Dynamic Multi-Level Capacitated and Uncapacitated Location Problems: an approach using primal-dual heuristics

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Abstract: In this paper several dynamic multi-level location problems are formulated as mixed-integer linear programs. Both uncapacitated and capacitated versions of the problem are studied. The models presented are more complete than the ones known from the literature: they are dynamic and consider the possibility of a facility being open, closed and reopen more than once during the planning horizon. They may include both upper and lower limits on the used capacity of each facility and may also consider the situation where there is no flow conservation in the intermediate facilities. Primal-dual heuristics were developed to solve efficiently the proposed models. Computational results are presented and discussed.

1 Introduction

Multi-level location problems have been widely studied in the literature. Authors studying and writing about these problems designate them differently: hierarchical, multi-level, multi-echelon, multi-stage, etc. Generally, the designation of the problem indicates the maximum number of levels considered: *k*-hierarchical, *k*-level, *k*-echelon location problems refer to problems with, at most, *k* levels of facilities.

The models developed in this research report are designated by multi-level, because it is the authors' understanding that this is the most general designation (a multi-level location problem may not necessarily represent a hierarchical location problem; in the latter case facilities have to be organized in a hierarchical structure, while in the first case this is not necessarily the case).

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There are several examples in our daily lives that show the importance of considering multi-level facility location problems: the hierarchical health service system, the hierarchical education system, the multi-level structure of bank and post-offices organizations, etc. These facility structures have some common features and some important differentiating characteristics. One can say that a multi-level facility location model is needed whenever the facilities to be located can be grouped in sets (levels) having different characteristics (by offering different services to clients, for instance) and interact with each other so that is not possible to locate facilities in each set independently from the others. The existing links between the different kinds of facilities can be implicit (in the form of a constraint that limits the global budget that can be spent globally), or explicitly determining the flow of clients between facilities (Daskin, 1995).

Narula (1984) attempts to classify the hierarchical location-allocation problems. He differentiates the problems through the facility hierarchy (successively inclusive – a facility in level m offers all services of facilities in levels 1 to m-1 plus services of level m; successively exclusive – a facility in level m offers services unique to it); arc and node flow discipline (flow is integrated if it is from any lower level to any higher level or is discriminating if the flow is from level m to level m+1; a unipath network is a network where the positive degree of every node is less than or equal to one, whereas in a multipath network the positive degree of at least one node is greater than one). To completely specify an hierarchical location-allocation problem it is needed to state the number of levels, the type of facility hierarchy, the arc and flow discipline. Nevertheless the classification scheme presented is not exhaustive, as the author remarks. Daskin (1995, pp 317-333) gives the basic notions of hierarchical facilities, and describes some models: median-based, coverage-based hierarchical location formulation and some extensions to those cases.

The models described in the literature are, almost exclusively, static problems. Some consider the uncapacitated others the capacitated version of the multi-level problem.

Moore and ReVelle (1982) describe a real case of successively inclusive hierarchical service location problem. They consider as objective the minimization of the population that lacks access to one or more components of service. A population is covered by a given level of service if some member of the facility hierarchy eligible to provide that service is located within a maximum established distance. The linear relaxation of the problem is solved, followed by a branch and bound procedure whenever necessary. Tien *et al* (1983) develop and solve two models with applications in the hierarchical health facility location-allocation problem. Tcha and Lee (1984) present a branch and bound algorithm based on a dual ascent procedure (similar to the one developed by Erlenkotter, 1978) for the multi-level uncapacitated facility location problem. Ten years later, a paper by Barros and Labbé (1994) prove that the multi-level uncapacitated facility location problem is not submodular, and question the validity of the branch and bound procedure described in Tcha and Lee

(1984). Ro and Tcha (1984) describe a branch and bound algorithm for resolution of the two-level uncapacitated facility location with additional constraints that represent the adjunct relationship of some warehouses to a certain plant (if a plant is open, there is a set of warehouses that have to be also open). Narula and Ogbu (1985) use lagrangean relaxation with subgradient optimisation to solve a two-hierarchical uncapacitated successively inclusive location problem. The model considers the location of p_1 facilities in level 1 and p_2 facilities in level 2. Only a fraction of the demand from level 1 facilities is referred to level 2 facilities. Gao and Robinson (1992) investigate the use of dual-based procedures similar to the ones develop by Erlenkotter (1978) in the resolution of the two-echelon uncapacitated facility location problem. Aardal *et al* (1996) investigate structural properties of the uncapacitated two-level facility location problem. Two different formulations are studied, that use different decision variables (the authors call these two formulations the single and the multi commodity flow formulations). Edwards (2001) studies several properties and describes approximation algorithms for the multi-level facility location problem. Bumb (2001) and Zhang and Ye (2002) describe approximation algorithms for the two level uncapacitated facility location problem with a maximization objective. Galvão *et al* (2002) treats a 3-level successively inclusive facility location model applied to a case study. There is an upper bound on the maximum number of facilities to locate at each level k, k=1,2,3. The authors do not consider the existence of fixed opening costs. Two heuristics are developed to find feasible solutions to the problem. Espejo *et al* (2003) treat the maximal covering two-level location problem. A combined Lagrangean-surrogate relaxation is imbedded into a subgradient optimization algorithm to find lower and upper bounds to the optimal solution.

Some references on capacitated multi-level problems are also available. Eitan *et al* (1991) present a mixed-integer linear programming model, which allows different hierarchical relationships to be simultaneously present, capacity constraints to be placed both on service types and service groups and the consideration of both fixed and variable costs. The model is applied to several literature-based problems and also to a new large-scale problem that fully illustrates the model capabilities. Bloemhof-Ruwaard *et al* (1996) present a capacitated version of the two-level facility location problem. They consider two different model formulations, and compare the quality of the lower bounds obtained by their linear relaxations with the lower bounds obtained with a lagrangean relaxation. The authors use LP round-off heuristics and also sequential capacitated facility location heuristics to find feasible primal solutions. Tragantalerngsak *et al* (1997) develop and compare six different lagrangean relaxations to the two-echelon, single source capacitated facility location problem. The lagrangean relaxations are used within a subgradient optimisation algorithm, and feasible solutions are constructed by heuristics based on a general assignment problem formulation. In a following paper (Tragantalerngsak *et al*, 2000), a branch and bound method based on the most efficient lagrangean relaxation is presented. Pirkul and Jayaraman (1998) develop a multi-commodity, capacitated, two-level facility location model. They consider the location of plants and warehouses. The problem is solved heuristically, constructing a primal feasible solution from the solution of a lagrangian relaxation (within a subgradient method optimisation). Chairdaire (1999) treats both uncapacitated and capacitated two-level location problems in the telecommunications network-planning context. Lagrangean relaxation and simulated annealing are both used to find feasible solutions. Klose (1995, 1998, 2000) considers a two-level single sourcing capacitated facility location problem (clients are served from depots that, in turn, are served by plants). The location decision variables consider only the problem of locating depots (it is considered that plants are already located at fixed sites). The problem is solved using lagrangean relaxation followed by a heuristic procedure. The author tries to solve the same problem using a LP-based heuristic (Klose, 1999): introducing several valid inequalities motivated by the subproblems contained in the multi-level problem, the LP relaxation is strengthen and solved. A heuristic procedure builds an admissible solution from the optimal linear solution calculated. Several computational tests where performed to assess the performance of both resolution approaches. Jayaraman *et al* (2003) consider a hierarchical facility structure where clients have two different kinds of demand: demand for basic and specialized services. The model presented intends to maximize the total demand coverage, allocating levels of services to open facilities and considering capacity constraints. A lagrangean relaxation followed by a primal heuristic is developed, and the viability of the approach is demonstrated through the results of extensive computational experiments.

Dynamic multi-level location problems are described in a few number of more recent papers. Melachrinoudis and Min (2000) present a real case of dynamic relocating an existing facility that belongs to a two-level facilities structure. The transition between the existent and the new location has to be done gradually. The authors consider both capacity and budget constraints and more than one objective function. No dedicated procedure is developed: the multi-objective mixed-integer linear problem is solved using a general solver. Hinojosa *et al* (2000) model the dynamic two-echelon multicommodity capacitated plant location problem. The model considers the possibility of opening a facility at the beginning of any time period (remaining open until the end of the planning horizon), and closing an already existing facility at the end of any time period (remaining closed until the end of the planning horizon). The calculation of admissible solutions is done using heuristics based on a lagrangean relaxation. Canel *et al* (2001) develop a model for the same kind of problem, but consider the possibility of a facility being open in more than one time period, not sequentially. The authors consider reopening and closing costs, but present a non-linear objective function. The paper describes an algorithm to solve the problem.

There are other interesting aspects of multi-level facility location problems that have also been treated by some authors. Hodgson (1981) studies a different problem that can be considered a kind of multi-level location problem: the location of public facilities intermediate to the journey to work. It is considered that clients have to travel daily from home to work. The objective is to locate public facilities such that the extra time needed to travel to those facilities is minimized. Madsen (1983) studies the problem of combined location-routing problems in a system composed of clients, depots and, at most, one factory. Serra *et al* (1992) study the problem of locating facilities with a hierarchical structure when there is competition in the region of interest. The model developed allows for both the location of new facilities and the relocation of existing facilities. Marianov and Serra (2001) study a hierarchical location-allocation model where the congestion problem is treated explicitly: there are situations where clients have to wait on queue for some time before being served. The authors describe a bi-level heuristic for construction of feasible solutions, and present the results of computational tests performed.

In this research report several dynamic multi-level location problems are addressed. In section 2 the uncapacitated version of the problem is described, in section 3 the capacitated version with maximum capacity restrictions and, in section 4, the capacitated version with maximum and minimum capacity restrictions but with no flow conservation at the intermediate facilities. In sections 2 to 4, primal-dual heuristics are described that can calculate admissible solutions for the corresponding problem, based on the work of Erlenkotter (1978), Van Roy and Erlenkotter (1982) and Guignard and Spielberg (1979). All problems treated in this research report have the following characteristics that distinguish them from the problems usually referred to in the literature:

1. The problems are dynamic and consider that a facility can be open, closed and reopen more than once during the planning horizon. It is possible to consider explicitly different fixed opening and reopening costs (that are, most of the times, clearly different). It is possible to consider the existence of already open facilities. It is possible to establish minimum functioning time intervals for a given facility (in some situations, a facility that is open in time period *t* should be open for a minimum number of time periods before being closed).
2. The models can be used for both successively inclusive and exclusive hierarchical problems. The model is also valid for all the arcs' and nodes' flow disciplines defined in Narula (1984).
3. The model can deal with paths of facilities with a different number of arcs. In a *k*-level facility location problem, it is possible to consider paths with a number of arcs from 1 to *k*. This is an interesting feature for both successively inclusive and exclusive problems. Imagine, for instance, a health facility service system. Patients are advised to go to their nearest health centre, from there they may be sent to a regional hospital, and from there to a central hospital (whenever it is

needed). Sometimes patients prefer to go directly to regional or central hospitals. In the models here presented, all these situations can be considered.

4. It is possible to consider more than one path constituted by exactly the same facilities, but with different characteristics. This is an important feature, especially if dealing with more than one objective function. Imagine a problem where there are two objectives: minimizing total cost and total travel time. Consider a path constituted by 2 locations. One can consider the locations connected by a highway and by a national road. The first option will be more expensive but faster, the second option will be less expensive but slower. Both paths can be simultaneously considered in the model formulation.

In section 5 some conclusions and future work directions are addressed.

2 Multi-Level Uncapacitated Location Problem

Figure 1 illustrates a possible 2-level network configuration. Clients are assigned to paths, and not to single facilities.

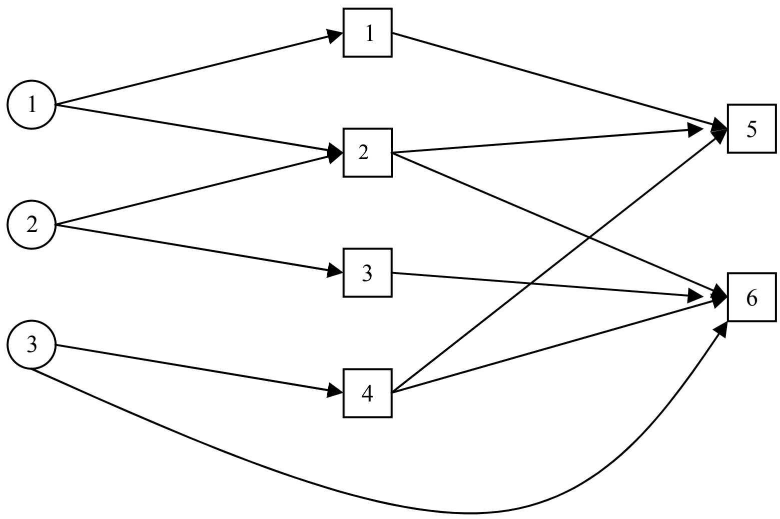

Figure 1: Example of a 2-level facility network. The symbol ○ represents clients, and □ represents facilities.

Consider the following definitions:

$J = \{1,..., j, \ldots, n\}$ set of indexes corresponding to the clients' locations;

$I = \{1,..., i, \ldots, m\}$ set of indexes corresponding to facilities' possible locations;

$P = \{1, \ldots, q\}$ set of all possible paths;

$P(i) = \{ p \in P: i$ belongs to path $p \}$;

T = number of time periods considered in the planning horizon ($1 \leq t \leq \xi \leq T$);

K = maximum number of levels in the multi-level facility structure;[1]

[1] K will also be the maximum path length.

c_{pj}^{t} = cost of fully assigning client j to path p in period t;

FA_{it}^{ξ} = fixed cost of opening a facility i at the beginning of period t, and closing it at the end of period ξ (the facility will be in operation from the beginning of t to the end of ξ);[2]

FR_{it}^{ξ} = fixed cost of reopening a facility i at the beginning of period t, and closing it at the end of period ξ (the facility will be in operation from the beginning of t to the end of ξ);[2]

and let us define the variables:

$$a_{it}^{\xi} = \begin{cases} 1 & \text{if facility } i \text{ is open at the beginning of period } t \text{ and stays open until the end} \\ & \text{of period } \xi \\ 0 & \text{otherwise} \end{cases}$$

$$r_{it}^{\xi} = \begin{cases} 1 & \text{if facility } i \text{ is reopen at the beginning of period } t \text{ and stays open until the end} \\ & \text{of period } \xi \\ 0 & \text{otherwise} \end{cases} \quad ,t > 1$$

$$x_{pj}^{t} = \begin{cases} 1 & \text{if client } j \text{ is assigned to path } p \text{ during period } t \\ 0 & \text{otherwise} \end{cases}.$$

The definition of variables x_{pj}^{t} was motivated by the work of Tcha and Lee (1984). The main difference has to do with the fact that each path p can have any number of arcs from 1 to K, while in the referred to paper all paths are of exactly the same length.

Each path can be constituted by one or more facilities. Each facility can belong to one or more paths. Consider the two-level example depicted in figure 1. All paths represented are valid.

A path p will be represented by an ordered set $(i_1, i_2,\ldots, i_k)$, with $k \le K$. If client j is assigned to path p, this means that client j is served by facility i_1, then goes to facility i_2, and so on, until reaching facility i_k.

Definition 1: A path p is said to be *open* during period t if and only if all facilities i belonging to p are open during period t.

Definition 2: A path p is said to be *partially open* if and only if p is not open and at least one facility i belonging to p is open during period t.

Definition 3: A path p is said to be *closed* during period t if and only if all facilities i belonging to p are closed during period t.

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[2] It is not necessary to consider all possible values for (t,ξ), $\xi \ge t$. If, for instance, a facility has to be operating for, at least, s time periods periods after being open, then only pairs (t,ξ) with $\xi - t \ge s$ should be considered.

2.1 *Primal Problem*

The dynamic, uncapacitated, multi-level location problem can be formulated as DUMLP:

DUMLP

$$Min\sum_{t}\sum_{p}\sum_{j}c_{pj}^{t}x_{pj}^{t}+\sum_{t}\sum_{i}\sum_{\xi=t}^{T}FA_{it}^{\xi}a_{it}^{\xi}+\sum_{t>1}\sum_{i}\sum_{\xi=t}^{T}FR_{it}^{\xi}r_{it}^{\xi} \quad (1)$$

subject to:

$$\sum_{p}x_{pj}^{t}=1, \qquad \forall\, j,t \quad (2)$$

$$\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{i\tau}^{\xi}+r_{i\tau}^{\xi}\right)-x_{pj}^{t}\geq 0, \qquad \forall\, i,j,t,p\in P(i) \quad (3)$$

$$\sum_{\tau=1}^{t-1}\sum_{\xi=\tau}^{t-1}a_{i\tau}^{\xi}-\sum_{\xi=t}^{T}r_{it}^{\xi}\geq 0, \qquad \forall\, i,t>1 \quad (4)$$

$$\sum_{t=1}^{T}\sum_{\xi=t}^{T}a_{it}^{\xi}\leq 1, \qquad \forall\, i \quad (5)$$

$$\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{i\tau}^{\xi}+r_{i\tau}^{\xi}\right)\leq 1, \qquad \forall\, i,t \quad (6)$$

$$\begin{aligned} &a_{it}^{\xi},x_{pj}^{t}\in\{0,1\}, && \forall p,j,t,\xi\geq t \\ &r_{it}^{\xi}\in\{0,1\}, && \forall i,t>1,\xi\geq t \end{aligned} \quad (7)$$

The objective function minimizes the total fixed and assignment costs. Constraints (2) guarantee that, in every time period, each client is fully assigned to exactly one path; constraints (3) assure that, in every time period, a client can only be assigned to open paths; constraints (4) guarantee that a facility can only be reopen at the beginning of period *t* if it has already been open earlier and is not in operation at the beginning of period *t*; constraints (5) impose that a facility can only be open once during the planning horizon; constraints (6) assure that, in every time period, only one facility can be open in each location. Constraints (5) and (6) need to be considered explicitly only when there are negative fixed costs. Constraints (3) could be replaced by the aggregated constraints:

$$\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{i\tau}^{\xi}+r_{i\tau}^{\xi}\right)-\sum_{p\in P(i)}x_{pj}^{t}\geq 0 \qquad \forall\, i,j,t \quad (3')$$

2.2 *Dual Problem and Complementary Conditions*

Multiplying constraints (5) and (6) by -1, and associating dual variables v_j^t with constraints (2), dual variables w_{ijp}^t with constraints (3), dual variables u_i^t with constraints (4), dual variables ρ_i

with constraints (5') and dual variables π_i^t with constraints (6'), the dual problem of DUMLP can be formulated as D-DUMLP:

D-DUMLP

$$Max\sum_t\sum_j v_j^t - \sum_i \rho_i - \sum_t\sum_i \pi_i^t \tag{8}$$

subject to:

$$v_j^t - \sum_{i\in p} w_{ijp}^t \le c_{pj}^t, \qquad \forall p, j, t \tag{9}$$

$$\sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)} w_{ijp}^{\tau} + \sum_{\tau=\xi+1}^{T} u_i^{\tau} - \rho_i - \sum_{\tau=t}^{\xi}\pi_i^{\tau} \le FA_{it}^{\xi}, \qquad \forall i, t, \xi = t,\cdots,T \tag{10}$$

$$\sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)} w_{ijp}^{\tau} - u_i^t - \sum_{\tau=t}^{\xi}\pi_i^{\tau} \le FR_{it}^{\xi}, \qquad \forall i, t > 1, \xi = t,\cdots,T \tag{11}$$

$$w_{ijp}^t, u_i^t, \rho_i, \pi_i^t \ge 0, \qquad \forall i, j, t, p \in P(i)$$

Considering $w_{ijp}^t = \eta_{ijp}^t \max\{0, v_j^t - c_{pj}^t\}$, with $\sum_{i\in p}\eta_{ijp}^t = 1$ and $\eta_{ijp}^t \ge 0$, $\forall i,j,p\in P(i)$, an equivalent condensed formulation is obtained:

CD-DUMLP

$$Max\sum_t\sum_j v_j^t - \sum_i \rho_i - \sum_t\sum_i \pi_i^t$$

subject to:

$$\sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)} \eta_{ijp}^{\tau} \max\{0, v_j^{\tau} - c_{pj}^{\tau}\} \le FA_{it}^{\xi} - \sum_{\tau=\xi+1}^{T} u_i^{\tau} + \rho_i + \sum_{\tau=t}^{\xi}\pi_i^{\tau}, \quad \forall i, t, \xi = t,\cdots,T \tag{12}$$

$$\sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)} \eta_{ijp}^{\tau} \max\{0, v_j^{\tau} - c_{pj}^{\tau}\} \le FR_{it}^{\xi} + u_i^t + \sum_{\tau=t}^{\xi}\pi_i^{\tau}, \qquad \forall i, t > 1, \xi = t,\cdots,T \tag{13}$$

$$u_i^t, \rho_i, \pi_i^t \ge 0, \qquad \forall i, j, t$$

Let us define:

$$SA_{it}^{\xi} = FA_{it}^{\xi} - \sum_{\tau=\xi+1}^{T} u_i^{\tau} + \rho_i + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - \sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)} \eta_{ijp}^{\tau} \max\{0, v_j^{\tau} - c_{pj}^{\tau}\}, \quad \forall i, t, \xi = t,\cdots,T \tag{14}$$

$$SR_{it}^{\xi} = FR_{it}^{\xi} + u_i^t + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - \sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)} \eta_{ijp}^{\tau} \max\{0, v_j^{\tau} - c_{pj}^{\tau}\}, \qquad \forall i, t > 1, \xi = t,\cdots,T \tag{15}$$

$$S_{it}^{\xi} = \min\{SA_{it}^{\xi}, SR_{it}^{\xi}\}, \qquad \forall i, t, \xi = t,\cdots,T \tag{16}$$

Considering the primal problem DUMLP and its dual CD-DUMLP, the following complementary conditions hold if in presence of optimal primal and dual solutions to the respective problems (when there is no duality gap):

$$\left(\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{i\tau}^{\xi}+r_{i\tau}^{\xi}\right)-x_{pj}^{t}\right)w_{ijp}^{t}=0, \qquad \forall i,j,t,p\in P(i) \qquad (17)$$

$$\left(\sum_{\tau=1}^{t-1}\sum_{\xi=\tau}^{t-1}a_{i\tau}^{\xi}-\sum_{\xi=t}^{T}r_{it}^{\xi}\right)u_{i}^{t}=0, \qquad \forall i,t>1 \qquad (18)$$

$$\left(\sum_{t=1}^{T}\sum_{\xi=t}^{T}a_{it}^{\xi}-1\right)\rho_{i}=0, \qquad \forall i \qquad (19)$$

$$\left(\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{i\tau}^{\xi}+r_{i\tau}^{\xi}\right)-1\right)\pi_{i}^{t}=0, \qquad \forall i,t \qquad (20)$$

$$SA_{it}^{\xi}\cdot a_{it}^{\xi}=0, \qquad \forall i,t,\xi=t,\dots,T \qquad (21)$$

$$SR_{it}^{\xi}\cdot r_{it}^{\xi}=0, \qquad \forall i,t,\xi=t,\dots,T \qquad (22)$$

2.3 *Primal-Dual Heuristic*

The primal-dual heuristic that has been developed builds admissible primal solutions based on admissible dual solutions to problem CD-DUMLP, trying to force the satisfaction of the complementary conditions. If the heuristic calculates a pair of admissible and complementary primal and dual solutions, then the primal optimal solution has been found. When this is not possible, the best dual solution found will establish a valid lower limit to the optimal value of the primal objective function. The heuristic functioning scheme is as follows:

1. Initialisation of dual variables;
2. Dual Ascent Procedure for dual variables v_j^t;
3. Primal Procedure;
4. Dual Adjustment Procedure for dual variables ρ_i. If the dual solution is changed go to 2;
5. Repeat the Primal-Dual Adjustment Procedure for variables v_j^t until there is no improvement in the dual objective function value;
6. Dual Adjustment Procedure for dual variables ρ_i. If the dual solution is changed go to 2;
7. Dual Ascent Procedure for dual variables u_i^t. If the dual solution is changed go to 2;
8. Dual Descent Procedure for dual variables u_i^t. If the dual solution is changed go to 2;
9. Dual Adjustment Procedure for variables π_i^t. If the dual solution is changed go to 2.

This heuristic functioning scheme is equal to the functioning scheme presented in Dias *et al* (2004b). In fact, the only procedures that are different from the ones described in Dias *et al* (2004b) are the Dual Ascent Procedure and the Primal-Dual Adjustment Procedure for variables v_j^t, and the Primal Procedure. For this reason, only these three procedures are going to be described here.

2.3.1 Dual Ascent Procedure for variables v_j^t

This procedure tries to increase all dual variables v_j^t, $j \in J^+$, $J^+ \subset J$. When this procedure is executed in step 2 of the primal-dual heuristic, then J^+ is equal to J. Otherwise, the set J^+ is defined before this procedure is executed. This procedure is a straightforward adaptation of the one described in Van Roy and Erlenkotter (1982). The only difference is in the updating step of slacks $SA_{i\tau}^{\xi}$ and $SR_{i\tau}^{\xi}$ with $\tau \leq t \leq \xi$ such that $v_j^t \geq c_{pj}^t$ and $p \in P(\ i\ )$. It is interesting to note that if $v_j^t \geq c_{pj}^t$ for more than one path $p \in P(\ i\ )$, slacks $SA_{i\tau}^{\xi}$ and $SR_{i\tau}^{\xi}$, with $\tau \leq t \leq \xi$, will be decremented more than once (this is a completely different behavior, when compared with the single-level case (Erkenkotter, 1978; Van Roy and Erlenkotter, 1982).

Dual Ascent Procedure for Variables v_j^t

1. Consider any admissible initial solution $\{v_j^t\}$ such that $v_j^t \geq c_j^{t1}, \forall (j,t), S_{it}^{\xi} \geq 0, \forall i,t,\xi \geq t$. For each (j,t), define $k(j,t) = min\{k : v_j^t \leq c_j^{tk}\}$. If $v_j^t = c_j^{tk(j,t)}$ then $k(j,t) \leftarrow k(j,t)+1$.
2. $(j,t) \leftarrow (j,t)_1; q \leftarrow 1; \delta \leftarrow 0$.
3. If $(j,t) \notin J^+$, then go to 7.
4. $\Delta_j^t \leftarrow \min_i \left\{ \frac{S_{i\tau}^{\xi}}{\sum_{\substack{p \in P(i) \\ v_j^t \geq c_{pj}^t}} \eta_{ijp}^t} : \tau \leq t \leq \xi \right\}$. If $\Delta_j^t = 0$, go to 7.
5. If $\Delta_j^t > c_j^{tk(j,t)} - v_j^t$ then $\Delta_j^t \leftarrow c_j^{tk(j,t)} - v_j^t; \delta \leftarrow 1; k(j,t) \leftarrow k(j,t)+1$.
6. For each facility i, $SR_{i\tau}^{\xi} \leftarrow SR_{i\tau}^{\xi} - \Delta_j^t \sum_{\substack{p \in P(i) \\ v_j^t \geq c_{pj}^t}} \eta_{ijp}^t$ and $SA_{i\tau}^{\xi} \leftarrow SA_{i\tau}^{\xi} - \Delta_j^t \sum_{\substack{p \in P(i) \\ v_j^t \geq c_{pj}^t}} \eta_{ijp}^t, \tau \leq t \leq \xi$.

   $v_j^t \leftarrow v_j^t + \Delta_j^t$.
7. If $q \neq \# J^+, (j,t) \leftarrow (j,t)_{q+1}; q \leftarrow q+1$. Go to 3.
8. If $\delta = 1$ go to 2. Else STOP.

The calculation of the η_{ijp}^t parameters can be done in several different ways. One very simple way is to consider $\eta_{ijp}^t = 1/n_p$, where n_p is equal to the number of services belonging to path p. Another calculation procedure is motivated by the analysis of step 4 of the dual ascent procedure. It is straightforward to conclude that η_{ijp}^t should take smaller values for services i such that the quotient between $S'^{\xi}_{i\tau} = min\{S_{i\tau}^{\xi}\}$, $\tau \leq t \leq \xi$, and the number of paths including i such that $v_j^t \geq c_{pj}^t$ is smaller. This procedure is more time consuming than the first, but, in general, calculates better dual solutions.

Calculation of η_{ijp}^{t} parameters

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1. $p \leftarrow 1$;
2. If $v_j^t \geq c_{pj}^t$ then go to 3. Else go to 9.
3. If $n_p = 1$, then $\eta_{ijp}^t \leftarrow 1,\ i \in P(i)$. Go to 8.
4. $i \leftarrow 1$.
5. If $p \in P(\ i\ )$ then go to 6, else go to 7.
6. $e_{pj}^t \leftarrow \begin{cases} 1, \text{ if } v_j^t \geq c_{pj}^t \\ 0, \text{ otherwise} \end{cases}$, $n_{ij}^t \leftarrow \sum_{p \in P(i)} e_{pj}^t$, $\eta_{ijp}^t \leftarrow \min_{\tau \leq t \leq \xi} \frac{S_{i\tau}^{\xi}}{n_{ij}^t}$.
7. $i \leftarrow i + 1$. If $i > m$, then go to 8. Else go to 5.
8. $D \leftarrow \sum_{i \in p} \eta_{ijp}^t$. Update $\eta_{ijp}^t \leftarrow \frac{\eta_{ijp}^t}{D}, \forall\ i \in p$.
9. $p \leftarrow p + 1$. If $p > q$ then stop. Else go to 2.

Example 1:

Consider a problem with four clients, two time periods and three services organized in two levels as depicted in figure 2.

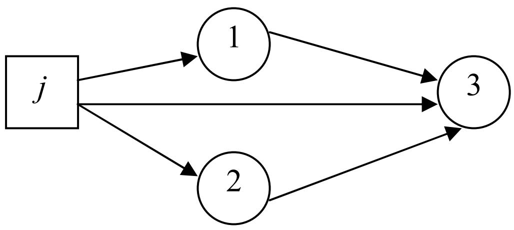

Figure 2: □ Client; ○ Service.

The admissible paths are: p_1=(1,3), p_2=(2,3) and p_3=(3). Tables 1 and 2 show the assignment costs for each time period and table 3 shows the fixed (re) opening costs.

Variables v_j^t are initialized as showed in table 4. The dual ascent procedure for variables v_j^t begins by considering t = 1 and j = 1. For path p equal to 2, the procedure calculates $\eta_{2jp}^t = \frac{10}{18} = 0.56$ and $\eta_{3jp}^t = \frac{8}{18} = 0.44$. Variable v_1^1 can be increased to 18.

For variable v_2^1, the procedure calculates $\eta_{222}^1 = 0.56$ and $\eta_{322}^1 = 0.44$. If v_2^1 was increased by 7 ($c_j^{tk(j,t)} - v_j^t$), then slack SA_{21}^1 would become equal to 1.11−7×0.56 that is less than zero, so Δ_2^1 is calculated as being equal to 2. No more dual variables v_j^1 are changed because slacks become as depicted in table 5.

p \ j	1	2	3	4
1	18	13	17	12
2	2	6	3	7
3	22	25	22	26

Table 1

p \ j	1	2	3	4
1	9	9	12	14
2	5	9	9	12
3	21	29	26	27

Table 2

i	FA_{i1}^{1}	FA_{i1}^{2}	FA_{i2}^{2}	FR_{i2}^{2}
1	3	11	20	26
2	10	11	45	29
3	8	9	71	16

Table 3

t \ j	1	2	3	4
1	2	6	3	7
2	5	9	9	12

Table 4

i	1	2	3
FA_{i1}^{1}	3	0	0
FA_{i1}^{2}	11	1	1

Table 5

For t equal to 2 and j equal to 1, $\eta_{212}^{2} = \eta_{312}^{2} = 0.5$. If the value $c_j^{tk(j,t)} - v_j^t = 4$ was considered, then slack SA_{21}^{2} would be changed to 1−0.5×4, that is less than zero. For this reason Δ_1^2 is calculated as being equal to 2. After the dual ascent procedure, the final result is depicted in tables 6 and 7.

t \ j	1	2	3	4
1	18	8	3	7
2	7	9	9	12

Table 6

i	FA_{i1}^{1}	FA_{i1}^{2}	FA_{i2}^{2}	FR_{i2}^{2}
1	3	11	20	26
2	0	0	44	28
3	0	0	70	15

Table 7

■

2.3.2 Primal-Dual Adjustment Procedure for variables v_j^t

The Primal-Dual adjustment procedure for variables v_j^t detects violations of the complementary conditions (17), and decreases the values of some variables v_j^t, increasing slacks and allowing other variables v_j^t to increase. This procedure tries to reduce the number of complementary conditions violations and, simultaneously, improve the value of the dual objective function.

Complementary condition (17) will be violated if there exists at least two open or partially open paths p_1 and p_2 such that $v_j^t > c_{p_1 j}^t$ and $v_j^t > c_{p_2 j}^t$. Notice that it isn't necessary that paths are open. They only need to be partially open to violate the complementary conditions.

Diminishing variable v_j^t will increase all slacks $SA_{i\tau}^{\xi}$ and $SR_{i\tau}^{\xi}$, with $\tau \leq t \leq \xi$, such that $v_j^t > c_{pj}^t$, with $p \in P(i)$. If there are two other dual variables v_j^t blocked exclusively due to slacks that are increased, then it is possible to improve the dual objective function value. It is interesting to notice that the violation may occur due to a single facility that belongs to more than one path (in the single case there had to be at least two different facilities involved).

In the single-level case, the dual ascent algorithm tries firstly to increase variables v_j^t blocked exclusively by slacks corresponding to a single facility i (Dias *et al*, 2004a). The dual adjustment procedure in the multi-level case tries firstly to increase all variables v_j^t blocked exclusively due to slacks that are going to be increased (even if corresponding to more than one facility). Consider the following definitions:

$I^* = \{ (i, \tau, \xi): S_{i\tau}^{\xi} = 0\}$

$I_t^* = \{ i: (i, \tau, \xi) \in I^* \text{ and } \tau \leq t \leq \xi\}$

$I_t^+ = \{i : \text{facility } i \text{ is open during period } t\}$

$I_j^{t*} = \left\{ i : \exists (\tau, \xi) \text{ with } \tau \leq t \leq \xi \text{ and } p \in P(i) \,|\, (i, \tau, \xi) \in I^* \text{ and } v_j^t \geq c_{pj}^t \right\}$

$I_j^{t+} = \left\{ i : i \in I_t^+ \text{ and } \exists p \in P(i) \text{ such that } v_j^t > c_{pj}^t \right\}$

$P_j^{t+} = \left\{ p : \exists i \in p \text{ such that } i \in I_t^+ \text{ and } v_j^t > c_{pj}^t \right\}$

$J^{t+} = \left\{ (j, \tau) : I_j^{\tau*} \subset I_j^{t+} \text{ and } (i, \gamma, \xi) \notin I^*, \forall i \in I_j^{t+}, \gamma \leq \tau \leq \xi < t \text{ or } t < \gamma \leq \tau \leq \xi \right\} \cup \left\{ (j, t) : I_j^{t*} \subset I_j^{t+} \right\}$

$c_j^{t-} = \max_p \left\{ c_{pj}^t : v_j^t > c_{pj}^t \right\}$

Set I^* corresponds to (i, τ, ξ), such that $SA_{i\tau}^{\xi}$ and/or $SR_{i\tau}^{\xi}$ are equal to zero. Set P_j^{t+} indicates, for each client j, all open or partially open paths such that v_j^t is greater than the assignment cost c_{pj}^t. A violation of the complementary conditions (17) is detected by the existence of, at least, one pair (j,t) such that the number of elements of P_j^{t+} is greater than one. The set I_j^{t+} indicates, for each client j, all operating facilities during period t that belong to any path p such that v_j^t is greater than the assignment cost c_{pj}^t. This means that all slacks $SA_{i\tau}^{\xi}$ and $SR_{i\tau}^{\xi}$, $\tau \leq t \leq \xi$, with $i \in I_j^{t+}$ will be increased with the decrease in v_j^t. The set I_j^{t*} corresponds to the set of all facilities i such that there exists at least one slack $SA_{i\tau}^{\xi}$ or $SR_{i\tau}^{\xi}$, $\tau \leq t \leq \xi$, blocking variable v_j^t. Set J^{t+} represents all dual variables that can be increased with the decrease of variable v_j^t. It is possible that variable v_j^t itself belongs to set J^{t+}. Nevertheless, this variable won't be considered in set J^+ at the first time the dual ascent procedure is called.

Consider variables v_j^t organized as a sequence of pairs (j,t).

Primal-Dual Adjustment Procedure for Dual Variables v_j^t

---

1. Initialize $(j, t) \leftarrow (j, t)_1$; $q \leftarrow 1$; $\delta \leftarrow 0$.
2. If $\# P_j^{t+} \leq 1$ then go to 9.
3. If $J^{t+} \setminus \{(j,t)\} = \varnothing$, then go to 9.
4. For each $(i,\tau,\xi), \tau \leq t \leq \xi$, $SA_{i\tau}^{\xi} \leftarrow SA_{i\tau}^{\xi} + \sum_{\substack{p \in P(i) \\ v_j^t > c_{pj}^t}} \left(v_j^t - c_j^{t-}\right)$, $SR_{i\tau}^{\xi} \leftarrow SR_{i\tau}^{\xi} + \sum_{\substack{p \in P(i) \\ v_j^t > c_{pj}^t}} \left(v_j^t - c_j^{t-}\right)$; $v_j^t \leftarrow c_j^{t-}$.
5. $J^+ = J^{t+} \setminus \{(j,t)\}$. Execute the dual ascent procedure for variable v_j^t.
   $J^+ = J^+ \cup \{(j,t)\}$. Execute the dual ascent procedure for variable v_j^t.
   $J^+ = J$. Execute the dual ascent procedure for variable v_j^t.
6. If v_j^t has been changed, go to 2.
7. Execute the primal procedure.
8. If there have been improvements in the dual or primal objective function value, then $\delta \leftarrow 0$. Else $\delta \leftarrow \delta + 1$.
9. If the primal objective function value is equal to the dual objective function value, or $\delta = \delta_{max}$, or $q \neq \# J \times T$ then stop; else $q \leftarrow q + 1$; $(j, t) \leftarrow (j, t)_q$, go to 2.

2.3.3 Primal Procedure

The primal procedure constructs primal feasible solutions based on dual feasible solutions, trying to force complementary conditions to be satisfied. Consider the sets I_t^*, I_t^+ and I^* defined in the previous section, and also: $I_A^+ = \{ (i, \tau, \xi) : a_{i\tau}^{\xi} = 1\}$, $I_R^+ = \{ (i, \tau, \xi) : r_{i\tau}^{\xi} = 1\}$, $P_t^+ = \{p$: p is open during period $t\}$.

Sets I_t^* and I_t^+ are not necessarily equal, because the primal procedure will always try to open the minimum number of services, guaranteeing that all clients will be assigned to one open path in every time period. Furthermore, it is often necessary to insert in set I_t^+ services that don't belong to I_t^*. Sets I_A^+ and I_R^+ are built during the primal procedure and determine which services will be (re) opened, when and for how long.

Definition 4: A path p is considered essential during period t if there is at least one client j that has to be assigned to path p during period t. This happens if and only if $\exists j \in J : v_j^t \geq c_{pj}^t \wedge v_j^t < c_{p'j}^t, \forall p' \in P, p' \neq p$.

Paths considered as essential are the first to be considered open. To open a path *p* at time period *t* all facilities belonging to that path have to be open. This is achieved by inserting all those facilities in set I_t^+. When opening a path, the primal procedure will often violate complementary restrictions (21) and (22), by including in set I_t^+ facilities $i \notin I_t^*$. Whenever path *p* is open, all other paths *p*' such that if $i \in p'$ then $i \in p$ will also be open.

Paths not considered essential will only be explicitly open during time period *t* if there are clients *j* that cannot be assigned to already open paths. In this case, the procedure will open the path that corresponds to the smallest assignment cost.

Sets I_A^+ and I_R^+ are build based on sets I_t^+, $\forall t$. These sets are built using exactly the same procedures described in Dias *et al* (2004a).

Primal Procedure

1. $I_A^+ = I_R^+ = \emptyset$. $I_t^+ = \emptyset, \forall t$. Build sets I^* and I_t^*. $Num = 0$;
2. For t=1 until T, include in set P_t^+ all paths p such that $\exists j : v_j^t \geq c_{pj}^t$ and $v_j^t < c_{p'j}^t, \forall p' \neq p$. Update sets P_t^+, $\forall t$, including in P_t^+ all open paths $p \notin P_t^+$.
3. For each client j such that $v_j^t < c_{pj}^t, \forall p \in P_t^+$, include in set P_t^+ path p such that $c_{pj}^t = \min_{v_j^t \geq c_{p'j}^t} c_{p'j}^t$. $Num \leftarrow Num+1$.
4. Include in set I_t^+ all facilities i belonging to path $p \in P_t^+$, $\forall t$.
5. If $Num = 1$ then $I_t^* \leftarrow I_t^+$ and $I_t^+ \leftarrow \emptyset$, $P_t^+ \leftarrow \emptyset$, $\forall t$, go to 2. Else go to 6.
6. Build sets I_A^+ and I_R^+. Update I_t^+ and P_t^+.
7. For t =1 until T, assign each client j to path $p' \in P_t^+$ such that $c_{p'j}^t = \min_{p \in P_t^+} \left\{ c_{pj}^t \right\}$. Calculate Z as being the primal objective function value.
8. $G = I_A^+ \cup I_R^+$.
9. Choose arbitrarily a variable $a_{i\tau}^{\xi}$ or $r_{i\tau}^{\xi}$ belonging to G and change its value from one to zero. If the solution remains admissible, recalculate the assignments of clients to open paths. Calculate Z' as the primal objective function value of the new solution. If $Z' < Z$, then remove variable $a_{i\tau}^{\xi}$ (or $r_{i\tau}^{\xi}$) from set I_A^+ (or I_R^+) and set $Z = Z'$.
10. Remove variable $a_{i\tau}^{\xi}$ (or $r_{i\tau}^{\xi}$) from set G. If $G = \emptyset$, go to 11. Else go to 9.
11. Test complementary conditions (19)-(21).

Steps 8-10 try to improve the primal solution calculated by decreasing the number of facility location variables that are considered equal to one. This has proved to decrease significantly the value of the primal objective function. These steps could also be replaced by a drop heuristic: from all variables equal to one, choose the one which, when its value is changed to zero, leads to the greatest improve in the primal objective function value. Repeat the process until there is no improvement in the objective function value. Such a drop heuristic was tested, but the results obtained showed that is much more time consuming and the value of the primal objective function obtained is the same as with the execution of steps 8-10. This is justified by the fact that the primal procedure considers more variables equal to one than the ones strictly needed (because the opening of a path consists in opening a set of facilities, that most of the times open implicitly other paths). These variables will always be considered equal to zero, even if chosen arbitrarily.

Step 11 tries to change the primal solution in order to guarantee the satisfaction of complementary conditions (18)-(20) that are being violated. This test is equal to the one already developed by the authors to the single-level case (Dias *et al*, 2004b).

Considering the dual solution presented in tables 6 and 7, the primal procedure would open path 2, considering facilities 2 and 3 open from period one to two ($a_{21}^{2} = a_{31}^{2} = 1$). This primal solution has an objective function value equal to 73, thus it is the optimal solution.

3 Including maximum capacity constraints

In almost all real situations, a facility has an upper limit on the demand it can serve. So, the inclusion of maximum capacity restrictions in the problem DUMLP is a natural extension. Consider:

Q_i = maximum capacity of facility i during a time period,

x_{pj}^{t} = fraction of customer j's demand served by path p, during time period t.

d_j^t = total demand of customer j during time period t.

The multi-level, dynamic, capacitated location problem (DCMLP) can be formulated simply by including in DUMLP constraint (23) below and changing (7) to (7'):

$$Q_i \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi} \right) - \sum_{j} \sum_{p \in P(i)} d_j^t x_{pj}^t \geq 0, \qquad \forall i,t \tag{23}$$

$$a_{i\tau}^{\xi} \in \{0,1\}, \qquad \forall i,\ \tau,\ \xi \geq \tau$$

$$r_{i\tau}^{\xi} \in \{0,1\}, \qquad \forall i,\ \tau > 1,\ \xi \geq \tau \tag{7'}$$

$$x_{pj}^{t} \geq 0, \qquad \forall j, p, t$$

The additional set of restrictions (23) establish an upper limit on the total flow that reaches an open facility i in each time period t.

To develop a primal-dual heuristic, it is necessary to observe the changes these additional restrictions bring to the dual condensed problem formulation. Associating dual variables λ_i^t to restrictions (23), the condensed dual problem formulation becomes:

CD-DCMLP

$$Max\sum_t\sum_j v_j^t - \sum_i \rho_i - \sum_t\sum_i \pi_i^t$$

subject to:

$$\sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)}\eta_{ijp}^{\tau}\max\left\{0, v_j^{\tau} - c_{pj}^{\tau} - d_j^{\tau}\sum_{i_1\in p}\lambda_{i_1}^t\right\} \le FA_{it}^{\xi} - \sum_{\tau=\xi+1}^{T}u_i^{\tau} + \rho_i + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - Q_i\sum_{\tau=t}^{\xi}\lambda_i^{\tau},$$

$$\forall i,t,\xi = t,\cdots,T \qquad (24)$$

$$\sum_j\sum_{\tau=t}^{\xi}\sum_{p\in P(i)}\eta_{ijp}^{\tau}max\left\{0, v_j^{\tau} - c_{pj}^{\tau} - d_j^{\tau}\sum_{i_1\in p}\lambda_{i_1}^t\right\} \le FR_{it}^{\xi} + u_i^t + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - Q_i\sum_{\tau=t}^{\xi}\lambda_i^{\tau},$$

$$\forall i,t,\xi = t,\cdots,T \qquad (25)$$

$$u_i^t, \rho_i, \pi_i^t, \lambda_i^t \ge 0, \qquad \forall i,j,t$$

A decrease or increase in dual variable λ_i^t will influence all slacks $S_{i'\tau}^{\xi}$, with $\tau\le t\le\xi$, such that $P(i) \cap P(i') \neq \emptyset$, $\forall i' \in I$ (in the calculation of $S_{i'\tau}^{\xi}$, the sum over all paths $p \in P(i')$ will consider all values of λ_i^t such that i belongs to p).

The primal-dual heuristic's functioning scheme presented for the uncapacitated case remains valid for the capacitated case, with two additional steps:

10. Dual Ascent Procedure for dual variables λ_i^t. If the dual solution is changed, go to 2.

11. Dual Descent Procedure for dual variables λ_i^t. If the dual solution is changed, go to 2.

The dual and descent procedures for variables λ_i^t will be described in the next two sections. The development of these procedures followed the work of Guignard and Spielberg (1979), Saldanha da Gama (2002) and Dias *et al* (2004b). The primal procedure will have to be changed (it is necessary to test the satisfaction of the additional capacity restrictions). The dual ascent and Primal-Dual adjustment procedures for variables v_j^t remain valid being only necessary to consider the assignment costs equal to $c_{pj}^t + d_j^t\sum_{i_1\in p}\lambda_{i_1}^t$. All the remaining procedures are not changed.

3.1 *Dual Ascent Procedure for Variables* λ_i^t

If variable λ_i^t is increased, then the left hand side of constraints (24) and (25) will diminish. The maximum change that should be considered is such that all those values became less than or equal to zero. Considering that variable λ_i^t is increased by δ, then δ should be less than or equal to $\Delta(i)$ such that:

$$\Delta(i) = \max_{\substack{j \in J \\ p \in P(i)}} \left\{ \frac{v_j^t - c_{pj}^t - d_j^t \sum_{i_1 \in p} \lambda_{i_1}^t}{d_j^t} \right\} \tag{26}$$

Consider the following definitions:

$$JP(i,i',\delta) = \left\{ (j,p): j \in J \wedge p \in P(i) \cap P(i') \wedge \frac{v_j^t - c_{pj}^t - d_j^t \sum_{i_1 \in p} \lambda_{i_1}^t}{d_j^t} \leq \delta \right\}, \tag{27}$$

$$\overline{JP}(i,i',\delta) = \left\{ (j,p): j \in J \wedge p \in P(i) \cap P(i') \wedge \frac{v_j^t - c_{pj}^t - d_j^t \sum_{i_1 \in p} \lambda_{i_1}^t}{d_j^t} > \delta \right\}. \tag{28}$$

Proposition 1: If dual variable λ_i^t is increased by $\delta \in]0,\Delta(i)]$, then slacks $SA_{i'\tau}^{\xi}$ and $SR_{i'\tau}^{\xi}$, $\tau \leq t \leq \xi$, $\forall i' \in I$ such that $P(i) \cap P(i') \neq \emptyset$, will be changed by:

$$\Omega(\delta,i,i') = \sum_{(j,p) \in JP(i,i',\delta)} \eta_{i'jp}^t \max\left\{ 0, v_j^t - c_{pj}^t - d_j^t \sum_{i_1 \in p} \lambda_{i_1}^t \right\} + \sum_{(j,p) \in \overline{JP}(i,i',\delta)} \eta_{i'jp}^t d_j^t \delta - E \tag{29}$$

where: $E = \begin{cases} Q_i \delta, \text{ if } i = i' \\ 0, \text{ otherwise} \end{cases}$.

This proposition follows similar results that can be found in Guignard and Spielberg (1979). This proposition motivates the dual ascent procedure that is now described.

Dual Ascent Procedure for Variables λ_i^t

1. $t \leftarrow 1$.
2. $i \leftarrow 1$.
3. $\delta \leftarrow 0$; $\delta' \leftarrow +\infty$.
4. $\delta \leftarrow \max_{\substack{j \in J \\ p \in P(i) \\ \frac{1}{d_j^t} \max\left\{0, v_j^t - c_{pj}^t - d_j^t \sum_{i_1 \in p} \lambda_{i_1}^t\right\} < \delta'}} \left\{ \frac{1}{d_j^t} \max\left\{ 0, v_j^t - c_{pj}^t - d_j^t \sum_{i_1 \in p} \lambda_{i_1}^t \right\} \right\}$.
5. Calculate sets $JP(i,i',\delta)$ and $\overline{JP}(i,i',\delta)$ as in (27) and (28), and $\Omega(i,i',\delta)$ as in (29), $\forall i' \in I$, with $P(i) \cap P(i') \neq \emptyset$.
6. If $\Omega(\delta,i,i') < 0$, $\forall i' \in I$, with $P(i) \cap P(i') \neq \emptyset$, then go to 11. Else go to 7.
7. If $\exists i' \in I$, with $P(i) \cap P(i') \neq \emptyset$, such that $SA_{i'\tau}^{\xi} + \Omega(\delta,i,i') < 0$ or $SR_{i'\tau}^{\xi} + \Omega(\delta,i,i') < 0$, $\tau \leq t \leq \xi$, then go to 8. Else go to 9.

8. If δ'= 0, then go to 11. Else δ'$\leftarrow\delta$ and go to 4.
9. $\lambda_i^t \leftarrow \lambda_i^t + \delta$; $SA_{i'\tau}^{\xi} \leftarrow SA_{i'\tau}^{\xi} + \Omega(\delta,i,i')$ and $SR_{i'\tau}^{\xi} \leftarrow SR_{i'\tau}^{\xi} + \Omega(\delta,i,i')$, $\tau \le t \le \xi$, $\forall$ $i' \in I$ such that $P(i) \cap P(i') \neq \varnothing$.
10. Execute the dual ascent procedure for variables v_j^t, with $J^+ = J$.
11. $i \leftarrow i+1$. If $i > M$ then go to 12. Else go to 3.
12. $t \leftarrow t+1$. If $t > T$ then stop. Else go to 2.

Example 2:

Consider a problem with six clients, three services organized as in figure 2, and two time periods. The clients have a known demand that is equal to 56, 22, 76, 58, 9 and 57 in time period 1 and equal to 59, 22, 78, 61, 8 and 58 in time period 2, for j = 1,…,6. Services have maximum capacities equal to 149, 216 and 451. Imagine that the first time the dual ascent procedure for variables λ_i^t is executed variables v_j^t take on the values presented in table 8, all other dual variables are equal to zero and slacks have values presented in table 9.

t \ j	1	2	3	4	5	6
1	3584	1474	6968.38	3770	531	2850
2	3220.29	990	4992	2623	336	2610

Table 8

i	SA_{i1}^1	SA_{i1}^2	SA_{i2}^2	SR_{i2}^2
1	605.09	654.66	5684.57	5797.57
2	126.30	0	4333.70	2170.70
3	3920.84	0	6559.16	6261.16

Table 9

Considering variable λ_1^1: δ'$\leftarrow +\infty$; $P(1)=\{1\}$. The value of δ is calculated as:

$$\delta \leftarrow max\left\{\frac{1}{56}max\{0,3584-2968\},\frac{1}{22}max\{0,1474-1034\},\frac{1}{76}max\{0,6968.38-4788\},\right.$$
$$\left.\frac{1}{58}max\{0,3770-3770\},\frac{1}{9}max\{0,531-531\},\frac{1}{57}max\{0,2850-2679\}\right\} = 28.69$$

$$\Omega(28.69,1,1) = (3584-2968)\eta_{111}^1 + (1474-1034)\eta_{121}^1 + (6968.38-4788)\eta_{131}^1 + 0 + 0 +$$
$$(2850-2679)\eta_{161}^1 - 28.69 \times 149$$

Considering $\eta_{111}^1 = 0.32$, $\eta_{121}^1 = 0.76$, $\eta_{131}^1 = 0.77$ and $\eta_{161}^1 = 0.87$, then $\Omega(28.69,1,1) = -1925.33$. Variable λ_1^1 cannot be increased by δ because the procedure detects that slack SA_{11}^1 would become less than zero. The value of δ' is changed to 28.69 and the procedure is repeated, obtaining δ equal to 20.

$$\Omega(20,1,1)=(3584-2968)\times 0.32+(1474-1034)\times 0.76+76\times 20\times 0.77+0+0+ (2850-2679)\times 0.87-20\times 149=-1137.27$$

This change is still not admissible, so δ' is changed to 20 and δ will become equal to 11.

$$\Omega(11,1,1)=(3584-2968)\times 0.32+22\times 0.76\times 11+76\times 11\times 0.77+0+0+ (2850-2679)\times 0.87-11\times 149=-470.97$$

All slacks $S_{1\tau}^{\xi}$ will continue greater than or equal to zero. Slacks $S_{2\tau}^{\xi}$ are not changed because $P(1)\cap P(2)=\varnothing$.

$$\Omega(11,1,3)=(3584-2968)\times 0.68+22\times 0.24\times 11+76\times 11\times 0.23+0+0+(2850-2679)\times 0.13 = 696.96$$

This means that slacks $S_{3\tau}^{\xi}$, $\tau \le 1 \le \xi$, are going to be increased. The updated slacks' values, are shown in table 10. The increase in variable λ_1^1 allows the improvement in the dual objective function value, because variable v_1^2 is increased to 3641.46. Table 11 shows the slacks' values after the execution of the dual ascent procedure for variables v_j^t.

i	SA_{i1}^1	SA_{i1}^2	SA_{i2}^2	SR_{i2}^2
1	134.13	183.69	5684.57	5797.57
2	126.30	0	4333.70	2170.70
3	4617.80	696.97	6559.16	6261.16

Table 10

i	SA_{i1}^1	SA_{i1}^2	SA_{i2}^2	SR_{i2}^2
1	134.13	38.32	5539.19	5652.19
2	126.30	0	4333.70	2170.70
3	4617.80	0	5862.20	5564.20

Table 11

■

3.2 *Dual Descent Procedure for Variables λ_i^t*

If dual variable λ_i^t is decreased all values $v_j^t - c_{pj}^t - d_j^t \sum_{i_1\in p} \lambda_{i_1}^t$, with $p\in P(i)$, will be increased. Consider a decrease in λ_i^t such that all those values that are less than zero remain that way. This means that λ_i^t can be decreased by δ such that:

$$0<\delta\le \min_{\substack{j\in J,\, p\in P(i) \\ v_j^t - c_{pj}^t - d_j^t \sum_{i_1\in p} \lambda_{i_1}^t <0}} \left\{ -\frac{1}{d_j^t}\left(v_j^t - c_{pj}^t - d_j^t \sum_{i_1\in p} \lambda_{i_1}^t \right)\right\} \quad (30)$$

Once again all slacks $SA_{i'\tau}^{\xi}$ and $SR_{i'\tau}^{\xi}$, $\tau \le t \le \xi$, $\forall i' \in I$ such that $P(i) \cap P(i') \ne \varnothing$, will be changed.

Proposition 2: If variable λ_i^t is decreased by a value δ in the interval defined by (30), then $SA_{i'\tau}^{\xi}$ and $SR_{i'\tau}^{\xi}$, $\tau \le t \le \xi$, $\forall i' \in I$ such that $P(i) \cap P(i') \ne \varnothing$, will be changed by:

$$\Phi(\delta,i',i)=\delta\left(E-\sum_{j\in J}\sum_{\substack{p\in P(i)\cap P(i')\\ v_j^t-c_{pj}^t-d_j^t\sum_{i_1\in p}\lambda_{i_1}^t\geq 0}} d_j^t\eta_{i'jp}^t\right), \tag{31}$$

where $E=\begin{cases}Q_i, & \text{if } i=i'\\ 0, & \text{otherwise}\end{cases}$.

This proposition follows similar results that can be found in Guignard and Spielberg, (1979).

Proposition 2 motivates the following dual descent procedure.

Dual Descent Procedure for Variables λ_i^t

1. $t \leftarrow 1$.
2. $i \leftarrow 1$.
3. $\delta \leftarrow min\left\{\lambda_i^t, \min_{\substack{j\in J,p\in P(i)\\ v_j^t-c_{pj}^t-d_j^t\sum_{i_1\in p}\lambda_{i_1}^t<0}}\left\{-\frac{1}{d_j^t}\left(v_j^t-c_{pj}^t-d_j^t\sum_{i_1\in p}\lambda_{i_1}^t\right)\right\}\right\}$.
4. Calculate $\Phi(\delta,i',i)$ as in (31), $\forall\ i' \in I$, with $P(i) \cap P(i') \neq \emptyset$.
5. If $\Phi(\delta,i',i) < 0$, $\forall\ i' \in I$, with $P(i) \cap P(i') \neq \emptyset$, then go to 10. Else go to 6.
6. If $\exists\ i' \in I$, with $P(i) \cap P(i') \neq \emptyset$, such that $SA_{i'\tau}^{\xi} + \Phi(\delta,i',i) < 0$ or $SR_{i'\tau}^{\xi} + \Phi(\delta,i',i) < 0$, $\tau \leq t \leq \xi$, then go to 7. Else go to 8.
7. $\delta \leftarrow \min_{\substack{\tau\leq t\leq\xi\\ S_{i'\tau}^{\xi}+\Phi(\delta,i',i)<0}}\left\{-\frac{S_{i'\tau}^{\xi}}{\Phi(\delta,i',i)/\delta}\right\}$. If $\delta = 0$ then go to 10. Else go to 8.
8. $\lambda_i^t \leftarrow \lambda_i^t - \delta$; $SA_{i'\tau}^{\xi} \leftarrow SA_{i'\tau}^{\xi} + \Phi(\delta,i',i)$ and $SR_{i'\tau}^{\xi} \leftarrow SR_{i'\tau}^{\xi} + \Phi(\delta,i',i)$, $\tau \leq t \leq \xi$, $\forall\ i' \in I$ such that $P(i) \cap P(i') \neq \emptyset$.
9. Execute dual ascent procedure for variables v_j^t, with $J^+ = J$.
10. $i \leftarrow i + 1$. If $i > M$ then go to 11. Else go to 3.
11. $t \leftarrow t + 1$. If $t > T$ then stop. Else go to 2.

Example 3:

Considering example 2 and variable λ_1^1 equal to 11. Then, for i' equal to 1 the procedure calculates:

$$\sum_{\substack{j\\ v_j^t-c_{1j}^t-d_j^t\geq 0}} d_j^t\eta_{1j1}^t = 56\times 0.37+76\times 0.77+57\times 0.87 \text{ and } \Phi(11,1,1)=149-128.83=20.17$$

Considering i' equal to 3 (it is not needed to consider i' equal to 2 because $P(1)\cap P(2)=\emptyset$) the procedure calculates $\Phi(11,3,1)=-60.17$. This means that slacks $S_{i'\tau}^{\xi}$, $\tau \leq t \leq \xi$, will increase for i'

equal to 1 and decrease for i' equal to 2. Slack SA_{31}^{2} is equal to zero, which means that δ will be equal to zero in step 7 of the procedure, so the dual variable is not changed.

3.3 *Primal Procedure*

The primal procedure that builds admissible solutions to DCMLP is very similar to the one describe for DUMLP. After building sets I_A^+ and I_R^+, it is necessary to test the satisfaction of the maximum capacity restrictions. The assignment of customers to paths is achieved by solving T transhipment problems. If the transhipment problem is not feasible for some time period t then it will be necessary to open more paths (notice that it is not possible to establish a necessary and sufficient condition of the form *total capacity of open facilities greater than or equal to client's total demand* to test the admissibility of a given solution, as happens in the single level case). To choose what path to open, the procedure calculates the minimum cost of opening a path p by considering the costs of opening all services belonging to p that are not operational during time period t. Consider:

F_i^t = minimum cost of opening facility $i \notin I_t^+$ during period t;

H_p^t = minimum cost of opening path $p \notin P_t^+$ during period t;

$$e_p^t = \begin{cases} 1, \text{ if } p \in P_t^+ \\ 0, \text{ otherwise} \end{cases}, \forall p, t;\ n_i^t = \sum_{p \in P(i)} e_p^t .$$

Primal Procedure

1. Execute step 1-6 of the DUMLP's primal procedure.
2. $t \leftarrow 1$.
3. Solve a transhipment problem considering as sources the set J of clients (with supplies d_j^t), as destinations the set of facilities $i \in I_t^+$ belonging to level K (with demands Q_i), and as transhipment points the set of facilities $i \in I_t^+$ belonging to levels 1 to K-1 (with demand and supply equal to Q_i). If the problem is not feasible go to 4. Else go to 8.
4. If $p \in P_t^+$, $\forall p$ then stop. The problem is infeasible. Else go to 5.
5. Calculate $H_p^t = \sum_{\substack{i \in p \\ i \notin I_t^+}} F_i^t$ and $Cap_p = \min_{i \in p} \left\{ \frac{Q_i}{n_i^t} \right\}$, $\forall p \notin P_t^+$.
6. Choose path p' such that $\frac{H_{p'}^t}{Cap_{p'}} = \min_{p \notin P_t^+} \left\{ \frac{H_p^t}{Cap_p} \right\}$.
7. Open path p', including p' in set P_t^+ and i in set I_t^+, $\forall i \in p$'. Go to 3.

8. $t \leftarrow t + 1$. If $t > T$ then go to 9. Else go to 3.
9. Calculate Z as the current primal objective function value.
10. Execute steps 8-10 of DUMLP's primal procedure.

The minimum cost of opening a facility i during period t (F_i^t) is calculated as described in Dias *et al* (2004b). Calculation of Cap_p tries to illustrate the maximum capacity of path p. The maximum capacity of path p is determined by the facility $i \in p$ with minimum capacity. If facility i belongs to more than one path, its capacity is considered to be equally divided by all open paths.

4 Paths Without Flow Conservation

Consider that facilities i belonging to levels $1,\ldots,K$-1 have a parameter θ_i such that if a flow of d_i units reaches i then this facility will only pass to the next facility a flow $d_i\theta_i$ (see figure 3). This is a generalisation of DCMLP, where θ_i is equal to one for all facilities.

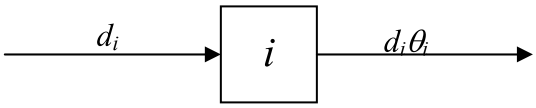

Figure 3

There are several examples where one can find this kind of behavior in intermediate facilities. In solid waste treatment systems, incinerators receive a certain amount of waste to burn, and only a small part of it (in the form of ashes) has to be placed in landfills. In a health facility structure, only a small group of clients that are served in local health centres have to be conducted to local hospitals.

The consideration of parameters θ_i associated with intermediate facilities complicates extremely the problem, specially the determination of optimal assignments of clients to paths (when the location variables are already fixed to one or zero). In fact, this assignment can no longer be calculated optimally through the resolution of transshipment problems.

In the next paragraphs, the primal and condensed-dual formulations are presented, and the primal-dual heuristic developed is described. The complementary conditions are not stated because they are very similar to (17)-(22).

The only difference between this problem and DCMLP lies in the maximum capacity restrictions. Consider the following definition:

Definition 5: Let i and i' be two facilities and p a path such that $i \in p$ and $i' \in p$. Then $i \underset{p}{<} i'$ if and only if facility i appears before facility i' in the ordered set p.

The multi-level, dynamic capacitated location problem without flow conservation on the intermediate facilities (TDCMLP) can be formulated as DCMLP with constraints (32) below instead of constraints (23).

$$Q_i \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi} \right) - \sum_{j} \sum_{p \in P(i)} d_j^t x_{pj}^t \prod_{\substack{i' < i \\ p}} \theta_{i'} \geq 0 , \qquad \forall i,t \qquad (32)$$

Considering the same dual variables as in CD-DCMLP, the condensed-dual problem becomes:

CD-TDCMLP

$$Max \sum_{t} \sum_{j} v_j^t - \sum_{i} \rho_i - \sum_{t} \sum_{i} \pi_i^t$$

subject to:

$$\sum_{j} \sum_{\tau=t}^{\xi} \sum_{p \in P(i)} \eta_{ijp}^{\tau} \, max \left\{ 0, v_j^{\tau} - c_{pj}^{\tau} - d_j^{\tau} \prod_{\substack{i' < i \\ p}} \theta_{i'} \sum_{i_1 \in p} \lambda_{i_1}^t \right\} \leq FA_{it}^{\xi} - \sum_{\tau=\xi+1}^{T} u_i^{\tau} + \rho_i + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - Q_i \sum_{\tau=t}^{\xi} \lambda_i^{\tau} ,$$

$$\forall i, t, \xi = t, \cdots, T \qquad (33)$$

$$\sum_{j} \sum_{\tau=t}^{\xi} \sum_{p \in P(i)} \eta_{ijp}^{\tau} \, max \left\{ 0, v_j^{\tau} - c_{pj}^{\tau} - d_j^{\tau} \prod_{\substack{i' < i \\ p}} \theta_{i'} \sum_{i_1 \in p} \lambda_{i_1}^t \right\} \leq FR_{it}^{\xi} + u_i^t + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - Q_i \sum_{\tau=t}^{\xi} \lambda_i^{\tau} ,$$

$$\forall i, t, \xi = t, \cdots, T \qquad (34)$$

$$u_i^t, \rho_i, \pi_i^t, \lambda_i^t \geq 0 , \qquad \forall i, j, t$$

The primal-dual heuristic functioning scheme is the same presented in section 3, with an additional step:

12. Calculate the optimal assignment of clients to paths considering sets I_t^+ and P_t^+ that correspond to the best primal solution calculated thus far.

Some procedures had to be slightly changed. Consider $d_{ijp}^t = d_j^t \prod_{\substack{i' < i \\ p}} \theta_{i'}$ $\forall i$, $p \in P(i)$, t. The dual ascent and primal-dual adjustment procedures for variables v_j^t do not need to be changed, being only necessary to consider assignment costs $c_{ijp}^t = c_{pj}^t + d_{ijp}^t \sum_{i' \in p} \lambda_{i'}^t$, $\forall i$, $p \in P(i)$, t. The changes in dual ascent and descent procedures for variables λ_i^t are a direct consequence of d_{ijp}^t definition. In the dual ascent procedure the definitions (26) to (28) should be replaced by (35) to (37):

$$\Delta(i)=\max_{\substack{j\in J\\ p\in P(i)\\ i'\in p}}\left\{\frac{v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t}{d_{i'jp}^t}\right\}, \quad (35)$$

$$JP(i,i',\delta)=\left\{(j,p):\ j\in J\wedge p\in P(i)\cap P(i')\wedge\frac{v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t}{d_{i'jp}^t}\le\delta\right\}, \quad (36)$$

$$\overline{JP}(i,i',\delta)=\left\{(j,p):\ j\in J\wedge p\in P(i)\cap P(i')\wedge\frac{v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t}{d_{i'jp}^t}>\delta\right\}, \quad (37)$$

In this case $\Omega(\delta,i,i')$ is given by:

$$\Omega(\delta,i,i')=\sum_{(j,p)\in JP(i,i',\delta)}\eta_{i'jp}^t\max\left\{0,v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t\right\}+\sum_{(j,p)\in\overline{JP}(i,i',\delta)}\eta_{i'jp}^t d_{i'jp}^t\delta-E \quad (38)$$

where: $E=\begin{cases}Q_i\delta, & \text{if } i=i'\\ 0, & \text{otherwise}\end{cases}$.

Step 4 of the dual ascent procedure for variables λ_i^t should be replaced by:

4. $\delta\leftarrow\max_{\substack{j\in J\\ p\in P(i)\\ i'\in p\\ \frac{1}{d_j^t}\max\left\{0,v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t\right\}<\delta'}}\left\{\frac{1}{d_{i'jp}^t}\max\left\{0,v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t\right\}\right\}$

In the dual descent procedure for variables λ_i^t, the following changes should be considered:

$$\Phi(\delta,i',i)=\delta\left(E-\sum_{j\in J}\sum_{\substack{p\in P(i)\cap P(i')\\ v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t\ge 0}}d_{i'jp}^t\eta_{i'jp}^t\right), \quad (39)$$

where:

$$0<\delta\le\min_{\substack{j\in J,p\in P(i)\\ i'\in p\\ v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t<0}}\left\{-\frac{1}{d_j^t}\left(v_j^t-c_{pj}^t-d_{i'jp}^t\sum_{i_1\in p}\lambda_{i_1}^t\right)\right\}. \quad (40)$$

Step 3 should be changed to:

$$3.\ \delta \leftarrow min\left\{\lambda_i^t,\ \min_{\substack{j\in J, p\in P(i)\\ i'\in p\\ v_j^t - c_{pj}^t - d_{i'jp}^t \sum_{i_1\in p} \lambda_{i_1}^t <0}} \left\{-\frac{1}{d_{i'jp}^t}\left(v_j^t - c_{pj}^t - d_{i'jp}^t \sum_{i_1\in p}\lambda_{i_1}^t\right)\right\}\right\}.$$

The primal procedure is very similar to the one already presented in section 3. Step 5 is changed to:

$$\textbf{5.}\ \text{Calculate } H_p^t = \sum_{\substack{i\in p\\ i\notin I_t^+}} F_i^t \text{ and } Cap_p = \min_{i\in p}\left\{\frac{Q_i}{n_i^t \prod_{\substack{i'<i\\ p}} \theta_{i'}}\right\},\ \forall p \notin P_t^+.$$

This change tries to account not only for the number of paths to which facility i belongs, but also for the relative position of facility i in those paths.

The assignment problem cannot be solved as a transhipment problem (step 3 of the primal procedure). After calculating sets I_t^+ and P_t^+, the following linear program should be solved to find the optimal assignments of clients to paths, for each time period t:

AP(*t*)

$$Min \sum_j \sum_p c_{pj}^t x_{pj}^t \qquad (41)$$

subject to:

$$\sum_{p\in P_t^+} x_{pj}^t = 1, \qquad \forall j \qquad (42)$$

$$\sum_j \sum_{p\in P(i)\cap P_t^+} x_{pj}^t d_j^t \prod_{\substack{i'<i\\ p}} \theta_{i'} \le Q_i, \qquad \forall i \in I_t^+ \qquad (43)$$

$$x_{pj}^t \ge 0, \qquad \forall j, p, t$$

Solving this problem optimally (using a general solver) in each execution of the primal procedure (and for all time periods) is very time consuming. So, a heuristic was developed to find feasible solutions to AP(t). This heuristic procedure is used in step 3 of the primal procedure (instead of the resolution of the transshipment problem). Problem AP(t), $\forall t$, is only solved optimally at step 12 of the primal-dual heuristic (using a general solver), considering the location variables that correspond to the best primal solution calculated by the heuristic until that moment.

Heuristic Procedure for the resolution of problem AP(*t*)

1. $D_j \leftarrow d_j^t, \forall j.\ Q_{ip} \leftarrow \frac{Q_i}{\prod_{\substack{i'<i \\ p}} \theta_{i'}}, \forall i, p \in P(i),\ Cap_p \leftarrow \min_{i \in p} \{Q_{ip}\}, \forall p \in P_t^+.\ x_{pj}^t \leftarrow 0, \forall j, p.$
2. If $Cap_p = 0, \forall p \in P_t^+$ and $\exists\, j$ such that $D_j > 0$, then stop: the problem is impossible.
3. For each client $j \in J$, with $D_j > 0$, calculate $c_{p1j}^t \leftarrow \min_{\substack{p \in P_t^+ \\ Cap_p > 0}} \{c_{pj}^t\}$ and $c_{p2j}^t \leftarrow \min_{\substack{p \in P_t^+ \\ Cap_p > 0}} \{c_{pj}^t : c_{pj}^t \geq c_{p1j}^t \wedge p_1 \neq p_2\}.$
4. Choose client j and path p_1 such that $c_{p2j}^t - c_{p1j}^t = \max_{j' \in J} \{c_{p2j'}^t - c_{p1j'}^t\}.$
5. $\delta \leftarrow \min\{D_j, Q_{p_1}\}, x_{p1j}^t \leftarrow x_{p1j}^t + \delta,\ Q_{ip1} \leftarrow Q_{ip1} - \delta, \forall i \in p_1,\ Cap_{p1} \leftarrow Cap_{p1} - \delta,\ D_j \leftarrow D_j - \delta.$
6. $\forall i, p \in P_t^+, p \neq p_1,$ such that $i \in p_1 \wedge i \in p$, $Q_{ip} \leftarrow Q_{ip} - \frac{\delta \prod_{\substack{i'<i \\ p1}} \theta_{i'}}{\prod_{\substack{i'<i \\ p}} \theta_{i'}}$ and $Cap_p \leftarrow \min\{Cap_p, Q_{ip}\}.$
7. If $D_j = 0, \forall j$, then stop. Else, go to 2.

Calculation of Q_{ip} expresses the maximum flow that can be assigned to path p due to the maximum capacity of facility i. The minimum of these capacities corresponds to the maximum capacity of path p designated by Cap_p.

Observation 1:

Consider two sets (S_1 and S_2) of location variables $a_{i\tau}^{\xi}$ and $r_{i\tau}^{\xi}$. Consider two admissible solutions to TDCML constructed by considering equal to one all location variables belonging to S_1 and S_2, respectively, being all the other location variables equal to zero. Let us consider that $Z_1^* < Z_2^*$, where Z_1^* and Z_2^* represent the optimal objective function value corresponding to solution S_1 and S_2, respectively. If Z_1' and Z'_2 represent the objective function value obtained by considering solutions S_1 and S_2, respectively, and solving the assignment problems $AP(t)$ heuristically, then $Z_1' \geq Z_1^*$ and $Z'_2 \geq Z_2^*$, but it is not possible to guarantee that $Z_1' < Z'_2$. This has an important consequence: it is possible that the primal-dual heuristic finds solutions better than the final solution presented, but that the heuristic is not able to identify as being better. This short come cannot be solved, unless the optimal solutions to $AP(t)$ are calculated for every time period t, whenever the primal procedure is executed. As has already been said, that would be prohibitive in terms of execution times.

Example 4:

Consider a problem with four clients and five potential services such that service 1 and 2 are in level 1, services 3 and 4 are in level 2 and service 5 is in level 3. There are five admissible paths:

p_1=(1,3,5), p_2=(1,4,5), p_3=(2,4,5), p_4=(2,5), p_5=(4,5). At period t the clients' demands are as follows: d_1^t=15; d_2^t=20; d_3^t=18; d_4^t=22. Services 1 to 5 have maximum capacities equal to 20, 50, 80, 70 and 100, respectively, and θ_i values equal to 0.8, 0.5, 0.5, 0.1.

Step 1: $Q_{11} = 20$; $Q_{31} = \dfrac{80}{0.8} = 100$; $Q_{51} = \dfrac{100}{0.8 \times 0.5} = 250$;

$$Cap_1 = min\{Q_{11}, Q_{31}, Q_{51}\} = 20;$$

$$Q_{12} = 20;\ Q_{42} = \frac{70}{0.8} = 87.5;\ Q_{52} = \frac{100}{0.1 \times 0.8} = 1250;$$

$$Cap_2 = min\{Q_{12}, Q_{42}, Q_{52}\} = 20;$$

$$Q_{23} = 50;\ Q_{43} = \frac{70}{0.5} = 140;\ Q_{53} = \frac{100}{0.1 \times 0.5} = 2000;$$

$$Cap_3 = min\{Q_{23}, Q_{43}, Q_{53}\} = 50;$$

$$Q_{24} = 50;\ Q_{54} = \frac{100}{0.5} = 200; Cap_4 = min\{Q_{24}, Q_{54}\} = 50;$$

$$Q_{45} = 70;\ Q_{55} = \frac{100}{0.1} = 1000; Cap_5 = min\{Q_{45}, Q_{55}\} = 70.$$

This information is summarized in table 12.

i \ p	1	2	3	4	5
1	20	20	-	-	-
2	-	-	50	50	-
3	100	-	-	-	-
4	-	87.5	140	-	70
5	250	1250	2000	200	1000
Cap_p	20	20	50	50	70

Table 12

Step 3:

j	c_{p1j}^t	p_1	c_{p2j}^t	p_2	$c_{p1j}^t - c_{p2j}^t$
1	2	1	2	3	0
2	2	2	4	1	2
3	1	2	4	5	3
4	2	3	2	5	0

Table 13

Step 4: Choose client j equal to 3 and path p equal to 2.

Step 5: $\delta \leftarrow min\{18, 20\}$=18. $D_3 \leftarrow 0$; Values Q_{ip} and Cap_p are changed according with table 14.

i \ p	1	2	3	4	5
1	20-18=2	20-18=2	-	-	-
2	-	-	50	50	-
3	100	-	-	-	-
4	-	87.5-18=69.5	$140-\frac{14.4}{0.5}$ $=111.20$	-	$70-\frac{1.44}{0.1}$ $=55.60$
5	$250-\frac{18\times0.1\times0.8}{0.5\times0.8}$ $=246.40$	1250-18=1232	$2000-\frac{1.44}{0.1\times0.5}$ $=1971.20$	$200-\frac{1.44}{0.5}$ $=197.12$	$1000-\frac{1.44}{0.1}$ $=985.60$
Cap_p	2	2	50	50	55.6

Table 14

Step 4: Choose client j equal to 2 and path p equal to 2.

Step 5: $\delta \leftarrow min\{20, 2\}=2$; $D_2 \leftarrow 18$; Values Q_{ip} and Cap_p are changed according with table 15 (the bold values are the ones that are changed).

i \ p	1	2	3	4	5
1	**0**	**0**	-	-	-
2	-	-	50	50	-
3	100	-	-	-	-
4	-	**67.5**	**108**	-	**54**
5	**246**	**1230**	**1968**	**196.80**	**984**
Cap_p	0	0	50	50	54

Table 15

Step 3:

j	$c^t_{p_1 j}$	p_1	$c^t_{p_2 j}$	p_2	$c^t_{p_1 j}-c^t_{p_2 j}$
1	2	3	3	4	1
2	4	5	5	5	1
4	2	3	2	5	0

Table 16

Step 4: Choose client j equal to 1 and path p equal to 3.

Steps 5 and 6 : $\delta \leftarrow min\{15, 50\}=15$ $D_1 \leftarrow 0$; Values Q_{ip} and Cap_p are changed according with table 17 (the bold values are the ones that are changed).

Step 4: Choose client j equal to 2 and path p equal to 5.

Steps 5 and 6: $\delta \leftarrow min\{18, 46.5\}=18$. $D_2 \leftarrow 0$; Values Q_{ip} and Cap_p are changed according with table 18.

Step 4: Choose client j equal to 4 and path p equal to 3

Steps 5 and 6: $\delta \leftarrow min\{22, 35\}=22$; $D_4 \leftarrow 0$;

Step 7: Values D_j are equal to zero, for all j, which means that an admissible solution has been found (the solution found is illustrated in figure 4):

I \ P	1	2	3	4	5
1	0	0	-	-	-
2	-	-	**35**	**35**	-
3	100	-	-	-	-
4	-	67.5	**93**	-	**46.50**
5	246	1230	**1953**	**195.30**	**976.50**
Cap_p	0	0	35	35	46.5

Table 17

i \ p	1	2	3	4	5
1	0	0	-	-	-
2	-	-	35	35	-
3	100	-	-	-	-
4	-	67.5	**75**	-	**28.50**
5	246	1230	**1917**	**191.70**	**958.50**
Cap_p	0	0	35	35	28.5

Table 18

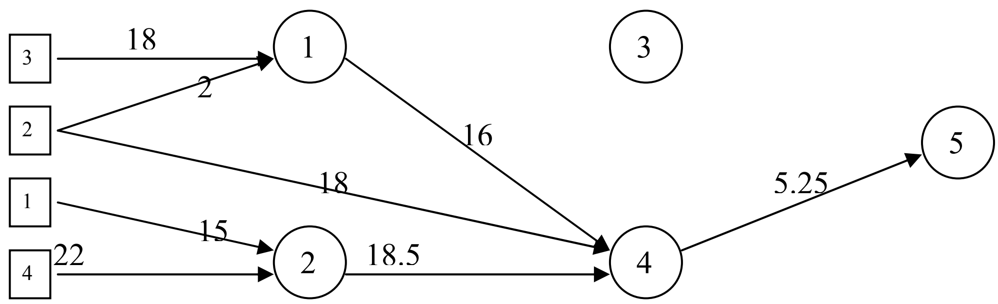

Figure 4

■

Problem TDCMLP can be generalized considering a new set of constraints imposing minimum capacity restrictions:

$$Q'_i \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{i\tau}^{\xi} + r_{i\tau}^{\xi}\right) - \sum_{j} \sum_{p \in P(i)} d_j^t x_{pj}^t \prod_{\substack{i' \in p \\ i' <_p i}} \theta_{i'} \le 0, \qquad \forall i,t \qquad (44)$$

where Q'_i represents the minimum flow that has to reach an open facility i, during the time periods this facility is operating. Associating dual variables β_i^t with constraints (44), the condensed dual formulation is slightly changed:

CD-TDCMLP2

$$Max \sum_t \sum_j v_j^t - \sum_i \rho_i - \sum_t \sum_i \pi_i^t$$

subject to:

$$\sum_j \sum_{\tau=t}^{\xi} \sum_{p\in P(i)} \eta_{ijp}^{\tau} \, max\left\{0, v_j^{\tau} - c_{pj}^{\tau} - d_j^{\tau} \prod_{\substack{i'<i\\p}} \theta_{i'} \left(\sum_{i_1\in p} \lambda_{i_1}^{t} + \sum_{i_1\in p} \beta_{i_1}^{t} \right)\right\} \le FA_{it}^{\xi} - \sum_{\tau=\xi+1}^{T} u_i^{\tau} + \rho_i + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - Q_i \sum_{\tau=t}^{\xi} \lambda_i^{\tau} + Q'_i \sum_{\tau=t}^{\xi} \beta_i^{\tau} \quad ,\forall i,t,\xi = t,\cdots,T \qquad \textbf{(45)}$$

$$\sum_j \sum_{\tau=t}^{\xi} \sum_{p\in P(i)} \eta_{ijp}^{\tau} \, max\left\{0, v_j^{\tau} - c_{pj}^{\tau} - d_j^{\tau} \prod_{\substack{i'<i\\p}} \theta_{i'} \left(\sum_{i_1\in p} \lambda_{i_1}^{t} - \sum_{i_1\in p} \beta_{i_1}^{t} \right)\right\} \le FR_{it}^{\xi} + u_i^{t} + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - Q_i \sum_{\tau=t}^{\xi} \lambda_i^{\tau} + Q'_i \sum_{\tau=t}^{\xi} \beta_i^{\tau} \quad , \forall i,t,\xi = t,\cdots,T \qquad \textbf{(46)}$$

$$u_i^t, \rho_i, \pi_i^t, \lambda_i^t, \beta_i^t \ge 0 , \qquad \forall i, j, t$$

As can be seen, the behavior of dual variables β_i^t and λ_i^t is symmetric, so the dual ascent procedure described for dual variables λ_i^t constitutes the dual descent procedure for β_i^t, and the dual descent procedure for dual variables λ_i^t constitutes the dual ascent procedure for variables β_i^t. The primal procedure has to be changed, because it is necessary to satisfy a new set of constraints. Nevertheless, the new primal procedure constructed is, in essence, similar to the one previously described. The only difference consists in the heuristic procedure to solve the assignment problem (problem AP(*t*) has also a new set of minimum capacity constraints), and also in some additional steps that try to close or interchange open and closed facilities, when an admissible solution is not found by opening new facilities (using the procedure already described). The optimal assignment of clients to facilities, considering the location variables fixed to one or zero, can be found by solving the following linear problem:

***AP*1(*t*)**

$$Min \sum_j \sum_p c_{pj}^t x_{pj}^t$$

subject to:

$$\sum_{p\in P_t^+} x_{pj}^t = 1 , \qquad \forall j \qquad \textbf{(42)}$$

$$Q'_i \le \sum_j \sum_{p\in P(i)\cap P_t^+} x_{pj}^t d_j^t \prod_{\substack{i'<i\\p}} \theta_{i'} \le Q_i , \qquad \forall i \in I_t^+ \qquad \textbf{(47)}$$

$$x_{pj}^t \ge 0 , \qquad \forall j, p, t$$

Heuristic to find an admissible solution to $AP1(t)$

1. Calculate an admissible solution to $AP(t)$. If the calculated solution is admissible to $AP1(t)$ stop. Else go to 2.
2. $x^t_{pj} \leftarrow 0, \forall p, j$; $Capmin_i \leftarrow Q'_i$ *and* $Capmax_i \leftarrow Q_i, \forall i \in I_t^+$; $D_j \leftarrow d_j^t, \forall j$.
3. If $Cap_i \leq 0$, $\forall\ i \in I_t^+$, then go to 10.
4. If $D_j = 0$, $\forall j$ and $\exists\ i \in I_t^+$ such that $Capmin_i > 0$, then stop: the heuristic cannot find an admissible solution. Else go to 5.
5. Choose facility i, with $Capmin_i > 0$, such that $\dfrac{Capmin_i}{Capmax_i} = \max_{i' \in I_t^+} \left\{ \dfrac{Capmin_{i'}}{Capmax_{i'}} \right\}$.
6. If $\exists\ p \in P_t^+ \cap P(i)$ such that $Capmin_i > 0$, $\forall i \in p$ then go to 7. Else go to 8.
7. Choose path $p \in P_t^+ \cap P(i)$ and client j such that $\dfrac{c^t_{pj}}{\prod_{\substack{i'<i \\ p}} \theta_{i'}} = \min_{\substack{p' \in P(i) \cap P_t^+ \\ Capmin_i > 0, \forall i \in p' \\ j' : D_{j'} > 0}} \left\{ \dfrac{c^t_{p'j'}}{\prod_{\substack{i'<i \\ p'}} \theta_{i'}} \right\}$. Go to 9.
8. Choose path $p \in P_t^+ \cap P(i)$ and client j such that $\dfrac{c^t_{pj}}{\prod_{\substack{i'<i \\ p}} \theta_{i'}} = \min_{\substack{p' \in P(i) \cap P_t^+ \\ j' : D_{j'} > 0}} \left\{ \dfrac{c^t_{p'j'}}{\prod_{\substack{i'<i \\ p'}} \theta_{i'}} \right\}$.
9. $\delta \leftarrow \min \left\{ D_j, \min_{i' \in p} \left\{ \dfrac{Capmax_{i'}}{\prod_{\substack{i''<i' \\ p}} \theta_{i''}} \right\}, \dfrac{Capmin_i}{\prod_{\substack{i'<i \\ p}} \theta_{i'}} \right\}$; $x^t_{pj} \leftarrow x^t_{pj} + \delta$; $D_j \leftarrow D_j - \delta$;

   $Capmin_i \leftarrow Capmin_i - \delta \prod_{\substack{i'<i \\ p}} \theta_{i'}$ and $Capmax_i \leftarrow Capmax_i - \delta \prod_{\substack{i'<i \\ p}} \theta_{i'}$, $\forall i \in p$. Go to 3.
10. If $\exists\ j$ such that $D_j > 0$, then solve heuristically problem $AP(t)$ considering Q_i equal to $Capmax_i$, $\forall i \in I_t^+$ and d_j^t equal to D_j, $\forall j$. Else stop.

If an admissible solution is not found in step 1 of this heuristic, then it tries to assign flow to paths in order to satisfy the minimum capacity constraints of open facilities. Step 5 of this heuristic chooses facility i such that the relation between its maximum and minimum capacities is more difficult in terms of satisfying both restrictions. The heuristic will try first to assign flow to paths $p \in P(i)$ such that no facility i' belonging to path p has its minimum capacity restriction satisfied (step 6). When this is not possible, the heuristic considers all open paths $p \in P(i)$ (step 7). In steps 6 or 7 the heuristic chooses a client j and an open path p that correspond to the minimum cost per unit that reaches facility i. The increase in the flow that is assigned to path p is calculated as being the minimum between the remaining demand of client j and the necessary flow to satisfy the minimum capacity restriction of facility i, guaranteeing that the maximum capacity restrictions for all other facilities belonging to path p remain admissible. When the heuristic reaches step 10, all minimum capacity restrictions for open facilities have been satisfied. If there are still clients j having a remaining demand greater than zero, then it will be necessary to assign these demands to paths using the already described heuristic for problem $AP(t)$. The described heuristic cannot guarantee the construction of an admissible solution to $AP1(t)$ whenever the problem is feasible. It is always possible to check if the problem $AP1(t)$ is infeasible by using a general solver (which will be more time consuming but will find out, for sure, if the problem is or is not infeasible, when the heuristic is unable to find an admissible solution).

Example 5

Consider the problem described in example 4, with facilities having the following minimum capacities: 5, 10, 0, 60, 5.

The minimum capacity restriction is not satisfied for facility 4, so the solution found in step 1 of the heuristic is not admissible.

Step 2: $Capmin_1 \leftarrow 5$; $Capmin_2 \leftarrow 10$; $Capmin_3 \leftarrow 0$; $Capmin_4 \leftarrow 60$; $Capmin_5 \leftarrow 10$.

$Capmax_1 \leftarrow 20$; $Capmax_2 \leftarrow 50$; $Capmax_3 \leftarrow 80$; $Capmax_4 \leftarrow 70$; $Capmax_5 \leftarrow 100$.

$D_1 \leftarrow 15$; $D_2 \leftarrow 20$; $D_3 \leftarrow 18$; $D_4 \leftarrow 22$.

Step 5: Choose facility 4 because $\frac{60}{70} = max\left\{\frac{5}{20}, \frac{10}{50}, \frac{60}{70}, \frac{5}{100}\right\}$.

Step 7: Choose path p_2, p_3 or p_5 and a client j according to the calculations presented in table 19. The heuristic will choose path 2 and client 3.

p \ j	1	2	3	4
2	$\frac{3}{0.8}$	$\frac{2}{0.8}$	$\frac{1}{0.8}$	$\frac{10}{0.8}$
3	$\frac{2}{0.5}$	$\frac{5}{0.5}$	$\frac{5}{0.5}$	$\frac{2}{0.5}$
5	4	5	4	2

Table 19

Step 9: $\delta \leftarrow min\left\{18, 20, \frac{70}{0.8}, \frac{100}{0.8 \times 0.1}, \frac{60}{0.8}\right\} = 18$; $x_{23}^{t} \leftarrow 18$; $D_3 \leftarrow 0$;

$Capmin_1 \leftarrow 5-18=-13$; $Capmin_4 \leftarrow 60-18 \times 0.8 = 45.60$; $Capmin_5 \leftarrow 5-18 \times 0.8 \times 0.1 = 3.56$.

$Capmax_1 \leftarrow 2$; $Capmax_4 \leftarrow 55.60$; $Capmax_5 \leftarrow 98.56$.

Step 5: Choose facility 4 because $\frac{45.60}{55.60} = max\left\{\frac{10}{50}, \frac{45.60}{55.60}, \frac{3.56}{98.56}\right\}$.

Step 7: The heuristic will choose path 5 and client 4.

Step 9: $\delta \leftarrow min\left\{22, 55.60, \frac{98.56}{0.1}, 45.60\right\} = 22$; $x^t_{54} \leftarrow 22$; $D_4 \leftarrow 0$;

$Capmin_4 \leftarrow 23.60$; $Capmin_5 \leftarrow 1.36$; $Capmax_4 \leftarrow 33.60$; $Capmax_5 \leftarrow 96.36$.

Step 5: Choose facility 4 because $\frac{23.60}{33.60} = max\left\{\frac{10}{50}, \frac{23.60}{33.60}, \frac{1.36}{96.36}\right\}$.

Step 7: The heuristic will choose arbitrarily between client 1 and path 3 or client 1 and path 5 (the heuristic will never choose client 2 and path 2 because path 2 has facilities with the minimum capacity restriction satisfied). Consider that the heuristic chooses client 1 and path 3.

Step 9: $\delta \leftarrow min\left\{15, 50, \frac{33.6}{0.5}, \frac{96.36}{0.5 \times 0.1}, \frac{23.60}{0.5}\right\} = 15$; $x^t_{31} \leftarrow 15$; $D_1 \leftarrow 0$;

$Capmin_2 \leftarrow -5$; $Capmin_4 \leftarrow 16.10$; $Capmin_5 \leftarrow 0.61$;

$Capmax_2 \leftarrow 35$; $Capmax_4 \leftarrow 26.10$; $Capmax_5 \leftarrow 95.61$.

Step 5: Choose facility 4 because $\frac{16.10}{26.10} = max\left\{\frac{16.10}{26.10}, \frac{0.61}{95.61}\right\}$.

Step 7: The heuristic chooses path 5, because it is the only path that is constituted by facilities whose minimum capacity restriction is not yet satisfied, and chooses client 2.

Step 9: $\delta \leftarrow min\left\{20, 26.1, \frac{95.61}{0.1}, 16.10\right\} = 16.10$; $x^t_{52} \leftarrow 16.10$; $D_2 \leftarrow 3.9$;

$Capmin_4 \leftarrow 0$; $Capmin_5 \leftarrow -1$;

$Capmax_4 \leftarrow 10$; $Capmax_5 \leftarrow 94$.

Step 10: The current solution is already admissible because $Capmin_i \leq 0, \forall i$. As $D_2 > 0$, then it will be necessary to use the heuristic to solve problem $AP(t)$.

■

If the minimum capacity restrictions were considered in problem DCMLP (with θ_i equal to one, for all facilities) then the assignment problem $AP1(t)$ would be a transshipment problem, with each intermediate facility corresponding to two transshipment points: one having demand and supply equal to its minimum capacity and the other having demand and supply equal to its maximum capacity minus its minimum capacity.

Consider:

f^t_i = smallest cost incurred by closing $i \in I^+_t$ during period t.

g_i^t=smallest cost incurred by closing $i \in I_t^+$ during period t and opening one or more services $i' \notin I_t^+$ such that problem $AP1(t')$, $t' \leq t$, is feasible. The set of services $i' \notin I_t^+$ that have to be open is designated by $I(i,t)$.

Primal Procedure for TDCMLP with minimum capacity restrictions

1. Execute steps 1-8 of the primal procedure for TDCMLP.
2. If the solution satisfies restrictions (47) then stop. Else go to 3.
3. $t \leftarrow 1$.
4. If $AP1(t)$ is feasible, then go to 14. Else go to 5.
5. Calculate f_i^t, $\forall i \in I_t^+$.
6. Choose facility i' such that $f_{i'}^t = \min_{i \in I_t^+} \{f_i^t\}$. Remove i' from set I_t^+. Update set P_t^+.
7. If problem $AP1(t)$ is feasible then go to 14. Else insert i' in set I_t^+, $f_{i'}^t \leftarrow +\infty$.
8. If $f_i^t = +\infty$ $\forall i \in I_t^+$, then update set P_t^+ and go to 9. Else go to 6.
9. Calculate g_i^t, $\forall i \in I_t^+$.
10. Consider i' such that $g_{i'}^t = \min_{i \in I_t^+} \{g_i^t\}$. Remove i' from set I_t^+. Insert in I_t^+ the set $I(i',t)$. Update set P_t^+.
11. If problem $AP1(t)$ is feasible then go to 14. Else $g_{i'}^t \leftarrow +\infty$, go to 12.
12. If $g_i^t = +\infty$, $\forall i \in I_t^+$, stop: the primal procedure cannot find an admissible solution. Else go to 10.
13. Rebuild sets I_A^+, I_R^+, I_t^+, P_t^+, $\forall t$.
14. $t \leftarrow t + 1$; If $t \leq T$ go to 4. Else stop.

In step 5, the calculation of f_i^t is done in the following way: if $i \in I_t^+$, then there exists $(i,a,b) \in I_A^+ \cup I_R^+$ such that $a \leq t \leq b$.

If $(i, a, b) \in I_A^+$ then: $f_i^t = \begin{cases} -FA_a^b, & \text{if } a = b \\ FA_a^{t-1} + FR_{t+1}^b - FA_a^b, & \text{otherwise} \end{cases}$.

If $(i, a, b) \in I_R^+$ then: $f_i^t = \begin{cases} -FR_a^b, & \text{if } a = b \\ FR_a^{t-1} + FR_{t+1}^b - FR_a^b, & \text{otherwise} \end{cases}$.

The calculation of g_i^t is more complicated. It tries to assess the cost of closing facility i, but opening more facilities so that the assignment problem is feasible in terms of maximum capacity restrictions.

Calculation of g_i^t, $\forall i \in I_t^+$

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1. $g_i^t \leftarrow f_i^t$, $\forall\, i \in I_t^+$. $I(i,t) \leftarrow \varnothing$. Remove i from set I_t^+.

2. Calculate $F_{i'}^t$, $\forall i' \notin I_t^+$.

3. If $F_{i'}^t = +\infty$, $\forall i \notin I_t^+$ then $g_i^t \leftarrow +\infty$ and stop.

4. Choose i' such that $F_{i'}^t = \min\limits_{i_1 \notin I_t^+} \left\{ F_{i_1}^t \right\}$. Include i in sets I_t^+ and $I(i,t)$. Update P_t^+. $g_i^t \leftarrow g_i^t + F_{i'}^t$.

   $F_{i'}^t \leftarrow +\infty$.

5. If $AP1(t)$ is feasible then insert i in set I_t^+ and remove $I(i,t)$ from set I_t^+; update set P_t^+ and stop. Else go to 3.

The calculation of $F_{i'}^t$, $\forall i' \notin I_t^+$ is done as described in Dias *et al* (2004b), but guaranteeing that problem $AP1(t')$ remains feasible for time periods $t' < t$.

It is important to notice that the inclusion or removal of a single facility i from set I_t^+ can be responsible for the inclusion or removal of more than one path p from set P_t^+. When removing facility i from I_t^+, all paths $p \in P(\, i\,)$ will have to be removed from P_t^+. The inclusion of facility i in I_t^+ will cause all paths $p \in P(\, i\,)$ such that p is open to be included in P_t^+.

To find out if problem $AP1(t)$ is feasible, for some t, there are two hypotheses: use a general solver to solve the linear programming problem or execute the heuristic procedure already described.

The primal procedure presented cannot guarantee the calculation of an admissible solution, even in the presence of feasible problems. Nevertheless this has not proved to be a problem, in the computational tests already performed.

5 Conclusions

The problems and the primal-dual heuristics presented in this research report are a generalization of the dynamic single-level location problems studied in Dias *et al* (2004a,b) to the multi-level case. The models developed have several features that distinguish them from the models generally studied in the literature.

All heuristics described were already programmed using C-language programming. To assess their performance (both in terms of time and quality of the primal solution) intensive computational tests over a set of random generated problems need to be performed. It will be important to observe the behavior of the heuristics when the number of maximum levels considered (K) increases.

Results are expected to be clearly worse for problems that consider the possibility of no flow conservation in path's intermediate facilities. It is important to notice that the described problems are very hard and difficult to solve.

Another generalization of the presented models would be obtained by considering the possibility of having facilities with maximum decreasing capacities (Dias *et al*, 2004c), and facilities that are constituted by one or more equipments of different discrete sizes (implying an associated discrete expansion location problem).

Another interesting generalization is to consider the existence of more than one objective function, and even the existence of several decision makers. It will also be interesting to compare the performance of primal-dual heuristics with the performance of metaheuristics (like evolutionary algorithms).

6 References

Aardal, K., Labbé, M., Leung, A., Queyranne, M. (1996), "On the Two-Level Uncapacitated Facility Location Problem", INFORMS Journal on Computing, 8, pp 289-301

Barros, A. I, Labbé, M. (1994), "The Multi-Level Uncapacitated Facility Location Problem is not Submodular", European Journal of Operational Research, 72, pp 607-609

Bloemhof-Ruwaard, J., Salomon, M., Van Wassenhove, L. N. (1996), "The Capacitated Distribution and Waste Disposal Problem", European Journal of Operational Research, 88, pp 490-503

Bumb, A. (2001), "An Approximation Algorithm for the Maximization Version of the Two-Level Uncapacitated Facility Location Problem", Operations Research Letters, 29, pp 155-161

Canel, C., Khumawala, B., Law, J., Loh, A. (2001), "An Algorithm for the Capacitated, Multi-Commodity, Multi-Period Facility Location Problem", Computers & Operations Research, 28, pp 411-427

Chairdaire, P. (1999), " Hierarchical Two Level Location Problems", in Sansò and Soriano (edts) Telecommunications Network Planning, Kluwer Academis Publishers, pp 33-54

Daskin, M. S. (1995), "Network and Discrete Location Models, Algorithms and Applications", John Wiley & Sons

Dias, J., Captivo, M.E., Clímaco, J., (2004a) "Efficient Primal-Dual Heuristic for a Dynamic Location Problem", submitted for publication

Dias, J., Captivo, M.E., Clímaco, J., (2004b) "Capacitated Dynamic Location Problems with Opening, Closure and Reopening of Facilities", submitted for publication

Dias, J., Captivo, M.E., Clímaco, J., (2004c) "A Dynamic Location Problem with Maximum Decreasing Capacities", submitted for publication

Edwards, N. (2001), “Approximation algorithms for the multi-level facility location problem”, PhD Thesis, Cornell University

Eitan, Y., Narula, S. C., Tien, J. (1991), “A Generalized Approach to Modelling the Hierarchical Location-Allocation Problem”, IEEE Transactions on Systems Man and Cybernetics, 21, pp 39-46

Erlenkotter, D. (1978), “A Dual-based Procedure for Uncapacitated Facility Location”, Operations Research, 26, pp 992-1009

Espejo, L. G. A, Galvão, R. D., Boffey, B. (2003), “Dual-based Heuristics for a Hierarchical Covering Location Problem”, Computers & Operations Research, 30, pp 165-180

Galvão, R. D., Espejo, L. G. A, Boffey, B. (2002), “A Hierarchical Model for the Location of Perinatal Facilities in the Municipality of Rio de Janeiro”, European Journal of Operational Research, 138 , pp 495-517

Gao, L.-L., Robinson Jr., E. P. (1992), “A Dual-Based Optimization Procedure for the Two-Echelon Uncapacitated Facility Location Problem”, Naval Research Logistics, 39, pp 191-212

Guignard, M., Spielberg, K., (1979), “A Direct Dual Method for the Mixed Plant Location Problem with Some Side Constraints”, Mathematical Programming, 17, 198-228

Hinojosa, Y., Puerto, J., Fernández, F.R. (2000), “A Multiperiod two-echelon multicommodity capacitated plant location problem”, European Journal of Operational Research, 123, pp 271-291

Hodgson, M. J. (1981), “The Location of Public Facilities Intermediate to the Journey to Work”, European Journal of Operational Research, 6, pp 199-204

Jayaraman, V., Gupta, R., Pirkul, H. (2003), “Selecting hierarchical facilities in a service-operations environment”, European Journal of Operational Research, 147, pp 613-628

Klose, A. (1995), “A Lagrangean Heuristic to Solve the Two-stage Capacitated Facility Location Problem”, Working Paper - Institut für Unternehmensforschung - Universität St. Gallen

Klose, A. (1998), “Obtaining Sharp Lower and Upper Bounds for Two-Stage Capacitated Facility Location Problems”, in B.Fleisechmann, J.A.E.Z Van Nunen, M.G.Speranza, P.Stähly (edts) Advances in Distribution Logistics, Lectures Notes in Economics and Mathematical Systems, 460, pp 185-214

Klose, A. (1999), “An LP-based heuristic for two-stage capacitated facility location problems”, Journal of the Operational Research Society, 50, pp 157-166

Klose, A. (2000), “A Lagrangean Relax-and-Cut Approach for the Two-Stage Capacitated Facility Location Problem”, European Journal of Operational Research, 126, pp 408-421

Madsen, O. (1983), “Methods for Solving Combined Two Level Location-Routing Problems of Realistic Dimensions”, European Journal of Operational Research, 12, pp 295-301

Marianov, V., Serra, D. (2001), “Hierarchical Location-Allocation Models for Congested Systems”, European Journal of Operational Research, 135, pp 195-208

Melachrinoudis, E., Min, H. (2000), “The Dynamic Relocation and Phase-Out of a Hybrid, Two-Echelon Plant/Warehouse Facility: A Multiple Objective Approach”, European Journal of Operational Research, 123, pp 1-15

Moore, G., ReVelle, C. (1982), “The Hierarchical Service Location Problem”, Management Science, 28, pp 775-780

Narula, S. (1984), “Hierarchical Location-Allocation Problems: A Classification Scheme”, European Journal of Operational Research, 15, pp 93-99

Narula, S., Ogbu, U. (1985), “Lagrangean Relaxation and Decomposition in an Uncapacitated 2-Hierarchical Location-Allocation Problem”, Computers & Operations Research, 12, pp 169-180

Pirkul, H., Jayaraman, V. (1998), “A Multi-Commodity, Multi-Plant, Capacitated Facility Location Problem: Formulation and Efficient Heuristic Solution”, Computers & Operations Research, 25, pp 869-878

Ro, H.-B., Tcha, D.-W. (1984), “A Branch and Bound Algorithm for the Two-Level Uncapacitated Facility Location Problem with Some Side Constraints”, European Journal of Operational Research, 18, pp 349-358

Saldanha da Gama, F., (2002), “Modelos e Algoritmos para o Problema de Localização Dinâmica”, PhD Thesis, Faculdade de Ciências, Universidade de Lisboa (in Portuguese)

Serra, D., Marianov, V., ReVelle, C. (1992), “The Maximum-Capture Hierarchical Location Problem”, European Journal of Operational Research, 62, pp 363-371

Tcha, D.-W., Lee, B.-I. (1984), “A Branch and Bound Algorithm for the Multilevel Uncapacitated Facility Location Problem”, European Journal of Operational Research 18, pp 35-43

Tien, J. M., El-Tell, K., Simons, G. R. (1983), “Improved Formulations to the Hierarchical Health Facility Location-Allocation Problem”, IEEE Transactions on Systems Man and Cybernetics, 13, pp 1128-1132

Tragantalerngsak, S., Holt, J., Rönnqvist, M. (2000), “An Exact Method for the Two-Echelon, Single-Source, Capacitated Facility Location Problem”, European Journal of Operational Research, 123, pp 473-489

Tragantalerngsak, S., Holt, J., Rönnqvist, M. (1997), “Lagrangian Heuristics for the Two-Echelon, Single-Source, Capacitated Facility Location Problem”, European Journal of Operational Research, 102, pp 611-625

Van Roy, T., Erlenkotter, D. (1982), “A Dual-Based Procedure for Dynamic Facility Location”, Management Science, 28, pp 1091-1105

Zhang, J., Ye, Y. (2002), “A note on the maximization version of the multi-level facility location problem”, Operations Research Letters, 30, pp 333-335