Planning a Solid Urban Waste Treatment System: Dynamic Location Model and Primal-Dual Heuristic

JOANA DIAS[(1)], M. EUGÉNIA CAPTIVO[(2)] *AND JOÃO CLÍMACO[(1)]
(1)Faculdade de Economia and INESC-Coimbra
Universidade de Coimbra
Av. Dias da Silva, 165
3004-512 Coimbra
Portugal
(2) Universidade de Lisboa, Faculdade de Ciências
Centro de Investigação Operacional
Campo Grande, Bloco C6, Piso 4
1749-016 Lisboa
Portugal

Abstract: A solid urban waste treatment system is generally composed of several facilities with different characteristics: waste transfer stations, incinerators and sanitary landfills. These facilities interact with each other, and the decision to locate one service should consider the locations of all the others. The planning of a solid urban waste treatment system is inherently dynamic, because all the services to be located will generally be in operation during a long period of time and it is necessary to consider the evolution in the production of waste (that generally increases with time). In this research report two dynamic location models are presented that consider the problem of locating these services. A primal-dual heuristic based on the dual problem of the second model developed is described.

1 Introduction

The problem of planning solid urban waste treatment systems is, more than ever, in the frontline. The development of our societies can presently be measured by the quantity of waste they produce. Each year there are more tons of wastes to treat than in the year before. Although there has been a great effort and investment in campaigns that lead people to separate the waste they produce (glass, paper, batteries, etc), and believing that recycling has to be, whenever possible, the best end of the line (or a new beginning) for urban solid waste, the truth is that there will always be a significant

---

* This research was partially supported by research project POCTI/ISFL-1/152 and POCTI/MAT/139/2001.

percentage of waste that has to be treated in other ways (it is generally incinerated or deposited in sanitary landfills).

In Portugal, the government understood that solid waste was beginning to be a serious problem in the ’80s, but the initial efforts were in creating modern fleets to collect the waste that was then disposed in open-air dumps (Antunes, 1999). In Antunes (1999), the author describes the situation in Portugal in the beginning of the 1990s, when the first studies were done to plan a serious solid urban waste system in the central region of the country. In the last decade, Portugal has witnessed the closure of all its open-air dumps and their substitution by sanitary landfills. This process has not been easy, because no one likes to live near a sanitary landfill. Whenever a specific location was chosen, strong public reaction against the construction of the facility was always heard. Nowadays, Portugal still faces serious problems in this sector, because there has not always been the political will to impose the necessary measures and the facilities that were built in the past decade are reaching their end of life.

A solid urban waste system is generally composed of three different services: solid waste transfer stations, incinerators and sanitary landfills. Solid waste transfer stations are facilities where the waste is temporarily disposed (and possibly compacted) before being transported to an incinerator or a sanitary landfill. They are mostly used by municipalities that are located far from an incinerator or sanitary landfill. Incinerators burn solid wastes, reducing the waste volume (the result of the incineration are ashes and scoria that have to be placed in a landfill). Sanitary landfills are the end of the line for the most part of solid urban waste.

The problem of planning a solid waste treatment system is dynamic and multi-objective in nature: all facilities to be located will be in operation for a given planning horizon; the location of facilities have to take into account the expected evolution of the amount of waste produced by populations that will have to be treated; most of the facilities to be located will be considered by the populations as obnoxious or semi-obnoxious, raising problems like the Not In My Backyard syndrome. This problem is also, by definition, a group decision problem: there are several decision makers whose opinion, believes and objectives have to be considered (a solid waste treatment system will serve several municipalities that have to count on the central government for, at least, part of the financing). Most of the times, these location decisions require some political courage to be implemented, and it is each time more important to have solid justifications for the solutions chosen.

In the present research report, two dynamic location models are presented that formulate the problem of planning a solid urban waste treatment system. The second model treats explicitly the multi-level nature of the location problem and motivated the development of a primal-dual heuristic that can find admissible primal solutions. The models presented consider the specific characteristics of the different kinds of facilities that are to be located. Having as special case the simple plant location problem it is straightforward to conclude that our problem is NP-hard.

2 The proposed models

The models developed try to consider explicitly the specific characteristics of the different facilities to be located.

Waste transfer stations: these facilities are generally composed of one or more equipments of equal or different dimensions. Equipments of greater dimensions have, generally, greater fixed opening costs but smaller unitary handling costs. These facilities are the ones that create the least opposition on the part of populations. Nevertheless it is advised to establish a maximum number of equipments to be located at each waste transfer station. The equipments, once placed at a waste transfer station, generally stay there until the end of their life-period (that is considered greater than the end of the planning horizon). They are generally equipped with technology that allows the compaction of waste, decreasing the transportation costs to incinerators or sanitary landfills.

Incinerators: these facilities require an enormous initial investment when compared with waste transfer stations or sanitary landfills. For this reason, it is considered that, once open, they will be operational until the end of the planning horizon. The incinerators have the capability of reducing the total amount of waste, and it is possible to define a constant α ($0< \alpha <1$) such that if an incinerator receives x tons of waste, then this waste is reduced to αx tons after incineration. The resulting waste (generally in the form of ashes and scoria) has to be placed in a landfill. It is possible to build a dedicated landfill near the incinerator that receives only the end products of the incineration (because the transportation of this waste is very expensive). If this is the case, the cost of building this landfill can be incorporated in the fixed cost of opening the incinerator, and this facility is the end of the line for all the waste it receives (it can be considered that the dimension of the dedicated landfill is planned in order to be able to receive all the resulting waste for all the incinerator operating time periods). Nevertheless, sometimes there is the need to deposit the resulting ashes and scoria in other non-dedicated landfills.[1]For this reason, in the models developed, the authors chose to consider explicitly the transportation of the waste from the incinerators to the landfills.[2]An incinerator has a maximum capacity of waste it can handle, and also a minimum quantity of waste it has to incinerate in order to be economically viable.

Sanitary landfills: This is the most common end of the line for solid urban waste. A sanitary landfill has a maximum capacity when it is open that decreases as the landfill receives waste. When its maximum capacity is reached, the landfill has to be closed and kept under observation to guarantee that it will not cause any health or environmental harm. The capacity of a sanitary landfill is limited, so there is the need to open more sanitary landfills as the ones already operating close. There is sometimes the possibility of reopening a sanitary landfill by, for instance, using more land near the existing landfill (being able to take advantage of all the infra-structures already built, and decreasing the possibilities of reactions against a new location from the populations). The reopening of a landfill has, obviously, costs completely different from its opening for the first time.

Consider the following definitions:

$J = \{1,\ldots,j,\ldots, n\}$ set of indexes corresponding to the locations that produce waste;

$I = \{1,\ldots, i,\ldots, m\}$ set of indexes corresponding to the possible locations of sanitary landfills;

$K = \{1,\ldots, k,\ldots, p\}$ set of indexes corresponding to waste transfer stations' possible locations;

$L = \{1, \ldots, l, \ldots, r\}$ set of indexes corresponding to incinerators' possible locations;

---

[1] See, for instance, the report of INR (Instituto Nacional dos Resíduos) "*Produção de Resíduos Sólidos Urbanos – 1999*" where the figures show exactly this.

[2] To adapt the models presented to the situation where there are only dedicated landfills to receive the ashes and scoria two approaches are possible: Considering explicitly these dedicated sanitary landfills, and considering all transportation costs from other locations or facilities to these dedicated landfills equal to $+\infty$, and the transportation costs from the related incinerator equal to zero; or else consider that incinerators are end-of-the-line facilities instead of transhipment facilities.

$S = \{1, \ldots, s, \ldots, q\}$ set of indexes corresponding to the possible dimensions of equipments that can be installed in waste transfer stations (ordered by ascending order of maximum capacity);

N_{max} = maximum number of equipments that can be installed in a waste transfer station;

d_j^t = amount of waste produced by location j in period t;

T = number of time periods to consider;

Q_i = maximum capacity of the sanitary landfill located at i when it is (re) opened;

Q^l_{min}, Q^l_{max} = minimum and maximum capacity of the incinerator located at l;

Q_s = maximum capacity of an equipment of dimension s, $s \in S$.

$(1-\alpha)$ = percentage of waste reduction that is achieved through incineration (if an incinerator receives x tons of waste, then only $\alpha \times x$ will have to be placed in a landfill).

FA_{kst} = fixed cost of opening for the first time the waste transfer station in k, installing an equipment of dimension s, in the beginning of period t (this equipment will be open until the end of the planning horizon, so this cost should consider all fixed operating costs during time periods from t to T);

FR_{kst} = fixed cost of opening an equipment of dimension s in the waste transfer station k at the beginning of time period t, knowing that there is already at least one equipment located at k (this fixed cost has to consider all fixed operating costs during time periods from t to T);[3]

$FA_{i\tau}^{\xi}$ = fixed cost of opening a sanitary landfill located at i in the beginning of period τ, knowing that it will be operating until the end of period ξ (this fixed cost should consider the fixed opening costs, plus the fixed operating costs during periods from τ to ξ, plus the costs of closing the sanitary landfill at the end of period ξ and also the costs of maintenance after closing);

$FR_{i\tau}^{\xi}$ = fixed cost of reopening the sanitary landfill located at i in the beginning of period τ, knowing that it will be operating until the end of period ξ (this fixed cost should consider the fixed reopening costs, plus the fixed operating costs during periods

[3] The fixed costs FA_{kst} should be equal to fixed costs FR_{kst} plus costs that are present only at the time of the opening of waste transfer station in k (like costs of land acquisition, construction of infra-structures, environment impact studies, etc).

from τ to ξ, plus costs of closing the sanitary landfill at the end of period ξ and also the costs of maintenance after closing);

FA_l^t = fixed cost of opening an incinerator located at l at the beginning of period t, including the fixed opening costs and also the fixed operating costs during time periods t to T;

c_{jks}^t = unitary transportation cost of waste from location j to an equipment of dimension s at waste transfer station located at k during period t, plus unitary handling costs;

c_{ji}^t = unitary transportation cost of waste from location j to the sanitary landfill located at i, plus unitary handling costs;

c_{jl}^t = unitary transportation cost of waste from location j to the incinerator located at l, plus unitary handling costs;

c_{ki}^t = unitary transportation cost of waste from transfer station k to sanitary landfill i, plus the unitary cost of treating the waste at i;

c_{kl}^t = unitary transportation cost of waste from waste transfer station k to the incinerator located at l, plus unitary handling costs;

c_{li}^t = unitary transportation cost of waste from incinerator l to the sanitary landfill located at i plus unitary handling costs;

P = set of possible paths;

$P(i)$ = subset of P that considers all paths p ending at sanitary landfill i;

$P(l)$ = subset of P that includes all paths p to which the incinerator l belongs;

$P(k,s)$ = subset of P that includes all paths p to which an equipment of dimension s located at waste transfer station k belongs;

c_{pj}^t = unitary cost of transporting waste from location j through path p during time period t.[4]

---

[4] The cost c_{pj}^t can be easily calculated from the costs defined previously. If, for instance, path p is composed by an equipment of dimension s at waste transfer station k, plus an incinerator l and a sanitary landfill i, then $c_{pj}^t = c_{jks}^t + c_{kl}^t + \alpha c_{li}^t$

Paths $p \in P$ can be constituted by: a sanitary landfill i; an incinerator l and a sanitary landfill i; an equipment of dimension s located at the waste transfer station k and a sanitary landfill i; an equipment of dimension s at waste transfer station k, an incinerator l and a sanitary landfill j. All solid urban waste should be subjected to a selection process, capable of reducing the total amount of waste that ends up in a sanitary landfill or incinerator, and redirecting it for recycling. This selection can be done in waste transfer stations, but it is often done in sanitary landfills. This means that, sometimes, not all the waste received is disposed in the sanitary landfill and even that some of the waste could be redirected to an incinerator. These observations are not considered in the present model, but can easily be accommodated through minor changes.

In the next sections the two models developed are presented.

2.1 Model 1

Consider the following decision variables:

$x^t_{jks} =$ fraction of waste transported from location j to equipments of dimension s located at the waste transfer station k during time period t, $s \in S$, $j \in J$, $k \in K$, $t = 1,...,T$.

$y^t_{ji} =$ fraction of waste transported from location j to the sanitary landfill located at i during time period t, $i \in I$, $j \in J$, $t = 1,...,T$.

$h^t_{jl} =$ fraction of waste transported from location j to the incinerator located at l during time period t, $j \in J$, $l \in L$, $t = 1,...,T$.

$z^t_{ki} =$ total amount of waste transported from the waste transfer station located at k to the sanitary landfill located at i, during time period t, $k \in K$, $i \in I$, $t = 1,...,T$.

$p^t_{kl} =$ total amount of waste transported from the waste transfer station located at k to the incinerator located at l, during time period t, $k \in K$, $l \in L$, $t = 1,...,T$.

$q^t_{li} =$ total amount of waste transported from the incinerator located at l to the sanitary landfill located at i, during time period t, $i \in I$, $l \in L$, $t = 1,...,T$.

$$a_{kst} = \begin{cases} 1, \text{if an equipment of dimension } s \text{ is installed} \\ \quad \text{at the waste transfer station in } k, \text{knowing} \\ \quad \text{that there is no equipment installed at } k, \text{at the} \\ \quad \text{beginning of time period } t \\ \\ 0, \text{otherwise} \end{cases}, \quad k \in K, s \in S, t = 1,...,T.$$

r_{kst} = number of equipments of dimension s located at the waste transfer station k at the beginning of time period t, knowing that at k there is at least one equipment installed at the beginning of time period $t' \leq t$; $k \in K$, $s \in S$, $t = 1,...,T$.

$$a_{it}^{\xi} = \begin{cases} 1, \text{if a sanitary landfill is open at } i \text{ in} \\ \quad \text{the beginning of time period } t \text{ and stays} \\ \quad \text{open until the end of time period } \xi \\ \\ 0, \text{otherwise} \end{cases}, \quad i \in I, t=1,...,T, \xi=t,\dots,T.$$

$$r_{it}^{\xi} = \begin{cases} 1, \text{if a sanitary landfill located at } i \\ \quad \text{reopens at the beginning of time period } t \\ \quad \text{and stays open until the end of time period } \xi \\ \\ 0, \text{otherwise} \end{cases}, \quad i \in I,\ t = 2,...,T,\ \xi = t,\dots,T.$$

$$y_l^t = \begin{cases} 1, \text{if an incinerator is located at } l \text{ at the beginning of time period } t \\ \\ 0, \text{otherwise} \end{cases}, \quad l \in L,\ t = 1,...,T.$$

The problem of planning a solid urban waste treatment system can be formulated in mixed integer linear programming as:

Minimize

$$\sum_t \sum_k \sum_s \left(FA_{kst} a_{kst} + FR_{kst} r_{kst} \right) + \sum_i \sum_{\tau=1}^{T} \sum_{\xi=\tau}^{T} FA_{i\tau}^{\xi} a_{i\tau}^{\xi} + \sum_i \sum_{\tau=2}^{T} \sum_{\xi=\tau}^{T} FR_{i\tau}^{\xi} r_{i\tau}^{\xi} + \sum_t \sum_l FA_l^t y_l^t +$$
$$\sum_t \sum_j \sum_k \sum_s c_{jks}^t x_{jks}^t d_j^t + \sum_t \sum_j \sum_i c_{ji}^t y_{ji}^t d_j^t + \sum_t \sum_j \sum_l c_{jl}^t h_{jl}^t d_j^t +$$
$$\sum_t \sum_k \sum_i c_{ki}^t z_{ki}^t + \sum_t \sum_k \sum_l c_{kl}^t p_{kl}^t + \sum_t \sum_l \sum_i c_{li}^t q_{li}^t$$

subject to:

$$\sum_k \sum_s x_{jks}^t + \sum_i y_{ji}^t + \sum_l h_{jl}^t = 1, \qquad \forall j,t \qquad (1)$$

$$\sum_i z_{ki}^t + \sum_l p_{kl}^t = \sum_j \sum_s x_{jks}^t d_j^t, \qquad \forall k,t \qquad (2)$$

$$\sum_{k} p_{kl}^{t} + \sum_{j} h_{jl}^{t} d_{j}^{t} = \frac{1}{\alpha} \sum_{i} q_{li}^{t}, \qquad \forall l,t, 0 < \alpha < 1 \quad (3)$$

$$\sum_{\tau=1}^{t} \left(\sum_{k} z_{ki}^{\tau} + \sum_{l} q_{li}^{\tau} + \sum_{j} y_{ji}^{\tau} d_{j}^{\tau} \right) \leq \sum_{\tau=1}^{t} \left(Q_i \sum_{\xi=\tau}^{T} a_{i\tau}^{\xi} + Q_i \sum_{\xi=\tau}^{T} r_{i\tau}^{\xi} \right), \qquad \forall i,t \quad (4)$$

$$\sum_{j} x_{jks}^{t} d_{j}^{t} \leq Q_s \sum_{\tau=1}^{t} \left(a_{ks\tau} + r_{ks\tau} \right), \qquad \forall k,s,t \quad (5)$$

$$\sum_{j} h_{jl}^{t} d_{j}^{t} + \sum_{k} p_{kl}^{t} \leq Q_{max}^{l} \sum_{\tau=1}^{t} y_{l}^{\tau}, \qquad \forall l,t \quad (6)$$

$$\sum_{j} h_{jl}^{t} d_{j}^{t} + \sum_{k} p_{kl}^{t} \leq Q_{max}^{l}, \qquad \forall l,t \quad (6')$$

$$x_{jks}^{t} \leq \sum_{\tau=1}^{t} \left(a_{ks\tau} + r_{ks\tau} \right), \qquad \forall j,k,s,t \quad (7)$$

$$y_{ji}^{t} \leq \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi} \right), \qquad \forall i,j,t \quad (8)$$

$$z_{ki}^{t} \leq Q_s N_{max} \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi} \right), \qquad \forall k,i,t \quad (9)$$

$$q_{li}^{t} \leq Q_{max}^{l} \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi} \right), \qquad \forall l,j,t \quad (10)$$

$$h_{jl}^{t} \leq \sum_{\tau=1}^{t} y_{l}^{\tau}, \qquad \forall l,j,t \quad (11)$$

$$p_{kl}^{t} \leq Q_{max}^{l} \sum_{\tau=1}^{t} y_{l}^{\tau}, \qquad \forall l,k,t \quad (12)$$

$$\sum_{j} h_{jl}^{t} d_{j}^{t} + \sum_{k} p_{kl}^{t} \geq Q_{min}^{l} \sum_{\tau=1}^{t} y_{l}^{\tau}, \qquad \forall l,t \quad (13)$$

$$\sum_{t=1}^{T} \sum_{\xi=t}^{T} a_{it}^{\xi} \leq 1, \qquad \forall i \quad (14)$$

$$\sum_{\xi=t}^{T} r_{it}^{\xi} \leq \sum_{\tau=1}^{t-1} \sum_{\xi=\tau}^{t-1} a_{i\tau}^{\xi}, \qquad \forall i, t>1 \quad (15)$$

$$\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{i\tau}^{\xi}+r_{i\tau}^{\xi}\right)\le 1, \qquad \forall i,t \qquad (16)$$

$$\sum_{t=1}^{T}\sum_{s} a_{kst} \le 1, \qquad \forall k \qquad (17)$$

$$r_{kst} \le N_{max}\sum_{\tau=1}^{t}\sum_{s} a_{ks\tau}, \qquad \forall k,s,t \qquad (18)$$

$$\sum_{\tau=1}^{T}\sum_{s}\left(a_{ks\tau}+r_{ks\tau}\right)\le N_{max}, \qquad \forall k \qquad (19)$$

$$\sum_{t=1}^{T} y_{l}^{t} \le 1, \qquad \forall l \qquad (20)$$

$$z_{ki}^{t}, p_{kl}^{t}, q_{li}^{t} \ge 0, \qquad \forall l,k,i,t=1,\ldots,T \qquad (21)$$

$$a_{it}^{\xi}, r_{it}^{\xi}, a_{kst}, y_{l}^{t} \in \{0,1\}, \qquad \forall j,k,s,l,t=1,\ldots,T,\ \xi=t,\ldots,T \qquad (22)$$

$$0 \le x_{jks}^{t}, y_{ji}^{t}, h_{jl}^{t} \le 1, \qquad \forall i,j,k,s,l,t=1,\ldots,T \qquad (23)$$

$$r_{kst} \ge 0 \text{ e inteiro}, \qquad \forall k,s,t=1,\ldots,T \qquad (24)$$

The meaning of each restriction presented is now stated:

(1): All the waste produced by location *j*, during time period *t*, will have to be transported to waste transfer stations, incinerators or sanitary landfills.

(2): All the waste that reaches waste transfer station *k*, during time period *t*, has to be transported to sanitary landfills or incinerators.

(3): The remaining waste (after incineration) that is produced at incinerator *l* has to be transported to a sanitary landfill.

(4): The total amount of waste deposited at the sanitary landfill located at *i* cannot be greater than its remaining capacity at each time period *t*.

(5): The total amount of waste that reaches equipments of dimension *s* located at the waste transfer station *k* cannot exceed their maximum capacity.

(6),(6’): The total amount of waste that reaches incinerator *l* cannot exceed its maximum capacity.[5]

[5] Restrictions (6) e (6’) are equivalent due to (11) and (12).

(7): It is possible to transport waste to equipments of dimension s at waste transfer station k if and only if these equipments are open at time period t.[6]

(8)-(10): The sanitary landfill located at i is able to receive waste from any location j, waste transfer station k or incinerator l (respectively) if and only if it is open during period t.

(11)-(12): The incinerator located at l can incinerate waste received from locations j or waste transfer stations k if it was open at the beginning of a time period less than or equal to t.[7]

(13): The incinerator located at l has a minimum functioning capacity.

(14): At location i it is possible to open at most one sanitary landfill.

(15): A sanitary landfill can be reopen at the beginning of time period t if and only if it had already been open earlier and is not functioning during period t.

(16): There cannot be overlaps between the functioning time periods regarding the opening and reopening sanitary landfills' decision variables.

(17): At location k there can be located at most one waste transfer station (independently of the number of equipments that are there installed).

(18): The addition of new equipments can be made at waste transfer stations that are already open.[8]

(19): The maximum number of equipments that can be located at each waste transfer station is limited.

(20): At each location l at most one incinerator can be open.

2.2 Model 2: Multi-level location model

Consider variables a_{kst}, r_{kst}, a_{it}^{ξ}, r_{it}^{ξ} and y_l^t already defined for model 1, and also:

x_{pj}^t = fraction of waste produced at location j that is assigned to path p;

---

[6] This restriction is redundant in presence of (5).

[7] Restriction (11) is redundant if restriction (6) is considered instead of (6').

[8] It is considered that a waste transfer station is composed of one or more equipments of equal or different dimensions. It is allowed that more than one equipment is installed at k at the same time. It is important to note that this restriction considers admissible the situation of opening a waste transfer station by simultaneously locating more than one equipment.

$$\theta_p = \begin{cases} \alpha, & \text{if } p \in P(l) \text{ for some } l \in L \\ 1, & \text{otherwise} \end{cases}.$$

The multi-level location model is formulated as:

Minimize

$$\sum_t \sum_k \sum_s \left(FA_{kst} a_{kst} + FR_{kst} r_{kst}\right) + \sum_i \sum_{\tau=1}^{T} \sum_{\xi=\tau}^{T} FA_{i\tau}^{\xi} a_{i\tau}^{\xi} + \sum_i \sum_{\tau=2}^{T} \sum_{\xi=\tau}^{T} FR_{i\tau}^{\xi} r_{i\tau}^{\xi} + \sum_t \sum_l FA_l^t y_l^t + \sum_t \sum_j \sum_p c_{jp}^t x_{jp}^t$$

subject to:

$$\sum_p x_{pj}^t = 1, \qquad \forall j,t \tag{25}$$

$$x_{pj}^t \le \sum_{\tau=1}^{t} \sum_{\xi=\tau}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi}\right), \qquad \forall j,t,p \in P(i),\ i \in I \tag{26}$$

$$x_{pj}^t \le \sum_{\tau=1}^{t} y_l^{\tau}, \qquad \forall j,t,p \in P(l),\ l \in L \tag{27}$$

$$x_{pj}^t \le \sum_{\tau=1}^{t} \left(a_{ks\tau} + r_{ks\tau}\right), \qquad \forall j,t,p \in P(k,s), k \in K, s \in S \tag{28}$$

$$Q_i \sum_{\tau=1}^{t} \sum_{\xi=\tau}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi}\right) \ge \sum_{\tau=1}^{t} \sum_j \sum_{p \in P(i)} d_j^t x_{pj}^t \theta_p, \qquad \forall i \in I, t \tag{29}$$

$$Q_{min}^l \sum_{\tau=1}^{t} y_l^{\tau} \le \sum_j \sum_{p \in P(l)} d_j^t x_{pj}^t, \qquad \forall l \in L, t \tag{30}$$

$$\sum_j \sum_{p \in P(l)} d_j^t x_{pj}^t \le Q_{max}^l \sum_{\tau=1}^{t} y_l^{\tau}, \qquad \forall l \in L, t \tag{31}$$

$$\sum_j \sum_{p \in P(k,s)} d_j^t x_{pj}^t \le Q_s \sum_{\tau=1}^{t} \left(a_{ks\tau} + r_{ks\tau}\right), \qquad \forall k \in K, s \in S, t \tag{32}$$

(14)-(20), (22), (24)

$$0 \le x_{jp}^t \le 1, \qquad \forall j,p,t = 1,\ldots,T \tag{33}$$

The meaning of each restriction presented is now stated:

(25): The total amount of waste produced at location *j* has to be assigned to one or more paths *p*, in each time period *t*.

(26)-(28): The waste can be assigned to path p only if all facilities belonging to path p are open during time period t.
(29): the total amount of waste deposited at sanitary landfill i cannot exceed its remaining capacity in any of the time periods.
(30),(31): The incinerator located at l has minimum and maximum functioning capacities.
(32): the total amount of waste transported to equipments of dimension s located at the waste transfer station k cannot exceed their maximum capacity, in any of the time periods.

In this model it is considered that services are organized by levels: waste transfer stations are in the first level, incinerators are in the second level and sanitary landfills are in the third level.[9]

3 Primal-Dual Heuristic

The primal-dual heuristic developed for the resolution of the problem of planning a urban solid waste treatment system is based on the second model presented and uses most of the work developed earlier by the same authors (Dias et al 2002, 2004a, 2004b, 2004c) and also by other authors (Erlenkotter, 1978, Van Roy and Erlenkotter, 1982, Guignard and Spielberg, 1979). In the next section, the dual problem will be formulated, and then the primal-dual heuristic is described. This heuristic tries to take advantage of the specific characteristics of the problem formulated, namely knowing that all services in level one have maximum capacities, all services in level two have maximum and minimum capacities and are the only services that have a parameter α associated, and that all services at level three have maximum decreasing capacities.

3.1 The dual problem

Consider set $I' = \{1,\cdots, p\times q, p\times q+1,\cdots, p\times q+r, p\times q+r+1,\cdots p\times q+r+m\}$, and its subsets:

$I_1 = \{1,\cdots, p\times q\}$; $I_2 = \{p\times q+1,\cdots p\times q+r\}$; $I_3 = \{p\times q+r+1,\cdots p\times q+r+m\}$.

[9] In the situation described in footnote 2, it is possible to consider incinerators and sanitary landfills as belonging to the second level.

In order to simplify the dual problem formulation, consider that set I_1 corresponds to equipments installed in waste transfer stations,[10] I_2 corresponds to incinerators and I_3 corresponds to sanitary landfills. We can also consider variables $a_{it}^{\xi}, i \in I_1,\ \xi \geq t$ such that $a_{it}^{\xi} = a_{kst}$, such that $i = (k-1) \times q + s$, and $a_{it}^{\xi} = 0$ [11] for all $\xi \neq T$. A similar procedure can be used for variables $a_{it}^{\xi}, i \in I_2$.

Associating dual variables v_j^t to restrictions (25), $\overline{w}_{ijp}^t$ to restrictions (26), $\overline{\overline{w}}_{ljp}^t$ to restrictions (27), $\overline{\overline{w}}_{(k,s)jp}^t$ to restrictions (28),[12] variables u_i^t to restrictions (15), variables π_i^t to restrictions (16), variables $\overline{\rho}_i$ to restrictions (14), variables $\overline{\overline{\rho}}_l$ to restrictions (20),[13] variables u_{ks}^t to restrictions (18),[14] variables $\overline{\pi}_k$ to restrictions (19) and $\overline{\overline{\overline{\rho}}}_k$ to restrictions (17), variables $\overline{\lambda}_i^t$ to restrictions (29), $\overline{\overline{\lambda}}_l^t$ and β_l^t to restrictions (30) and (31) and λ_{ks}^t to restrictions (32),[15] then the dual problem can be formulated as:

$$Max \sum_t \sum_j v_j^t - \sum_{i \in I' \backslash I_1} \rho_i - \sum_k \overline{\overline{\overline{\rho}}}_k - \sum_t \sum_{i \in I_3} \pi_i^t + \sum_k N_{max} \overline{\pi}_k \tag{34}$$

subject to:

---

[10] An equipment of dimension s installed in location k corresponds to facility $i = (k-1) \times q + s$, $i \in I_1$.

[11] Considering the respective fixed cost equal to $+\infty$.

[12] Considering set I', is equivalent to say that w_{ijp}^t, $i \in I_3$, are associated with restrictions (26), w_{ijp}^t, $i \in I_2$, are associated with restrictions (27) and w_{ijp}^t $i \in I_1$ with $i = (k-1) \times q + s$ are associated with restrictions (28). Both notations will be used indistinctly.

[13] Considering set I', is equivalent to say that ρ_i, $i \in I_3$, are associated with restrictions (14), and, ρ_i, $i \in I_2$, are associated with restrictions (20).

[14] They can also be represented by u_i^t, $i \in I_1$.

[15] Considering set I', it is equivalent to say that λ_i^t, $i \in I_3$ are associated with constraints (29), λ_i^t and β_i^t, $i \in I_2$ are associated with constraints (30) and (31), respectively, and λ_i^t, $i \in I_1$ are associated with constraints (32).

$$v_j^t - \sum_{i \in p} w_{ijp}^t - d_j^t \left(\sum_{\substack{i \in p \\ i \in I_1 \cup I_2}} \lambda_i^t + \theta_p \sum_{\substack{i \in p \\ i \in I_3}} \sum_{\psi = t}^{T} \lambda_i^{\psi} - \sum_{\substack{i \in p \\ i \in I_2}} \beta_{i_1}^t \right) \le c_{pj}^t, \ \forall p, j, t \tag{35}$$

$$\sum_j \sum_{\tau = t}^{\xi} \sum_{p \in P(i)} w_{ijp}^{\tau} + \sum_{\tau = \xi + 1}^{T} u_i^{\tau} - \rho_i - \sum_{\tau = t}^{\xi} \pi_i^{\tau} + Q_i \sum_{\tau = t}^{T} \lambda_i^{\tau} \le FA_{it}^{\xi}, \quad \forall i \in I_3, t, \xi = t, \cdots, T \tag{36}$$

$$\sum_j \sum_{\tau = t}^{\xi} \sum_{p \in P(i)} w_{ijp}^{\tau} - u_i^t - \sum_{\tau = t}^{\xi} \pi_i^{\tau} + Q_i \sum_{\tau = t}^{T} \lambda_i^{\tau} \le FR_{it}^{\xi}, \ \ \forall i \in I_3, t > 1, \xi = t, \cdots, T \tag{37}$$

$$\sum_j \sum_{\tau = t}^{T} \sum_{p \in P(i)} w_{ijp}^{\tau} - \rho_i + Q_{max}^l \sum_{\tau = t}^{T} \beta_i^{\tau} - Q_{min}^l \sum_{\tau = t}^{T} \lambda_i^{\tau} \le FA_i^t, \qquad \forall i \in I_2, t \tag{38}$$

$$\sum_j \sum_{\tau = t}^{T} \sum_{p \in P(k,s)} w_{(k,s)jp}^{\tau} + \sum_{\tau = t}^{T} N_{max} u_{ks}^{\tau} - \overline{\overline{\rho}}_k - \overline{\pi}_k + Q_s \sum_{\tau = t}^{T} \lambda_{ks}^{\tau} \le FA_{kst}, \quad \forall k, s, t, \xi = t, \cdots, T \tag{39}$$

$$\sum_j \sum_{\tau = t}^{T} \sum_{p \in P(k,s)} w_{(k,s)jp}^{\tau} - u_{ks}^t - \overline{\pi}_k + Q_s \sum_{\tau = t}^{T} \lambda_{ks}^{\tau} \le FR_{kst}, \qquad \forall k, s, t, \xi = t, \cdots, T \tag{40}$$

$$\overline{\overline{\rho}}_k, w_{ijp}^t, u_i^t, \rho_i, \pi_i^t, \overline{\pi}_k, u_{ks}^t, w_{(k,s)jp}^t, \beta_i^t, \lambda_i^t, \lambda_{ks}^t \ge 0, \qquad \forall i, j, k, s, t, p$$

To build the condensed dual problem, the same steps described in Dias et al (2004c) can be followed. It is necessary to consider:

$$w_{ijp}^t = \eta_{ijp}^t \, max \left\{ 0, v_j^t - c_{pj}^t - d_j^t \left(\sum_{\substack{i_1 \in p \\ i_1 \in I_1 \cup I_2}} \lambda_{i_1}^t + \theta_p \sum_{\substack{i_1 \in p \\ i_1 \in I_3}} \sum_{\psi = t}^{T} \lambda_{i_1}^{\psi} - \sum_{\substack{i_1 \in p \\ i_1 \in I_2}} \beta_{i_1}^t \right) \right\},$$

with $\sum_{i \in p} \eta_{ijp}^t = 1$ and $\eta_{ijp}^t \ge 0$, $\forall i \in I', j, p \in P(i)$, perform the corresponding substitutions in restrictions (36)-(40) and drop restriction (35).

Let us also define:

$$d_{pj}^t = c_{pj}^t + d_j^t \left(\sum_{\substack{i_1 \in p \\ i_1 \in I_1 \cup I_2}} \lambda_{i_1}^t + \theta_p \sum_{\substack{i_1 \in p \\ i_1 \in I_3}} \sum_{\psi = t}^{T} \lambda_i^{\psi} - \sum_{\substack{i_1 \in p \\ i_1 \in I_2}} \beta_{i_1}^t \right), \qquad \forall p, j, t \tag{41}$$

$$SA_{it}^{\xi} = FA_{it}^{\xi} - \sum_{\tau=\xi+1}^{T} u_i^{\tau} + \rho_i + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - Q_i \sum_{\tau=t}^{T} \lambda_i^{\tau} - \sum_j \sum_{\tau=t}^{\xi} \sum_{p \in P(i)} \eta_{ijp}^{\tau} \max\left\{0, v_j^{\tau} - d_{pj}^{\tau}\right\},$$

$$\forall i \in I_3, t, \xi = t, \cdots, T \qquad (42)$$

$$SR_{it}^{\xi} = FR_{it}^{\xi} + u_i^{t} + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - Q_i \sum_{\tau=t}^{T} \lambda_i^{\tau} - \sum_j \sum_{\tau=t}^{\xi} \sum_{p \in P(i)} \eta_{ijp}^{\tau} \max\left\{0, v_j^{\tau} - d_{pj}^{\tau}\right\},$$

$$\forall i \in I_3, t > 1, \xi = t, \cdots, T \qquad (43)$$

$$S_{it}^{\xi} = \min\left\{SA_{it}^{\xi}, SR_{it}^{\xi}\right\}, \qquad \forall i \in I_3, t, \xi = t, \cdots, T \qquad (44)$$

$$S_i^{t} = FA_i^{t} + \rho_i - Q_{max}^{i} \sum_{\tau=t}^{T} \beta_i^{\tau} - Q_{min}^{i} \sum_{\tau=t}^{T} \lambda_i^{\tau} - \sum_j \sum_{\tau=t}^{\xi} \sum_{p \in P(i)} \eta_{ijp}^{\tau} \max\left\{0, v_j^{\tau} - d_{pj}^{\tau}\right\},$$

$$\forall i \in I_2, t, \xi = t, \cdots, T \qquad (45)$$

$$SA_{kst} = FA_{kst} - \sum_{\tau=t}^{T} N_{max} u_{ks}^{\tau} + \overline{\overline{\rho}}_k + \overline{\pi}_k - Q_s \sum_{\tau=t}^{T} \lambda_{ks}^{\tau} - \sum_j \sum_{\tau=t}^{T} \sum_{p \in P(k,s)} \eta_{(k,s)jp}^{\tau} \max\left\{0, v_j^{\tau} - d_{pj}^{\tau}\right\}$$

$$\forall k, s, t, \xi = t, \cdots, T \qquad (46)$$

$$SR_{kst} = FR_{kst} + u_{ks}^{t} + \overline{\pi}_k - Q_s \sum_{\tau=t}^{T} \lambda_{ks}^{\tau} - \sum_j \sum_{\tau=t}^{T} \sum_{p \in P(k,s)} \eta_{(k,s)jp}^{\tau} \max\left\{0, v_j^{\tau} - d_{pj}^{\tau}\right\},$$

$$\forall k, s, t > 1, \xi = t, \cdots, T \qquad (47)$$

$$S_{kst} = \min\left\{SA_{kst}, SR_{kst}\right\}, \qquad \forall k, s, t, \xi = t, \cdots, T \qquad (48)$$

The complementary conditions between the primal and dual problems are very similar to the ones presented in Dias et al (2004c), so they will be omitted.

3.2 The heuristic

The heuristic functioning scheme is the following:

1. Initialisation of dual variables;
2. Dual Ascent Procedure for dual variables v_j^t;
3. Primal Procedure;
4. Dual Adjustment Procedure for dual variables ρ_i, $i \in I_2 \cup I_3$.
5. Dual Adjustment Procedure for dual variables $\overline{\overline{\rho}}_k$.
6. If the dual solution is changed in step 4 and/or 5 go to 2;
7. Repeat the Dual-Primal Adjustment Procedure for variables v_j^t until there is no improvement in the dual objective function value;

8. Repeat steps 4-6.
9. Dual Ascent Procedure for dual variables u_i^t, $i \in I'$. If the dual solution is changed go to 2;
10. Dual Descent Procedure for dual variables u_i^t, $i \in I'$. If the dual solution is changed go to 2;
11. Dual Adjustment Procedure for variables π_i^t, $i \in I_3$.
12. Dual Adjustment Procedure for variables $\overline{\pi}_k$.
13. If the dual solution is changed in step 11 and/or step 12, then go to 2.
14. Dual Ascent Procedure for dual variables λ_i^t, $i \in I'$. If the dual solution is changed, go to 2.
15. Dual Descent Procedure for dual variables λ_i^t, $i \in I'$. If the dual solution is changed, go to 2.
16. Dual Ascent Procedure for dual variables β_i^t, $i \in I_2$. If the dual solution is changed, go to 2.
17. Dual Descent Procedure for dual variables β_i^t, $i \in I_2$. If the dual solution is changed, go to 2.
18. Solve the assignment problem optimally (using a general solver) considering the best primal solution found so far.

As it is easily seen, the decision variables that correspond to the location of incinerators (y_l^t) can be treated as variables a_{lt}^{ξ} and r_{lt}^{ξ} such that r_{lt}^{ξ} is equal to zero, for all (t,ξ) and variables a_{lt}^{ξ} are equal to zero for $\xi \neq T$. In Dias et al (2004c) the authors have already described a primal-dual heuristic that is able to tackle multi-level capacitated location problems with no flow conservation where the facilities can be characterized by maximum and minimum capacities. In a similar way, the variables a_{kst} and r_{kst} can be interpreted as variables a_{kst}^{ξ} and r_{kst}^{ξ} (as defined in Dias et al, 2004b), being all variables equal to zero except those with $\xi = T$. Taking this observations into account, most of the procedures described in Dias et al (2004b, 2004c) can be directly applied in the present heuristic.

The procedures that have to be changed are the primal procedure and the dual ascent and descent procedures for variables λ_i^t, $i \in I_3$. The primal procedure is based on the primal procedures described in Dias et al (2004c). The other two procedures are generalisations to the multi-level case of the procedures described in Dias et al (2004a).

3.3 Dual Ascent Procedure for Variables λ_i^t, $i \in I_3$

Consider the following definitions:

$$\Delta(i) = \max_{\substack{j \in J \\ \tau \le t \\ p \in P(i)}} \left\{ \frac{v_j^\tau - d_{pj}^\tau}{d_j^\tau} \right\} \qquad (49)$$

$$JP(i,i',\delta,t) = \left\{ (j,p) : j \in J \wedge p \in P(i) \cap P(i') \wedge \frac{v_j^t - d_{pj}^t}{d_j^t} \le \delta \right\}, \qquad (50)$$

$$\overline{JP}(i,i',\delta,t) = \left\{ (j,p) : j \in J \wedge p \in P(i) \cap P(i') \wedge \frac{v_j^t - d_{pj}^t}{d_j^t} > \delta \right\}. \qquad (51)$$

Proposition 1: If dual variable λ_i^t is increased by $\delta \in]0,\Delta(i)]$, then slacks $SA_{i'\tau}^{\xi}$ and $SR_{i'\tau}^{\xi}$, $\tau \le t \le \xi$, $\forall i' \in I'$ such that $P(i) \cap P(i') \neq \emptyset$, will be changed by:

$$\Omega(\delta,i,i',\tau,\xi) = \sum_{s=\tau}^{\min\{\xi,t\}} \left(\sum_{(j,p) \in JP(i,i',\delta,s)} \eta_{i'jp}^s \max\left\{0, v_j^s - d_{pj}^s\right\} + \sum_{(j,p) \in \overline{JP}(i,i',\delta,s)} \eta_{i'jp}^s d_j^s \delta \right) - E \qquad (52)$$

where: $E = \begin{cases} Q_i \delta, & \text{if } i = i' \\ 0, & \text{otherwise} \end{cases}$.

The proof follows directly from similar results that are presented in Dias et al (2004a, 2004c), therefore is omitted.

Dual Ascent Procedure for Variables λ_i^t, $i \in I_3$

---

1. $t \leftarrow 1$.
2. $i \leftarrow 1$.
3. $\delta \leftarrow 0$; $\delta' \leftarrow +\infty$.
4. $\delta \leftarrow \max_{\substack{j \in J \\ \tau \le t \\ p \in P(i) \\ \frac{1}{d_j^\tau} \max\{0, v_j^\tau - d_{pj}^\tau\} < \delta'}} \left\{ \frac{1}{d_j^\tau} \max\left\{0, v_j^\tau - d_{pj}^\tau\right\} \right\}$.

5. Calculate sets $JP(i,i',\delta,t)$ and $\overline{JP}(i,i',\delta,t)$ as in (50) and (51), and $\Omega(\delta, i,i',\tau, \xi)$ as in (52), $\forall\ i' \in I'$, with $P(i) \cap P(i') \neq \emptyset$, $\forall \xi \geq t$.
6. If $\Omega(\delta,i,i',\tau, \xi) < 0$, $\forall\ i' \in I'$, with $P(i) \cap P(i') \neq \emptyset$, then go to 11. Else go to 7.
7. If $\exists\ i' \in I$, with $P(i) \cap P(i') \neq \emptyset$, such that $SA_{i'\tau}^{\xi} + \Omega(\delta,i,i',\tau,\xi) < 0$ or $SR_{i'\tau}^{\xi} + \Omega(\delta,i,i',\tau, \xi) < 0$, $\tau \leq t$, then go to 8. Else go to 9.
8. If $\delta' = 0$, then go to 11. Else $\delta' \leftarrow \delta$ and go to 4.
9. $\lambda_i^t \leftarrow \lambda_i^t + \delta$; $SA_{i'\tau}^{\xi} \leftarrow SA_{i'\tau}^{\xi} + \Omega(\delta,i,i',\tau, \xi)$ and $SR_{i'\tau}^{\xi} \leftarrow SR_{i'\tau}^{\xi} + \Omega(\delta,i,i',\tau,\xi)$, $\tau \leq t \leq \xi$, $\forall\ i' \in I'$ such that $P(i) \cap P(i') \neq \emptyset$.
10. Execute dual ascent procedure for variables v_j^t, with $J^+ = J$.
11. $i \leftarrow i + 1$. If $i > M$ then go to 12. Else go to 3.
12. $t \leftarrow t + 1$. If $t > T$ then stop. Else go to 2.

3.4 Dual Descent Procedure for Variables λ_i^t, $i \in I_3$

Proposition 2: If variable λ_i^t is decreased by a value δ in the interval defined by:

$$0 < \delta \leq \min_{\substack{j \in J,\, p \in P(i) \\ \tau \leq t \\ v_j^\tau - d_{pj}^\tau < 0}} \left\{ -\frac{1}{d_j^\tau}\left(v_j^\tau - d_{pj}^\tau\right) \right\},$$

then $SA_{i'\tau}^{\xi}$ and $SR_{i'\tau}^{\xi}$, $\tau \leq t \leq \xi$, $\forall i' \in I'$ such that $P(i) \cap P(i') \neq \emptyset$, will be changed by:

$$\Phi(\delta, i', i, \tau, \xi) = \delta \left(E - \sum_{\psi=\tau}^{\min\{\xi,t\}} \sum_{j \in J} \sum_{\substack{p \in P(i) \cap P(i') \\ v_j^\tau - d_{pj}^\tau \geq 0}} d_j^\tau \eta_{i'jp}^\tau \right), \qquad (53)$$

where $E = \begin{cases} Q_{i'}, & \text{if } i = i' \\ 0, & \text{otherwise} \end{cases}$.

The proof follows similar results presented in Dias et al (2004a, 2004c), therefore is omitted.

Dual Descent Procedure for Variables λ_i^t, $i \in I_3$

1. $t \leftarrow 1$.
2. $i \leftarrow 1$.
3. $\delta \leftarrow min\left\{ \lambda_i^t, \min_{\substack{j \in J, p \in P(i) \\ \tau \le t \\ v_j^\tau - d_{pj}^\tau < 0}} \left\{ -\frac{1}{d_j^\tau}\left(v_j^\tau - d_{pj}^\tau\right) \right\} \right\}$.
4. Calculate $\Phi(\delta, i', i, \tau, \xi)$ as in (53), $\forall\, i' \in I$, with $P(i) \cap P(i') \neq \varnothing$, $\tau \le t \le \xi$.
5. If $\Phi(\delta, i', i, \tau, \xi) < 0$, $\forall\, i' \in I$, with $P(i) \cap P(i') \neq \varnothing$, $\tau \le t \le \xi$ then go to 10. Else go to 6.
6. If $\exists\; i' \in I$, with $P(i) \cap P(i') \neq \varnothing$, such that $SA_{i'\tau}^\xi + \Phi(\delta, i', i, \tau, \xi) < 0$ or $SR_{i'\tau}^\xi + \Phi(\delta, i', i, \tau, \xi) < 0$, $\tau \le t \le \xi$, then go to 7. Else go to 8.
7. $\delta \leftarrow \min_{\substack{\tau \le t \\ S_{i'\tau}^\xi + \Phi(\delta, i', i, \tau, \xi) < 0}} \left\{ -\frac{S_{i'\tau}^\xi}{\Phi(\delta, i', i, \tau, \xi)/\delta} \right\}$. If $\delta = 0$ then go to 10. Else go to 8.
8. $\lambda_i^t \leftarrow \lambda_i^t - \delta$; $SA_{i'\tau}^\xi \leftarrow SA_{i'\tau}^\xi + \Phi(\delta, i', i, \tau, \xi)$ and $SR_{i'\tau}^\xi \leftarrow SR_{i'\tau}^\xi + \Phi(\delta, i', i, \tau, \xi)$, $\tau \le t \le \xi$, $\forall\, i' \in I$ such that $P(i) \cap P(i') \neq \varnothing$.
9. Execute dual ascent procedure for variables v_j^t, with $J^+ = J$.
10. $i \leftarrow i + 1$. If $i > M$ then go to 11. Else go to 3.
11. $t \leftarrow t + 1$. If $t > T$ then stop. Else go to 2.

3.5 Primal Procedure

The primal procedure constructs primal feasible solutions based on dual feasible solutions, trying to force complementary conditions to be satisfied. Consider:

$I^* = \{\, (i, \tau, \xi): i \in I'$ and $S_{i\tau}^\xi = 0 \,\}$,

$I_t^* = \{\, i: (i, \tau, \xi) \in I^*$ and $\tau \le t \le \xi \,\}$,

$I_t^+ = \{ i \in I'$: facility i is open during period $t \}$,

$I_A^+ = \{\, (i, \tau, \xi) : a_{i\tau}^\xi = 1 \,\}$,

$I_R^+ = \{ (i,\tau,\xi) : i \in I_1 \cup I_3 \text{ and } r_{i\tau}^{\xi} \geq 1\}$,

$P_t^+ = \{p: p \text{ is open during period } t\}$.

Sets I_t^* and I_t^+ are not necessarily equal, because the primal procedure will always try to open the minimum number of services, guaranteeing that all clients will be assigned to one open path in every time period. Furthermore, it is often necessary to insert in set I_t^+ services that do not belong to I_t^*. Sets I_A^+ and I_R^+ are built during the primal procedure and determine which services will be (re) opened, when and for how long.

Definition 1: A path *p* is considered essential during period *t* if there is at least one client *j* that has to be assigned to path *p* during period *t*. This happens if and only if $\exists j \in J : v_j^t \geq c_{pj}^t \wedge v_j^t < c_{p'j}^t, \forall p' \in P, p' \neq p$.

Paths considered as essential are the first to be considered open. To open a path *p* at time period *t* all facilities belonging to that path have to be open. This is achieved by inserting all those facilities in set I_t^+. When opening a path, the primal procedure will often violate complementary restrictions, because it includes in set I_t^+ facilities $i \notin I_t^*$. Whenever path *p* is open, all other paths *p*' such that if $i \in p'$ then $i \in p$ will also be open.

Paths not considered essential will only be explicitly open during time period *t* if there are clients *j* that cannot be assigned to already open paths. In this case, the procedure will open the path that corresponds to the smallest assignment cost.

Sets I_A^+ and I_R^+ are build based on sets I_t^+, $\forall t$. These sets are built in step 6 of the primal procedure using exactly the same procedures described in Dias *et al* (2002) for services $i \in I_2 \cup I_3$, and described in Dias et al (2004b) for services $i \in I_1$.[16]

Primal Procedure

---

1. $I_A^+ = I_R^+ = \emptyset$. $I_t^+ = \emptyset, \forall t$. Build sets I^* and I_t^*. $Num = 0$;
2. For t=1 until T, include in set P_t^+ all paths p such that $\exists j : v_j^t \geq c_{pj}^t$ and $v_j^t < c_{p'j}^t, \forall p' \neq p$. Update sets P_t^+, $\forall t$, including in P_t^+ all open paths $p \notin P_t^+$.

[16] In the referred to research report, it is defined the set I^+ that corresponds to $I_A^+ \cup I_R^+$ for $i \in I_3$.

3. For each client j such that $v_j^t < c_{pj}^t, \forall p \in P_t^+$, include in set P_t^+ path p such that $c_{pj}^t = \min_{v_j^t \geq c_{p'j}^t} c_{p'j}^t$. $Num \leftarrow Num+1$.
4. Include in set I_t^+ all facilities i belonging to path $p \in P_t^+$, $\forall t$.
5. If $Num = 1$ then $I_t^* \leftarrow I_t^+$ and $I_t^+ \leftarrow \varnothing$, $P_t^+ \leftarrow \varnothing$, $\forall t$, go to 2. Else go to 6.
6. Build sets I_A^+ and I_R^+. Update I_t^+ and P_t^+.
7. $t \leftarrow 1$.
8. Test the admissibility of the present solution in period t. If the solution is admissible go to 10, else go to 9.
9. Change the present solution, trying to achieve the admissibility for period t. If the admissibility is achieved go to 10. Else stop: it is not possible to find an admissible solution.
10. $t \leftarrow t+1$. If $t \geq T$ go to 11. Else go to 8.
11. Solve the assignment problem heuristically for all time periods.
12. $G = I_A^+ \cup I_R^+$.
13. Choose arbitrarily a variable $a_{i\tau}^{\xi}$ or $r_{i\tau}^{\xi}$ belonging to G and change its value to zero. If the solution remains admissible, recalculate heuristically the assignments of clients to open paths. Calculate Z' as the primal objective function value of the new solution. If $Z'<Z$, then remove variable $a_{i\tau}^{\xi}$ (or $r_{i\tau}^{\xi}$) from set I_A^+ or I_R^+ and $Z = Z'$.
14. Remove variable $a_{i\tau}^{\xi}$ (or $r_{i\tau}^{\xi}$) from set G. If $G = \varnothing$, stop. Else go to 13.

Steps 8, 9 and 11 require further explanations. After fixing the value of location decision variables, it is necessary to solve the assignment problem that cannot be decomposed in t separate linear programming problems due to the maximum decreasing capacities of sanitary landfills. The computational time needed to solve optimally the assignment problem (using a general solver) is prohibitive, at least to be used in a heuristic procedure.

Step 8 of the primal procedure tries to assess the admissibility of the current solution without solving the assignment problem. The procedure that is executed to test the present solution's admissibility is based on the following observations:

Observation 1: The total amount of waste that is necessary to treat in time period t has to be "absorbed" by incinerators and sanitary landfills.

Observation 2: The total amount of waste that is necessary to treat in period t has to be greater than or equal to the total minimum capacities of incinerators that are open during period t.

Observation 3: If there are admissible paths from locations j to incinerators and sanitary landfills that do not pass through waste transfer stations, then the number and dimensions of equipments installed in waste transfer stations do not influence the admissibility of a solution, in terms of the maximum quantity of waste that can be treated.

These observations lead to the conclusion that, to test the admissibility of a solution, it is necessary to consider only the incinerators and sanitary landfills installed.[17]

To calculate the maximum amount of waste the solid urban waste system can treat, one has to consider the incinerators' capability of reducing the total quantity of existing waste and also the maximum capacity of landfills. To use at the maximum the systems' capacities, one should send to the incinerators the maximum amount of waste they can incinerate (that can be inferior to their maximum capacity because the resulting ashes and scoria have to be sent to landfills).

Consider set $L_t \subseteq L$, that is composed of all incinerators that are open during time t, and set $I_t \subseteq I$ that is composed of all the sanitary landfills that are open in period t (being C_i^t the remaining capacity of sanitary landfill i at beginning of period t and $CAP_{sl}^t = \sum_{i \in I_t} C_i^t$).

Proposition 3: If all incinerators $l \in L_t$ have the same α value, then the maximum amount of waste the set L_t of incinerators can receive is given by:

$$CAP_{inc}^t = min\left\{ \sum_{l \in L_t} Q_{max}^l, \frac{CAP_{sl}^t}{\alpha} \right\} \tag{54}$$

The total amount of waste that is absorbed by incinerators can be calculated as $CAP_{inc}^t(1-\alpha)$.

---

[17] Throughout this work it is considered that all possible paths are admissible (this assumption is always valid because, if necessary, a path can have an assignment cost of $+\infty$).

Proposition 4: The total amount of waste that can be treated during period t by the described solid waste treatment system characterized by sets L_t and I_t can be calculated as:[18]

$$C = CAP_{inc}^{t}(1-\alpha) + CAP_{sl}^{t} \tag{55}$$

Proposition 5: For an urban solid waste treatment system to be able to treat the total amount of waste produced during period t it is necessary and sufficient that:[19]

$$\sum_{l \in L_t} Q_{min}^{l} \leq \sum_{j} d_j^t \leq CAP_{inc}^{t}(1-\alpha) + CAP_{sl}^{t} \tag{56}$$

Example 1:

Consider a system composed of two incinerators (α=10%) with maximum capacities equal to 100 and 200 and minimum capacities equal to 10 and 15, respectively, and one sanitary landfill with remaining capacity at the beginning of period t equal to 25. The maximum amount of waste this system will be able to treat can be calculated as $min\left\{100+200, \frac{25}{0.1}\right\} \times (1-0.1) + 25 = 250$. Notice that the total maximum capacity of incinerators cannot be used because the sanitary landfill does not have sufficient remaining capacity to receive the resulting incinerated waste. If the sanitary landfill had a remaining capacity equal to 100, then C would be equal to $min\left\{100+200, \frac{100}{0.1}\right\} \times (1-0.1) + 100 = 370$.

■

Proposition 3 can be changed in order to consider only admissible paths from incinerators to sanitary landfills:

Proposition 6: If all incinerators $l \in L_t$ have the same α value, then the maximum amount of waste the set L_t of incinerators can receive, considering only paths such that $\exists\, j$: $c_{pj}^{t} < +\infty$, is given by:

[18] If incinerators are considered as end-of-the-line facilities then the total system's capacity is much easier to calculate: $CAP_{sl}^{t} + \sum_{l \in L_t} Q_{max}^{l}$.

[19] Proposition 5 states a necessary and sufficient condition, but does not take into account the possible assignment costs of such a solution that can be equal to $+\infty$ if not all possible paths p are admissible.

$$CAP_{inc}^{t} = min\left\{ \sum_{l \in L_t} min\left\{ Q_{max}^{l}, \frac{\sum\limits_{\substack{i \in I_t \\ i:P(i)\cap P(l)\neq 0}} C_i^t}{\alpha} \right\}, \frac{\sum\limits_{\substack{i \in I_t \\ i:P(i)\cap P(l)\neq 0}} C_i^t}{\alpha} \right\} \qquad (57)$$

Propositions 4 and 5 could be changed accordingly.

Propositions 3 to 5 can be generalised to the situation where there are different incinerators with different α values (α_l). If this is the case, in order to take the most advantage of the systems' capabilities, one should first send waste to the open incinerator with the smallest α_l.

Proposition 7: The maximum amount of waste the set L_t of incinerators can receive is given by:

$$CAP_{inc}^{t} = \sum_{l \in L_t} min\left\{ Q_{max}^{l}, \frac{max\left\{0, CAP_{sl}^{t} - \sum\limits_{l':\alpha_{l'} < \alpha_l} Q_{max}^{l'} \alpha_{l'} \right\}}{\alpha_l} \right\} \qquad (58)$$

The total amount of waste that is absorbed in incinerators is given by:

$$\sum_{l \in L_t} (1-\alpha_l) min\left\{ Q_{max}^{l}, \frac{max\left\{0, CAP_{sl}^{t} - \sum\limits_{l':\alpha_{l'} < \alpha_l} Q_{max}^{l'} \alpha_{l'} \right\}}{\alpha_l} \right\}. \qquad (59)$$

Proposition 4 and 5 could be changed accordingly.

Example 2:

Consider a system composed of three incinerators, with maximum capacities, minimum capacities and α_l values as given in table 1, and a sanitary landfill with remaining capacity at the beginning of period t equal to 100.

The maximum system's capacity is calculated as:

$$(1-0.20)min\left\{110,\frac{max\{0,100-300\times 0.10\}}{0.20}\right\}+$$

$$(1-0.50)min\left\{200,\frac{max\{0,100-300\times 0.10-110\times 0.20\}}{0.50}\right\}+(1-0.10)min\left\{300,\frac{100}{0.10}\right\}+100=506$$

	l_1	l_2	l_3
Q^l_{max}	110	200	300
Q^l_{min}	10	20	5
α_l	0.20	0.50	0.10

Table 1

This capacity is achieved if the maximum capacity of incinerator l_3 is consumed (needing to deposit in the landfill 300×0.10 units of waste), then the maximum capacity of incinerator l_1 is used (needing to use 110×0.20 units of the landfill's capacity) and finally the incinerator l_2 receives the maximum amount of waste it can receive considering that the sanitary landfill has a remaining capacity that is equal to 48.

■

Steps 8 and 9 of the primal procedure are based on proposition 5 and execute the procedure that tests and tries to achieve the admissibility of the current primal solution. Consider the following notation:

$F_i^t, i \in I_2$ and $i \notin I_t^+$: minimum cost of opening facility i during period t (this value is calculated using the procedure described in Dias et al, 2004a, for the C2-DLPOCR primal procedure);

$F_i^t, i \in I_1$: minimum cost of opening an equipment of dimension s at waste transfer station k during period t (this value is calculated as described in Dias et al, 2004b);

$h_i^t, i \in I_3$ and $i \notin I_t^+$: minimum cost incurred by (re)opening a facility i during period t (calculated as described in Dias et al, 2004a - steps 1-10);

$p_i^t, i \in I_3$ and $i \in I_t^+$: minimum cost incurred by (re)opening a facility i at the beginning of time period $t' < t$ (calculated as described in Dias et al, 2004a - steps 1-6).

Procedure to test and achieve the admissibility of a given solution

1. $t \leftarrow 1$; $CAP_{sl}^t \leftarrow 0$; $CAP_{sl}^0 \leftarrow 0$; *extra* $\leftarrow +\infty$; *admissible* $\leftarrow$ *true*.

2. $CAP_{sl}^{t} \leftarrow CAP_{sl}^{t-1} + \sum_{\xi=t}^{T}\sum_{i} Q_i\left(a_{it}^{\xi} + r_{it}^{\xi}\right)$; Calculate CAP_{inc}^{t} as in (54);

$C_{min}^{t} \leftarrow \sum_{\tau=1}^{t}\sum_{l} Q_{min}^{l} y_l^{\tau}$; $D \leftarrow \sum_{j} d_j^{t}$. Calculate C as in (55).

3. If $C_{min}^{t} \le D \le C$, then go to 4. Else go to 5.

4. If $CAP_{inc}^{t} > D$ then $CAP_{sl}^{t} \leftarrow CAP_{sl}^{t} - D\alpha$; else

   $CAP_{sl}^{t} \leftarrow CAP_{sl}^{t} - D + CAP_{inc}^{t}(1-\alpha)$. If *extra* > CAP_{sl}^{t}, then *extra* $\leftarrow CAP_{sl}^{t}$. $t \leftarrow t+1$.

   If $t \ge T$ then stop. Else go to 2.

5. If $D < C_{min}^{t}$ then go to 6. Else go to 9.

6. For each $l \in L_t$ calculate $g_l^{t} \leftarrow \begin{cases} FA_l^{t+1} - FA_l, \text{if } y_l^{t} = 1 \text{ and } t < T \\ -FA_l^{t}, \text{if } y_l^{t} = 1 \text{ and } t = T \\ +\infty, \text{if } y_l^{t} = 0 \end{cases}$. If $g_l^{t} = +\infty$,

   $\forall l \in L_t$, then *admissible* $\leftarrow$ *false* and stop: it is not possible to achieve an admissible solution through this procedure. Else go to 7.

7. Consider set:

   $L_t^{+} = \left\{l \in L_t : g_l^{t} \ne +\infty \text{ and } C > D, \text{considering } L_t = L_t \setminus \{l\} \text{ and using (55)}\right\}$. If $L_t^{+} \ne \emptyset$,

   choose $l' = \underset{l \in L_t^{+}}{arg\,min}\left\{g_l^{t}\right\}$. Else consider $l' = arg\,min\left\{g_l^{t}\right\}$.

8. Change $y_{l'}^{t}$ from one to zero and $y_{l'}^{t+1}$ from zero to one. Recalculate C_{min}^{t}, CAP_{inc}^{t} and C. Go to 5.

9. If $D \le C$ then go to 4. Else go to 10.

10. Calculate F_i^{t} , $\forall i \notin I_t^{+}$ and $i \in I_2$. If $Q_{min}^{i} + C_{min}^{t} > D$, then $F_i^{t} \leftarrow +\infty$; else

    $F_i^{t} \leftarrow F_i^{t} \times \left(1 + \dfrac{Q_{min}^{l}}{Q_{max}^{l}}\right)$, $\forall i \notin I_t^{+}$ and $i \in I_2$. Calculate F_i^{t} , $\forall i \in I_1$. Calculate

    h_i^{t} ,$\forall i \notin I_t^{+}$ and $i \in I_3$ and p_i^{t} , $\forall i \in I_t^{+}$ and $i \in I_3$.

11. Calculate $H_p^t = \sum_{\substack{i\in p \\ i\notin I_t^+ \\ i\in I_2}} F_i^t + \sum_{\substack{i\in p \\ i\in I_1}} F_i^t + \sum_{\substack{i\in p \\ i\in I_t^+ \\ i\in I_3}} p_i^t + \sum_{\substack{i\in p \\ i\notin I_t^+ \\ i\in I_3}} h_i^t$,$\forall p$. For each p such that $H_p^t < +\infty$, calculate C_p as in (55), considering the facilities that are open and belong to p. If $H_p^t = +\infty$, $\forall p$ then *admissible* $\leftarrow$ *false* and stop: the problem is infeasible. Else $H_p^t \leftarrow H_p^t \times (1+\Theta)$, $\Theta = \begin{cases} 0, & \text{if } C'_p \geq D \\ \dfrac{D - C'_p}{C'_p - C}, & \text{if } C'_p < D \end{cases}$.

12. Choose path p' such that $H_{p'}^t = \min_p \{H_p^t\}$. Open path p', including p' in set P_t^+ and changing the solution according with the calculation of H_p^t. Recalculate C_{min}^t, CAP_{inc}^t, CAP_{sl}^t and C and go to 9.

Steps 6 to 8 consider the situation when the solution is not admissible at period t due to the minimum capacities of open incinerators. This procedure tries to close incinerators that are open at the beginning of period t. If all incinerators $l \in L_t$ were open earlier, then this procedure is not capable of changing the current solution. It would be possible to close incinerators open at the beginning of time periods $t' < t$, but it would be necessary to verify if the solution remained admissible (in terms of maximum capacity restrictions) for periods from t' to t–1. This would increase immensely the complexity of the described procedure. As the primal-dual heuristic is dealing with an urban waste treatment system, it is natural to assume that the D value will increase (or, at least, not decrease) with t, so an incinerator open at the beginning of period t' will not be responsible for violations of the minimum capacity restrictions in periods $t > t'$.

At the end of this procedure, if variable *admissible* is true then the current solution is admissible and it is necessary to determine heuristically the assignments of waste to paths. Variable *extra* represents the maximum amount of waste that can be deposited in sanitary landfills in addition to the amount of waste that has to be transported to the landfills (the end products of incineration plus the waste that exceeds the total maximum capacity of open incinerators). This procedure considers that all sanitary landfills with remaining capacities greater than zero at the beginning of period t are open during that period (as can be seen by the way the CAP_{sl}^t value is updated in step

2). As can be seen by the restrictions of the primal problem, there are no guarantees that the current solution will satisfy this assumption. That is why the heuristic that finds an admissible assignment solution is sometimes forced to change the current solution by increasing the number of time periods the sanitary landfills with remaining capacities are open.

Example 3:

Consider a system with n = 3, m = 3, p = 0, r = 3, α = 10%, T = 3, Q_{max}^1=5000, Q_{max}^2=10.000, Q_{max}^3=1000, $Q_{min}^1 = Q_{min}^2 = Q_{min}^3 = 50$, Q_1=900, Q_2=1100, Q_3=1300, $d_1^1 = d_1^2 = d_1^3 = 2000$, $d_2^1 = d_2^2 = d_2^3 = 3000$ and $d_3^1 = d_3^2 = d_3^3 = 4000$. Consider the following solution: $y_1^1 = y_2^1 = 1$, $a_{11}^3 = a_{21}^3 = 1$. The fixed costs of opening incinerator 3 are 100, 80 and 50 for time periods 1, 2 and 3 respectively. The fixed open and reopen costs for sanitary landfills are given in table 2.

	FA_{i1}^1	FA_{i1}^2	FA_{i1}^3	FA_{i2}^2	FA_{i2}^3	FA_{i3}^3	FR_{i2}^2	FR_{i2}^3	FR_{i3}^3
i =1	5	10	15	5	10	10	5	10	6
i =2	10	11	12	10	15	10	10	10	10
i =3	20	25	25	20	25	10	20	25	30

Table 2

There are 12 admissible paths:
$p_1 = \{1,4\}$; $p_2 = \{1,5\}$; $p_3 = \{1,6\}$; $p_4 = \{2,4\}$; $p_5 = \{2,5\}$; $p_6 = \{2,6\}$; $p_7 = \{3,4\}$; $p_8 = \{3,5\}$; $p_9 = \{3,6\}$; $p_{10} = \{4\}$; $p_{11} = \{5\}$; $p_{12} = \{6\}$.

Period t = 1: CAP_{sl}^1= 2000; CAP_{inc}^1=15000; C_{min}^1=100; D = 9000; C =15500. As 100 ≤ 9000 ≤ 15500 and 15000>9000 then CAP_{sl}^1=2000–9000×0.1=1100. *Extra*=1100.

Period t = 2: CAP_{sl}^2 = 1100; CAP_{inc}^2 = 11000; C_{min}^2=100; D = 9000; C = 11000. As 100 ≤ 9000 ≤ 11000 and 11000>9000 then CAP_{sl}^2=1100–9000×0.1=200 *Extra*=200.

Period t = 3: CAP_{sl}^{3} = 200; CAP_{inc}^{3} = 2000; C_{min}^{3}=100; D = 9000; C = 2000. As 9000>2000 the current solution is not admissible. It is needed to open more facilities. The procedure calculates: $F_3^1 = 50 \times \left(1 + \frac{50}{1000}\right) = 52.50$; $h_6^3 = FA_3^3 = 10$; $p_4^3 = min\{5+10-15, 10+6-15\} = 0$; $p_5^3 = min\{10+10-12, 11+10-12\} = 8$.

It then calculates: $H_1^3 = 0$; $H_2^3 = 8$; $H_3^3 = 10$; $H_4^3 = 0$; $H_5^3 = 8$; $H_6^3 = 10$; $H_7^3 = 52.5$; $H_8^3 = 60.5$; $H_9^3 = 62.5$; $H_{10}^3 = 0$; $H_{11}^3 = 8$; $H_{12}^3 = 10$. For each and every path $p \in P$ we have $C_p > D$, so values H_p^3 are not changed. The procedure chooses arbitrarily between p_1, p_4 or p_{10}, and changes the solutions as follows: $a_{11}^1 = r_{12}^3 = 1$ and $a_{11}^3 = 0$. CAP_{sl}^{3} = 200+900 = 1100 and C = 11000. As $C \geq D$ and 11000>9000, CAP_{sl}^{3}=1100-900=200. The current solution is admissible.

■

At step 11 of the primal procedure, the following procedure is used to solve heuristically the assignment problem:

<u>Heuristic that finds an admissible solution to the assignment problem</u>

1. $t \leftarrow 1$; $C_i^0 \leftarrow 0, \forall i$.
2. Calculate CAP_{inc}^{t}; $C_i^t \leftarrow C_i^{t-1} + \sum_{\xi=t}^{T} Q_i \left(a_{it}^{\xi} + r_{it}^{\xi}\right); D \leftarrow \sum_j d_j^t$.
3. If $i \in I_t$ then $B_i \leftarrow C_i^t$; else $B_i \leftarrow 0$. If CAP_{inc}^{t} >0 then go to 4; else go to 5.
4. Calculate $B_{total} \leftarrow \sum_i B_i$. If $CAP_{inc}^{t} > D$, then $Sum \leftarrow extra$; else $Sum \leftarrow D + extra - CAP_{inc}^{t}$. If $Sum > B_{total}$, then $Sum \leftarrow B_{total}$. $B_i \leftarrow \frac{B_i}{B_{total}} Sum$, $\forall i$.
5. Solve a transshipment problem considering as sources the set J of clients (with supplies d_j^t), as destinations the set of facilities $i \in I_3$ (with demands B_i), and also two destinations for each open incinerator ($i \in I_2$): one has demand equal to its minimum capacity (destination i) and the other has demand equal to its maximum

capacity minus minimum capacity (destination i'). The transshipment points are the set of facilities $i \in I_1$ (with demand and supply equal to their maximum capacities). Consider variables x_{ji} represent the total amount of waste that is transported from location j to location i in the optimal solution. If the problem is not feasible go to 7. Else go to 6.

6. Update the values of C_i^t: $C_i^t \leftarrow C_i^t - \sum_j x_{ji} - \sum_{i_1 \in I_1} x_{i_1 i}$, $\forall i \in I_3$.

   $X_i^t \leftarrow \sum_j x_{ji} + \sum_{i_1 \in I_1} x_{i_1 i}$, $\forall i \in I_3$. $X_i^t \leftarrow \sum_j x_{ji} + \sum_j x_{ji'} + \sum_{i_1 \in I_1} x_{i_1 i} + \sum_{i_1 \in I_1} x_{i_1 i'}$, $\forall i \in I_2$.

   Go to 8.
7. Consider all services $i \in I_3$ such that $C_i^t > 0$ and $B_i = 0$. Calculate the minimum cost of opening facility i during period t, by changing the lower or upper limits of variables $a_{i\tau}^{\xi}$ or $r_{i\tau}^{\xi}$ ($i \in I_3$) that are equal to one in the current solution. Choose the facility that corresponds to the least of these costs, change the solution and go to 2.
8. If $CAP_{inc}^t > 0$ then go to 9; else go to 10.
9. Solve a transportation problem considering as sources the open incinerators $i \in I_2$ with supplies equal to αX_i^t, and destinations $i \in I_3$ with demands C_i^t.
10. Update the values of C_i^t: $C_i^t \leftarrow C_i^t - \sum_{i' \in I_2} x_{i'i}$ and $X_i^t \leftarrow X_i^t + \sum_{i' \in I_2} x_{i'i}$, $i \in I_3$.
11. If $CAP_{inc}^t > D$ then $extra \leftarrow extra - \left(\sum_{i \in I_3} X_i^t - D\alpha \right)$; else

    $extra \leftarrow extra - \left(\sum_{i \in I_3} X_i^t - \left(D - (1-\alpha) \times CAP_{inc}^t \right) \right)$;
12. $t \leftarrow t + 1$. If $t > T$ then stop. Else go to 2.

The described procedure solves heuristically the assignment problem through the resolution of one or two transportation problems in each time period t (depending on existing or not an open incinerator during period t) [20]. The first transportation problem does not consider the transportation of waste from incinerators to landfills. Step 4 of the

[20] If the incinerators are considered as end-of-the-line facilities, this procedure would naturally be much easier, being only necessary to solve one transportation problem in each time period.

procedure calculates the demands of the landfills guaranteeing that they will have sufficient remaining capacity to receive the waste that results from incineration. The second transportation problem considers the transport of waste from incinerators to landfills. The calculation and update of variable *extra* gives some flexibility to the transportation problems solved: it represents the total sanitary landfills' capacity that can be used in addition to the capacity that really has to be consumed.

Example 4:

Consider the problem described in example 3.

Period $t = 1$: $CAP_{inc}^{1} = 15000$; $C_4^1 = 900$; $C_5^1 = 1100$; $C_6^1 = 0$; $D = 9000$.

$B_4 = 900$; $B_5 = 1100$; $B_6 = 0$. $B_{total} = 2000$; $Sum = 200$.

$B_4 = \dfrac{900}{2000} \times 200 = 90$; $B_5 = \dfrac{1100}{2000} \times 200 = 110$.

As there are no waste transfer stations, step 5 solves a transportation problem with three sources with supplies 2000, 3000 and 4000 and six destinations (table 3).

Destination	1	1'	2	2'	4	5
Demand	50	4950	50	9950	90	110

Table 3

It is necessary to consider a fictitious origin with supply equal to 6200 (the costs from this origin to destinations 1' and 2' are equal to $+\infty$). Consider that the optimal solution to this transportation problem is given in table 4.

The procedure updates $C_4^1 = 900 - 90 = 810$; $C_5^1 = 1100$; $X_1^1 = 5000$; $X_2^1 = 3910$; $X_4^1 = 90$ and $X_5^1 = 0$.

	1	1'	2	2'	4	5
1	50	1860			90	
2			50	2950		
3		3090		910		

Table 4

It is necessary to solve another transportation problem considering as sources the incinerators 1 and 2 with supplies equal to 500 and 391, respectively, and destinations 4 and 5 with demands equal to 810 and 1100. Suppose the optimal solution is given in table 5.

	4	5
1	500	
2	310	81

Table 5

$C_4^1 = 810 - 500 - 310 = 0$; $C_5^1 = 1100 - 81 = 1019$; $X_4^1 = 90 + 810 = 900$;

$X_5^1 = 81$.

As 15000 > 9000 then *extra* is equal to 200 − (981 − 9000 × 0.1) = 119. This means that there were (200 − 119) units of waste that could have been sent to the incinerators but were sent to sanitary landfills.

Period $t = 2$: CAP_{inc}^1 = 15000; C_4^1 = 900; C_5^1 = 1019+1100 (because r_{12}^3 = 1); C_6^1 = 0; D = 9000, and the procedure would be repeated.

■

4 Conclusion

In the present research report two models were developed that can be used in the process of planning a solid urban waste treatment system. A primal-dual heuristic was described that can calculate primal admissible solutions to the problem. The computational tests already performed show that the heuristic performs well if the maximum capacity restrictions are not too tight in relation with the total amount of waste. If the maximum capacity restrictions are tighter, then only a small group of location variables that are found by the heuristic (the ones considered at step 6 of the primal procedure) belong to the optimal solution.

The problems considered in this report are multi-objective and multi-decision maker by nature. The heuristic proposed can be used in a multi-objective environment if the several objective functions considered are previously weighted (the primal admissible solutions calculated by the heuristic are not guaranteed to be non-dominated, because they are not, in principle, optimal solutions). Nevertheless, it is the authors' opinion that the complexity of the described problem justifies the use of a dedicated decision aiding system.

5 References

Antunes, A. P. (1999) “Location Analysis Helps Manage Solid Waste in Central Portugal”, Interfaces, 29, pp 32-43

Dias, J., Captivo, M.E., Clímaco, J. (2002) “Problema de Localização Dinâmica com Abertura, Fecho e Reabertura de Serviços: Formulação e Heurística Primal-Dual”, Research Report n. 6, Inesc-Coimbra

Dias, J., Captivo, M.E., Clímaco, J. (2004a) “Capacitated Dynamic Location Problems with Opening, Closure and Reopening of Facilities”, Research Report n. 2/2004 Inesc-Coimbra

Dias, J., Captivo, M.E., Clímaco, J. (2004b) “Dynamic Location Problems with Discrete and Reduction Sizes of Available Capacities”, Research Report n. 18/2004 Inesc-Coimbra

Dias, J., Captivo, M.E., Clímaco, J. (2004c), “Dynamic Multi-Level Capacitated and Uncapacitated Location Problems: an Approach Using Primal-Dual Heuristics”, Research Report n. 26/2004 Inesc-Coimbra

Erlenkotter, D. (1978) “A Dual-Based Procedure for Uncapacitated Facility Location”, Operations Research 26, pp 992-1009.

Guignard, M., Spielberg, K. (1979) “A Direct Dual Method for the Mixed Plant Location Problem with Some Side Constraints”, Mathematical Programming, 17, pp 198-228

Van Roy, T., Erlenkotter, D., (1982) “A Dual-Based Procedure for Dynamic Facility Location”, Management Science, 28, pp 1091-1105.

Acknowledgments

The authors would like to thank Doutor António Antunes, professor in the Civil Engineering Department of the University of Coimbra, Eng. Cordeiro Santo from ERSUC and Dr. António Manuel Tenreiro da Cruz from Associação de Municípios da Região do Planalto Beirão, for all the informations kindly provided.