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AN INVERSE APPROACH FOR ELECTRE III

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Abstract

ELECTRE III is a well-known multiple criteria decision aiding (MCDA) method, which has a long history of successful real-life applications. One of the most important parameters of ELECTRE III is the preference information of the decision maker expressed as weights for criteria. The difficulty of eliciting weights from the decision makers is well-known, and several methods for weight elicitation have been proposed in literature in the past. In this paper, we introduce an inverse weight-space analysis for the ELECTRE III. The analysis is based on a modified version of the Stochastic Multicriteria Acceptability Analysis (SMAA). We also derive robust conclusions of an existing ELECTRE III application by applying the inverse approach. In this application 11 actions are evaluated in terms of 11 criteria, and the analysis aims at ranking the actions and selecting the first one. Our re-analysis illustrates the usefulness of the presented method: the recommendation of the original case study seems to be not a “robust” one. The inverse analysis shows that by allowing 10% weight alterations, the action selected in the original case study obtains the first rank with only 26% of the feasible weights, and almost in all the remaining cases the first rank was shared with another action.

Keywords: *Multiple Criteria Decision Aiding (MCDA); ELECTRE III; Stochastic Multicriteria Acceptability Analysis (SMAA); Inverse Analysis*

1 Introduction

ELECTRE III (Roy, 1978) is a well-established multiple criteria decision aiding (MCDA) method that has a history of successful real-world applications. It has been applied in past to various types of decision making situations (see e.g. some recent applications: Georgopoulou et al., 1997; Hokkanen and Salminen, 1997; Karagiannidis and Moussiopoulos, 1997; Rogers et al., 2000). For a comprehensive description of ELECTRE methods the reader can consult (Figueira et al., 2005; Roy and Bouyssou, 1993).

ELECTRE III requires an input of criteria evaluations for the actions, preference information expressed as weights, thresholds, and other parameters. The criteria evaluations can usually be determined with “certain accuracy”, and the imperfect knowledge about the evaluations can be taken into account when defining the thresholds for the model. For preference information the situation is even worse: if the decision makers (DMs) cannot provide precise and complete weight information, or if there are multiple DMs with conflicting preferences, ELECTRE methods cannot be used for decision aiding without some external method for transforming the preferences to deterministic weight values.

There are numerous weight elicitation techniques proposed for ELECTRE methods, the following being among the most recent:

1. The method proposed by Mousseau (1995) produces intervals for weights, and these intervals should be used for analysing the “robustness” of the results.
2. Hokkanen and Salminen (1997) used two different weight elicitation procedures, and found that the normalized sets of weights had only minor differences. Also in this case the stability of the ranks should have been analyzed by applying intervals defined by these two sets of weights.
3. The revised Simos’ procedure by Figueira and Roy (2002) allows weight elicitation in a user-friendly manner by using a pack of “playing” cards based technique to determine the criteria coefficients.
4. An approach proposed by Rogers and Bruen (1998b) uses pairwise comparisons to elicit the weights. When using one of the previous two weight elicitation techniques, the stability should be analyzed by using intervals for the weights, because the difficulty of expressing beliefs in mathematical terms causes inaccuracy in the evaluations.

In this paper, we introduce an inverse weight-space analysis to the ELECTRE III method. The analysis is based on exploring the weight space in order to describe which weights result in certain ranks for the actions, and it allows ELECTRE III to be used with weight information of arbitrary type. We will concentrate on weight information provided as weight intervals, because:

1. this type of weight information can be provided by the existing weight elicitation techniques,
2. it allows a particular kind of easily comprehensible “robustness analysis” also in the case when the weights are deterministic, and

3. if there are multiple DMs whose preferences need to be taken into account, the weight intervals can be determined to contain the preferences of all DMs.

Our method is based on Stochastic Multicriteria Acceptability Analysis (SMAA), which is a family of decision support methods to aid DMs in discrete decision making problems with multiple DMs. Instead of giving direct answers to the decision making problem, SMAA methods are based on inversely analyzing and characterising the problem, leaving the final decision for DMs. Among of the main results of SMAA are the rank acceptability indices, which can be interpreted as measurements of some kind of robustness for alternatives to obtain certain ranks. There are some links between inverse analysis and robustness concerns in the sense that from the results of an inverse analysis we can derive robust conclusions.

A *robustness analysis (or concerns)* consists of the aptitude of a *conclusion* to remain unchanged in the presence of imperfect knowledge about data (Roy, 2002, 2004). A *conclusion* in decision aiding problems is said to be *robust* with respect to some possible values for parameters, if there is no particular set of parameters, which clearly invalidates the conclusion (see Figueira et al., 2005, Ch. 4, Sect. 4.1). In the case of analyzing robustness of ELECTRE III methods, the parameters are the weights, thresholds, etc. In this paper, we will analyze robustness with respect to the weights, and consider the remaining parameters (thresholds, cutting levels, ...) fixed. Usually the “robustness” in the field of MCDA is analyzed by altering only a discrete set of weights for a criterion (sensitivity analysis), or by considering only the extremes of the feasible weight space (Dias et al., 2002). In our inverse weight-space analysis, we consider all possible weight vectors in the feasible weight space.

The SMAA family consists of numerous methods. In the original SMAA method by Lahdelma et al. (1998) the weight space analysis is performed based on an additive utility or value function and stochastic criteria evaluations. The SMAA-2 method (Lahdelma and Salminen, 2001) generalized the analysis to a general utility or value function, to include various kinds of preference information and to consider holistically all ranks. The SMAA-3 method (Lahdelma and Salminen, 2002) applies ELECTRE III type pseudo-criteria in the analysis. The SMAA-O method (Lahdelma et al., 2003) extends SMAA-2 for treating mixed ordinal and cardinal criteria in a comparable manner. The SMAA-A method models the preferences using reference points and achievement scalarizing functions (Lahdelma et al., 2004a). SMAA-D applies, instead of a value function, the efficiency score of Data Envelopment Analysis (DEA) (Lahdelma and Salminen, 2004). The SMAA methods have successfully been applied to real-life decision making problems (see e.g. Hokkanen et al., 1998, 1999, 2000; Lahdelma and Salminen, 2005; Lahdelma et al., 2002, 2001).

This paper is organized as follows: a comprehensive description of ELECTRE III is introduced in Section 2. The inverse approach is presented in Section 3. Section 4 contains a re-analysis of a case-study applying the novel method. The paper ends with conclusions and avenues for future research in Section 5.

2 ELECTRE III

The contents of this section can be found in a similar form in Figueira and Greco (2004). The ELECTRE III method is based on two phases. First, the outranking relation between pairs of actions is formed. This results in an outranking matrix. The second phase consists of exploiting this relation, producing a partial pre-order.

In this paper, we will use the following notation:

- $F = \{g_1, \dots, g_j, \dots, g_n\}$ is the set or family of *criteria*. Let \mathcal{J} denote the set of criteria indices.
- $A = \{a_1, \dots, a_i, \dots, a_m\}$ is the set of *actions* (alternatives).
- $w = (w_1, \dots, w_j, \dots, w_n)$ is the *weight vector* modelling the preferences of the DM. Let us assume that $\sum_{j \in \mathcal{J}} w_j = 1$.
- $g_j(a_i)$ is the *evaluation* of criterion g_j for action a_i .

And the following comprehensive binary relational operators, allowing to compare two alternatives, a and b :

- P is the *strong preference* relation, that is aPb denotes the relation “ a is strongly preferred over b ”.
- I is the *indifference* relation, that is aIb denotes the relation “ a is indifferent to b ”.
- Q is the *weak preference* relation, that is aQb denotes the relation “ a is weakly preferred over b , which means hesitation between indifference and preference.
- R is the *incomparability* relation, that is aRb denotes that action a and b are incomparable.
- S is the *outranking* relation, that is aSb denotes that “ a is at least as good as b ”.
- \succ is the *preference* relation, that is $a \succ b$ denotes that a is preferred (strongly or weakly) over b .

When the relational operator is superscripted (for example, \succ^{Z_1}) it denotes that the relation holds in the set of the superscript. When the relational operator is subscripted (for example, S_j) it denotes that the relation holds with respect to the criterion indexed by the subscript. The thresholds of the ELECTRE III model are denoted as follows:

- q_j is the *indifference threshold* for the criterion g_j .
- p_j is the *preference threshold* for the criterion g_j .
- v_j is the *veto threshold* for the criterion g_j .

These thresholds can also vary (directly or inversely) along the scale of each criterion. In what follows we will consider variable thresholds, the ones that take into account less good performance of the two actions. We consider the two actions a and b to be such, that $g_j(a)$ is less good than $g_j(b)$. Therefore, all thresholds are defined by taking into account $g_j(a)$.

2.1 The construction of an outranking relation

The construction of an outranking relation requires the definition of a *credibility index* for the outranking relation aSb ; let $\rho(a, b)$ denote such an index. It is defined by using both a comprehensive concordance index, $c(a, b)$, and a discordance index for each criterion $g_j \in F$, that is, $d_j(a, b)$, for all $j \in J$.

2.1.1 Computing the concordance of the assertion aSb

The concordance index is computed by considering individually for each criterion g_j the support it provides for the assertion aS_jb . For computing the partial concordance indices, we first define sets for two coalitions of criteria (assuming that all the criteria are to be maximised).

1. concerning the coalition of criteria in which aSb

$$\mathcal{J}^S = \left\{ j \in J : g_j(a) + q_j(g_j(a)) \geq g_j(b) \right\} \quad (1)$$

2. concerning the coalition of criteria in which bQa

$$\mathcal{J}^Q = \left\{ j \in J : g_j(a) + q_j(g_j(a)) < g_j(b) \leq g_j(b) + p_j(g_j(b)) \right\} \quad (2)$$

Using these two sets, we can compute the partial concordance indices $c_j(a, b)$, as follows,

$$c_j(a, b) = \begin{cases} 1 & \text{if } j \in \mathcal{J}^S \\ \frac{g_j(a) + p_j(g_j(a)) - g_j(b)}{p_j(g_j(a)) - q_j(g_j(a))} & \text{if } j \in \mathcal{J}^Q \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Thus $c_j(a, b)$ decreases linearly from 1 to 0, when g_j describes the range

$$[g_j(a) + q_j(g_j(a)), g_j(a) + p_j(g_j(a))].$$

After computing the partial concordance indices, the comprehensive concordance index is calculated as a weighted sum,

$$c(a, b) = \sum_{j \in J} w_j c_j(a, b). \quad (4)$$

2.1.2 Computing the discordance of the assertion aSb

The discordance of a criterion g_j describes the veto effect that the criterion provides against the assertion aS_jb . The discordance indices are computed separately for all criteria. A discordance index reaches its maximal value when criterion g_j puts its veto to the outranking relation; it is minimal when the criterion g_j is not discordant with that relation. To define the value of the

discordance index on the intermediate zone a linear interpolation is used. The partial discordance indices are computed as follows, for all $j \in \mathcal{J}$

$$d_j(a, b) = \begin{cases} 1 & \text{if } g_j(b) > g_j(a) + v_j(g_j(a)) \\ 0 & \text{if } g_j(b) \leq g_j(a) + p_j(g_j(a)) \\ \frac{g_j(b) - g_j(a) - p_j(g_j(a))}{v_j(g_j(a)) - p_j(g_j(a))}, & \text{otherwise} \end{cases} \quad (5)$$

2.1.3 Computing the credibility index (fuzzy outranking relation)

The outranking relation is constructed by defining the credibility of the assertion aSb as follows

$$\rho(a, b) = c(a, b) \prod_{j \in V} \frac{1 - d_j(a, b)}{1 - c(a, b)}, \quad (6)$$

where

$$V = \{j \in \mathcal{J} : d_j(a, b) > c(a, b)\}. \quad (7)$$

Notice that when $d_j(a, b) = 1$, it implies that $\rho(a, b) = 0$.

2.2 The exploitation procedure

The exploitation of the outranking procedure consists of two phases. In the first phase, we need to build two complete pre-orders, descending (Z_1) and ascending (Z_2). In the second phase, the partial pre-order Z is obtained as the intersection of the previous two complete pre-orders.

The complete pre-order Z_1 is defined as a partition of the set A into L ordered classes

$$C_1, \dots, C_l, \dots, C_L,$$

where C_1 is the head class in Z_1 . Each class C_l may be composed of *ex æquo* (ties) elements according to Z_1 . The complete pre-order Z_2 is determined in a similar way, where A is partitioned into K ordered classes

$$E_1, \dots, E_l, \dots, E_K,$$

where E_K is the head class in Z_2 .

2.2.1 The discrimination threshold

The pre-orders are obtained as output from two *distillation procedures*, Z_1 from ascending (upward) distillation, and Z_2 from descending (downward) distillation. The distillations are computed based on a fuzzy credibility index $\rho(a, b)$ describing the credibility of the assertion aSb . However, since the arbitrariness when computing such an index is frequent, in particular due to the linear interpolation involved in that computation, the value of $\rho(a, b)$ is not absolute. Thus, it requires an adjustment. For example, when $\rho(a, b) > \rho(c, d)$ it does not mean that the outranking of a over

b is more credible than the one of c over d . This is why a *discrimination threshold* $s(\lambda)$ should be introduced in order to avoid such a kind of drawbacks. The discrimination threshold $s(\lambda)$ is a function such that

$$\begin{aligned} & \lambda \in [0, 1] \\ & \text{and if} \\ & \rho(a, b) = \lambda \\ & \text{and} \\ & \rho(c, d) = \lambda - \eta \text{ with } \eta > s(\lambda), \end{aligned}$$

we can say that aSb is strictly more credible than cSd . Usually $s(\lambda)$ is defined to be of form

$$s(\lambda) = \alpha + \beta\lambda, \quad (8)$$

where α and β are discrimination parameters.

2.2.2 The cutting levels and the definition of the preference relation

The *cutting levels* λ are used to define the successive λ cuts of the fuzzy relation. The distillation algorithms proceeds by lowering the cutting level λ from an initial value, λ_0 , to zero. After determining the cutting level, λ , we can define the preference relation $a \succ b$ for all pairs $(a, b) \in D \subset A$ at the cutting level λ , as follows:

$$a \succ b \text{ iff } \begin{cases} \rho(a, b) - \rho(b, a) > s(\rho(a, b)) \\ \rho(a, b) > \lambda \end{cases} \quad (9)$$

2.2.3 The qualification of each action

The qualification score $Q(a)$ of each action $a \in D \subset A$ is calculated based on the strength of the action $P(a)$ and the weakness of the action $F(a)$ as follows:

$$\begin{aligned} Q(a) &= P(a) - F(a), \text{ where} \\ P(a) &= |\{b \in A : a \succ b\}|, \text{ and} \\ F(a) &= |\{b \in A : b \succ a\}|. \end{aligned} \quad (10)$$

2.2.4 Construction of the complete pre-orders and the final partial pre-order

Descending distillation. The procedure designed to compute Z_1 starts (first distillation) by defining an initial set $D_0 = A$; it leads to the first final distilled C_1 . After getting C_l , in the distillation $l + 1$, the procedure sets $D_0 = A \setminus (C_1 \cup \dots \cup C_l)$. The actions in class C_l are, according to Z_1 , preferable to those of class C_{l+1} ; for this reason, distillations that lead to these classes are called descendants. The descending distillation procedure is presented in Algorithm 1.

Algorithm 1 The descending algorithm: produces the complete pre-order Z_1

```
1:  $l \leftarrow 0$ 
2:  $A_l \leftarrow A, C_{l+1} \leftarrow \emptyset$ 
3: repeat
4:    $k \leftarrow 0$ 
5:    $D_k \leftarrow A_l$ 
6:    $\lambda_k \leftarrow \max_{(a,b) \in A_l} \{ \rho(a,b) \}$  with  $a \neq b$ 
7:   repeat
8:      $\lambda_{k+1} \leftarrow \max_{(a,b) \in D_k} \{ \rho(a,b) \}$  with  $a \neq b$  and  $\rho(a,b) < \lambda_k - s(\lambda_k)$ 
9:     if  $\rho(a,b) > \lambda_k - s(\lambda_{k+1}), \forall (a,b) \in D_k$  then
10:        $\lambda_{k+1} \leftarrow 0$ 
11:     end if
12:     Compute the qualification  $Q(a)$ , for all  $a \in D_k$ 
13:      $Q^{max} \leftarrow \max_{a \in D_k} \{ Q(a) \}$ 
14:      $D_{k+1} \leftarrow \{ a \in D_k : Q(a) = Q^{max} \}$ 
15:      $k \leftarrow k + 1$ 
16:   until  $|D_k| = 1$  or  $\lambda_k = 0$ 
17:    $C_{l+1} \leftarrow D_k$ 
18:    $A_{l+1} \leftarrow A_l \setminus C_{l+1}$ 
19:    $l \leftarrow l + 1$ 
20: until  $A_l = \emptyset$ 
```

Ascending distillation. The procedure leading to Z_2 is almost similar to descending distillation, but now the actions in class E_{l+1} are preferred to those in class E_l ; these distillations are called ascendants. The ascending distillation procedure is presented in Algorithm 2.

Algorithm 2 The ascending algorithm: produces the complete pre-order Z_2 .

```

1:  $l \leftarrow 0$ 
2:  $A_l \leftarrow A, E_{l+1} \leftarrow \emptyset$ 
3: repeat
4:    $k \leftarrow 0$ 
5:    $D_k \leftarrow A_l$ 
6:    $\lambda_k \leftarrow \max_{(a,b) \in A_l} \{ \rho(a,b) \}$  with  $a \neq b$ 
7:   repeat
8:      $\lambda_{k+1} \leftarrow \max_{(a,b) \in D_k} \{ \rho(a,b) \}$  with  $a \neq b$  and  $\rho(a,b) < \lambda_k - s(\lambda_k)$ 
9:     if  $\rho(a,b) > \lambda_k - s(\lambda_{k+1}) \forall (a,b) \in D_k$  then
10:        $\lambda_{k+1} \leftarrow 0$ 
11:     end if
12:     Compute the qualification  $Q(a)$  for all  $a \in D_k$ 
13:      $Q^{min} \leftarrow \min_{a \in D_k} \{ Q(a) \}$ 
14:      $D_{k+1} \leftarrow \{ a \in D_k : Q(a) = Q^{min} \}$ 
15:      $k \leftarrow k + 1$ 
16:   until  $|D_k| = 1$  or  $\lambda_k = 0$ 
17:    $E_{l+1} \leftarrow D_k$ 
18:    $A_{l+1} \leftarrow A_l \setminus E_{l+1}$ 
19:    $l \leftarrow l + 1$ 
20: until  $A_l = \emptyset$ 

```

The final partial pre-order. The partial pre-order Z will be computed as the intersection of Z_1 and Z_2 . In the final partial pre-order, the following hold:

$$\begin{aligned}
(a \succ^{Z_1} b \wedge a \succ^{Z_2} b) \vee (aI^{Z_1} b \wedge a \succ^{Z_2} b) \vee (a \succ^{Z_1} b \wedge aI^{Z_2} b) &\iff a \succ b \\
(aI^{Z_1} b \wedge aI^{Z_2} b) &\iff aIb \\
(a \succ^{Z_1} b \wedge b \succ^{Z_2} a) \vee (b \succ^{Z_1} a \wedge a \succ^{Z_2} b) &\iff aRb.
\end{aligned}$$

3 The SMAA methodology

The SMAA methodology has been developed for discrete stochastic MCDA problems with multiple DMs. The SMAA-2 method (Lahdelma and Salminen, 2001) applies inverse weight space analysis to describe for each action what kind of preferences make it the most preferred one, or place it on any particular rank. In SMAA, the criteria evaluations can be generated based on arbitrary distributions, or they can be sampled from an external source. Because of the possibility of using samples as input, the SMAA approach is suitable for weight space analysis of all MCDA methods which have weights among the input parameters (for example, ELECTRE methods). We will next introduce a SMAA approach for inverse weight space analysis of ELECTRE III. In this approach the other free parameters (thresholds, cutting levels, ...) of ELECTRE III are considered to be fixed.

3.1 The inverse approach

The weights of ELECTRE III are represented by a weight distribution with joint density function $f_W(w)$ in the feasible weight space W . Total lack of preference information is represented in “Bayesian” spirit by a uniform weight distribution in W , that is,

$$f_W(w) = 1/\text{vol}(W). \quad (11)$$

The weights are non-negative and normalized: the weight space is an $n - 1$ dimensional simplex in n dimensional space:

$$W = \left\{ w \in R^n : w \geq 0 \text{ and } \sum_{j=1}^n w_j = 1 \right\}. \quad (12)$$

The inverse weight space analysis is based on exploring the weight space in order to describe which weights result in certain ranks for the actions. The ranks are defined according to the partial pre-order of ELECTRE III. The rank of an action which is not outranked by any other action in the final partial pre-order is 1, the one that is outranked by one action is 2, etc. We denote by $e_i(\hat{w})$ the function providing the ranking of action a_i with weight vector \hat{w} . The inverse weight space analysis is then based on analyzing the sets of favourable rank weights:

$$W_i^r = \{ \hat{w} \in W : e_i(\hat{w}) = r \}. \quad (13)$$

Any weight $\hat{w} \in W_i^r$ results in such values for different actions, that action a_i obtains rank r . It should be noticed, that the rank information is not equal to the information present in the final partial pre-order of ELECTRE III: we transform the partial pre-order to a complete pre-order, because the rank acceptability indices cannot be computed for a partial pre-order.

3.2 Descriptive measures

The inverse weight space analysis for ELECTRE III produces three descriptive measures: (1) rank acceptability indices, (2) pair-wise winning indices, and (3) central weight vectors.

3.2.1 The rank acceptability index

The rank acceptability index (b_i^r) measures the variety of different weights that grant action a_i rank r . It is the share of all feasible weights that make the action acceptable for a particular rank, and it is most conveniently expressed percentage-wise. In our opinion, this is a particular case of robustness concerns.

The rank acceptability index b_i^r is computed numerically as a multidimensional integral over the favourable rank weights as

$$b_i^r = \int_{w \in W_i^r} f_W(w) dw. \quad (14)$$

The most acceptable (best) actions are those with high acceptabilities for the best ranks. Evidently, the rank acceptability indices are in the range $[0,1]$, where 0 indicates that the action will never obtain a given rank and 1 indicates that it will obtain the given rank always with any choice of weights. Thus, the rank acceptability indices can be interpreted as a some kind of measure of robustness.

3.2.2 The pair-wise winning index

The pair-wise winning index (o_i^k) (introduced by Kangas et al. (2004)) describes the variety of weights that give an action a better rank than another. An action a_i that has $o_i^k = 1$ for some k always obtains a better rank than action a_k , and can thus be said to *dominate* it.

The pair-wise winning index o_i^k is computed numerically as a multidimensional integral over the favourable rank weights as

$$o_i^k = \int_{w \in W: e_i(w) > e_k(w)} f_W(w) dw. \quad (15)$$

From the pair-wise winning indices we can construct a *pair-wise winning matrix*. In the position (i,k) of the matrix is the value o_i^k , so the matrix consists of all pair-wise winning indices.

3.2.3 The central weight vector

The central weight vector (w_i^c) is the expected centre of gravity (centroid) of the favourable first rank weights of an action a_i . The central weight vector represents the preferences of a “typical” DM supporting this action. The central weights of different actions can be presented to the DMs in order to help them understand how different weights correspond to different choices.

The central weight vector w_i^c is computed numerically as a multidimensional integral over the criteria distributions and the favourable first rank weights using

$$w_i^c = \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^1(\xi)} f_W(w) w dw d\xi / a_i. \quad (16)$$

4 Case study: Experiments and results

ELECTRE III has been used to choose the best waste incineration strategy for the Eastern Switzerland region. In this case study, there were 11 alternative strategies (actions) that were evaluated in terms of 11 criteria. There were 6 interest groups whose preferences were taken into account by executing ELECTRE III analysis 6 times, with preference information of the different interest groups. We will not present the complete study here. The interested reader should refer to (Rogers et al., 2000, Sect. 6).

4.1 Results of the original case study

We will re-analyze this case study for the “baseline run” with weight information from the Switzerland's Federal Agency for the Environment. In this initial run, the veto thresholds were not used. The ranking from the “baseline run” is presented in Table 1. The weights were elicited by using two methods, the revised Simos procedure by Figueira and Roy (2002) and the method by Rogers and Bruen (1998a). According to (Rogers et al., 2000), the differences in the weights obtained by using the two methods were minor. To see how small differences in the weights can cause alteration of the results, we have chosen to perform an inverse analysis for this case study by using intervals for weights rather than exact ones.

Table 1: Ranking from the “baseline run” with exact weights.

Rank	Alternatives
1	S3.1, S4.1
2	S2.2
3	S4.2
4	S2.3
5	S1.1, S2.1, S4.3
6	S1.2, S3.2
7	S2.4

4.2 Results of the analysis

We are analyzing the robustness of the case study by performing inverse weight space analysis. The feasible weight space is defined to be the original weights $\pm 10\%$. The names of the criteria, the original weights of the case study, as well as the intervals used in the case study are presented in Table 2.

Table 2: Names, original weights and weight intervals of the criteria.

Criterion	Original weight	Weight interval
C1.1	0.16	[0.144, 0.176]
C1.2	0.033	[0.029, 0.036]
C1.3	0.033	[0.029, 0.036]
C2.1	0.097	[0.087, 0.106]
C2.2	0.097	[0.087, 0.106]
C3.1	0.16	[0.144, 0.176]
C3.2	0.097	[0.087, 0.106]
C3.3	0.16	[0.144, 0.176]
C4.1	0.033	[0.029, 0.036]
C4.2	0.033	[0.029, 0.036]
C4.3	0.097	[0.087, 0.106]

We implemented ELECTRE III and the inverse weight space analysis by using the C++ programming language. We performed the inverse weight space analysis by executing 100000 Monte Carlo simulations, which provides sufficient accuracy for the results (according to Lahdelma et al., 2004b). The resulting rank acceptability indices are presented in Table 3, and the pair-wise winning matrix in Table 4. The central weight vectors are defined only for the two alternatives that obtained the first rank with some weights; they are presented in Table 5.

The effect of variable number of ranks in different simulations can be noticed by looking at the rank acceptability indices in Table 3. For example, the rank acceptabilities of alternatives S1.1 (rank 7) and S1.2 (rank 8) clearly depend on preference of S1.1 over S1.2 in simulation runs. In such situations, the pair-wise winning matrix provides valuable information: by looking at the first row of it (in Table 4), we notice that S1.1 always obtains a better rank than S1.2.

Table 3: Rank acceptability indices of the re-analysis (in %).

Alternative	b_i^1	b_i^2	b_i^3	b_i^4	b_i^5	b_i^6	b_i^7	b_i^8	b_i^9	b_i^{10}	b_i^{11}
S1.1	0	0	0	0	27	22	51	0	0	0	0
S1.2	0	0	0	0	0	27	22	51	0	0	0
S2.1	0	0	1	9	46	43	0	0	0	0	0
S2.2	0	29	71	0	0	0	0	0	0	0	0
S2.3	0	0	0	29	51	20	0	0	0	0	0
S2.4	0	0	0	0	0	0	27	22	51	0	0
S3.1	99	1	0	0	0	0	0	0	0	0	0
S3.2	0	0	0	0	14	23	19	44	0	0	0
S4.1	26	72	2	0	0	0	0	0	0	0	0
S4.2	0	0	34	64	1	0	0	0	0	0	0
S4.3	0	0	0	0	27	18	16	40	0	0	0

Table 4: Pair-wise winning matrix of the re-analysis (in %).

Alternative	S1.1	S1.2	S2.1	S2.2	S2.3	S2.4	S3.1	S3.2	S4.1	S4.2	S4.3
S1.1	-	100	0	0	0	100	0	78	0	0	44
S1.2	0	-	0	0	0	100	0	0	0	0	0
S2.1	63	100	-	0	30	100	0	79	0	0	63
S2.2	100	100	100	-	100	100	0	100	2	92	100
S2.3	100	100	70	0	-	100	0	100	0	1	100
S2.4	0	0	0	0	0	-	0	0	0	0	0
S3.1	100	100	100	100	100	100	-	100	74	100	100
S3.2	8	22	0	0	0	100	0	-	0	0	8
S4.1	100	100	100	97	100	100	1	100	-	100	100
S4.2	100	100	96	0	99	100	0	100	0	-	100
S4.3	0	56	0	0	0	100	0	42	0	0	-

Table 5: Central weight vectors of the re-analysis (in %).

Alternative	C1.1	C1.2	C1.3	C2.1	C2.2	C3.1
S3.1	16.09	3.25	3.25	9.68	9.68	16.09
S4.1	16.31	3.28	3.25	10.07	9.65	16.03
	C3.2	C3.3	C4.1	C4.2	C4.3	
	9.68	16.09	3.25	3.25	9.68	
	9.22	16.04	3.25	3.25	9.65	

The results of our analysis are the following:

1. On the original case study, alternatives S3.1 and S4.1 shared the best rank. Their study based on analyzing 6 different sets of weights led to recommend S4.1 as the primary choice, and S3.1 as the secondary choice. By looking at the rank acceptability indices in Table 3, it can be noticed that both of these alternatives are good candidates, if we want to separate the best two. But based on our inverse analysis, alternative S4.1 seems not the most adequate to “recommend” as the most favourable option, because its rank acceptability index for the first rank is only 26%. And with 99% of these weights, it shares the first rank with alternative S3.1. Likewise, by looking at the pair-wise winning matrix in Table 4, we notice that not only it is often (74% of the feasible weights) ranked lower than S3.1, but sometimes (2% of the feasible weights) even lower than S2.2.
2. On the other hand, S3.1 obtains lower rank than S4.1 with only 1% of the feasible weights, and is always ranked higher than the other alternatives (excluding S4.1). Thus, a possible robust conclusion in this example would not be to “recommend” S4.1, because even small variations in the weights drop it below S3.1 in the ranking. Our conclusion would have been to select S3.1, and S4.1 as a “back-up” strategy, if for some reason S3.1 could not have been chosen.

The authors of the case study also describe results from a robustness analysis point of view. But in their study, the robustness is analyzed by altering only a single weight at a time, which is more like a sensitivity analysis. A different way to analyze robustness in ELECTRE III should always be considered by looking at the whole feasible weight space, otherwise nonlinearity of the ranking function, which ELECTRE III represents, can produce surprising results.

5 Conclusions and avenues for future research

In this paper, we have presented a novel method of inverse weight-space analysis for ELECTRE III. We presented a re-analysis of a case study showing the usefulness of the method. Our method allows the weights of ELECTRE III to be of arbitrary type: no deterministic weights are required. This has numerous advantages, especially in the context of MCDA with multiple DMs, because the weights can be determined as intervals which contain the preferences of all DMs.

The inverse weight space analysis can be applied to a wide range of MCDA methods. In particular, it can be applied to PROMETHEE and other outranking methods for ranking problem statements (see Figueira et al., 2005). Our method is the first allowing inverse analysis to be performed on ELECTRE III. As such, its usefulness to real-life cases should be studied. Re-analyses should be done by analysts who originally performed the analysis with deterministic weights. We hope that the presented method will be applied by researchers, analysts, and engineers alike, to achieve robust conclusions when applying ELECTRE III to MCDA problems.

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