0536397
<i>Minimum</i>	0,0236989	0,0236489	0,0236086	*0,0234200	0,0234784	0,0249238

Table 10: Results for the maximum time series (Problem 3)

(see the tangent point in Figure 6, as an illustration of this aspect).

Table 14 displays the coefficient of determination for the best models, showing that the explanation ability of the models remains high.

6 Conclusions

This paper presented methodological approaches aiming at providing the investors several levels of information about the intraday market prices: approximation of the time series, a bandwidth where the intraday prices can vary and a trust bound for the maximum or minimum prices.

Nonlinear models, using Gaussian functions, were used to represent the maximum and minimum prices and also related bounds. The scatter search metaheuristic was used to estimate the "best" set of parameters. The model and the procedure revealed to be quite effective in the representation of the maximum and the minimum values and also their bounds. Its flexibility is useful to answer the new demands of information from the investors. The requested number of autoregressive terms was small, revealing that these variables dependent of their most recent values. The number of Gaussian functions necessary to get an adequate representation was also small. Nevertheless, the most adequate model for one problem is not necessarily the most adequate to answer to the other estimation problems.

From the approximation point of view, Problem 3 was a more difficult problem, compared with Problems 1 and 2. It deserves more additional research. Several approaches could have been followed to incorporate the bound constraint. An additive penalty function was adopted. While this option revealed to lead to models that respect the limit constraints in the training data, the deviation from the actual values was high. A compromise between having a model

<i>DataSet</i>	<i>MSE</i>	<i>RMSE</i>	<i>MAE</i>	<i>MaxAE</i>	<i>MaxAPE</i>	<i>> Sup</i>
<i>NASDAQ</i>						
<i>Serie</i>	0,0004489	0,0211868	0,0197291	0,0479254	7,6932906	9
<i>Training</i>	0,0004517	0,0212525	0,0200143	0,0475286	5,8203218	0
<i>Test</i>	0,0004211	0,0205209	0,0168889	0,0479254	7,6932906	9
<i>DAX</i>						
<i>Serie</i>	0,0005580	0,0236211	0,0224235	0,0450106	5,4481689	0
<i>Training</i>	0,0005736	0,0239500	0,0227251	0,0450106	5,4481689	0
<i>Test</i>	0,0004594	0,0214334	0,0205231	0,0340316	3,8768068	0
<i>CAC40</i>						
<i>Serie</i>	0,0005114	0,0226131	0,0208703	0,0552816	6,9919264	21
<i>Training</i>	0,0005376	0,0231854	0,0216381	0,0552816	6,9919264	0
<i>Test</i>	0,0002519	0,0158708	0,0132692	0,0333606	4,7356527	21
<i>S&P</i>						
<i>Serie</i>	0,0005392	0,0232203	0,0219544	0,0473675	5,6099601	1
<i>Training</i>	0,0005574	0,0236086	0,0223478	0,0473675	5,6099601	0
<i>Test</i>	0,0004246	0,0206061	0,0194760	0,0304937	3,4229440	1

Table 11: Evaluation measures for the maximum times series (Problem 3)

<i>RMSE</i>	$\text{NGAR}_{(1,1)}^+$	$\text{NGAR}_{(1,2)}^+$	$\text{NGAR}_{(1,3)}^+$	$\text{NGAR}_{(2,1)}^+$	$\text{NGAR}_{(2,2)}^+$	$\text{NGAR}_{(2,3)}^+$
<i>NASDAQ</i>						
<i>Average</i>	0,0348626	0,0335493	0,0347316	0,0387853	0,0349079	0,0343110
<i>Maximum</i>	0,1089464	0,0771242	0,0725518	0,1005042	0,0437518	0,0418411
<i>Minimum</i>	0,0229354	0,0260358	0,0280211	0,0276517	0,0293224	0,0266351
<i>DAX</i>						
<i>Average</i>	0,0273088	0,0258452	0,0247995	0,0362486	0,0328533	0,0329289
<i>Maximum</i>	0,0474048	0,0534752	0,0298211	0,1060084	0,0498018	0,0499019
<i>Minimum</i>	0,0220832	0,0218686	0,0219253	0,0222559	0,0223592	0,0217842
<i>CAC40</i>						
<i>Average</i>	0,0366929	0,0296096	0,0421938	0,0328965	0,0349365	0,0371987
<i>Maximum</i>	0,0576381	0,0581266	0,2211351	0,0510154	0,0483501	0,0590414
<i>Minimum</i>	0,0220945	0,0217543	0,0219703	0,0218454	0,0219922	0,0220748
<i>S&P</i>						
<i>Average</i>	0,0300276	0,0296186	0,0295247	0,0343467	0,0327500	0,0337978
<i>Maximum</i>	0,0317973	0,0408526	0,0324046	0,0758488	0,0402641	0,0486944
<i>Minimum</i>	0,0265680	0,0266878	0,0262067	0,0265978	0,0265820	0,0270774

Table 12: Results for the minimum time series (Problem 3)

<i>DataSet</i>	<i>MSE</i>	<i>RMSE</i>	<i>MAE</i>	<i>MaxAE</i>	<i>MaxAPE</i>	<i>< inf</i>
<i>NASDAQ</i>						
<i>Serie</i>	0,0015483	0,0393481	0,0306117	0,1706727	34,1874613	0
<i>Training</i>	0,0006779	0,0260358	0,0243505	0,0539973	6,3985809	0
<i>Test</i>	0,0102176	0,1010819	0,0929725	0,1706727	34,1874613	0
<i>DAX</i>						
<i>Serie</i>	0,0005030	0,0224284	0,0206762	0,0813372	9,8040585	0
<i>Training</i>	0,0004807	0,0219253	0,0200782	0,0813372	9,8040585	0
<i>Test</i>	0,0006436	0,0253693	0,0244437	0,0404705	5,1192208	0
<i>CAC40</i>						
<i>Serie</i>	0,0006515	0,0255246	0,0222541	0,0837380	16,1155393	0
<i>Training</i>	0,0004730	0,0217477	0,0198029	0,0647794	8,4533416	0
<i>Test</i>	0,0024191	0,0491844	0,0465205	0,0837380	16,1155393	0
<i>S&P</i>						
<i>Serie</i>	0,0007429	0,0272565	0,0254854	0,0551040	6,8205346	0
<i>Training</i>	0,0006868	0,0262067	0,0244147	0,0551040	6,8205346	0
<i>Test</i>	0,0010965	0,0331137	0,0322302	0,0492771	6,1987144	0

Table 13: Evaluation measures for the minimum times series (Problem 3)

<i>Data Set</i>	<i>Time series</i>	<i>Nasdaq</i>	<i>Dax</i>	<i>CAC40</i>	<i>S&P</i>
<i>Entire data</i>	<i>Maximum</i>	0,9892	0,9864	0,9914	0,9734
	<i>Minimum</i>	0,9635	0,9839	0,9946	0,9696
<i>Test data</i>	<i>Maximum</i>	0,9716	0,9661	0,9657	0,9514
	<i>Minimum</i>	0,9532	0,9639	0,9471	0,9679

Table 14: Coefficients of determination (Problem 3)

that verify the bounds and a model with low average deviations must be assumed. Additional research is needed to develop flexible models to manage the needs of the estimation of Problem 3.

For all the problems studied in this paper, it would also be very interesting to define the most flexible nonlinear model, since the models used also create a specific functional relationship among variables.

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