Instituto de Engenharia de Sistemas e Computadores de Coimbra
INESC - Coimbra

Rita Girão-Silva, José Craveirinha, João Clímaco

Hierarchical Multiobjective Routing in MPLS Networks with Two Service Classes – Report on a Pareto Archived Strategy

18/2009 Dec. 2009

Instituto de Engenharia de Sistemas e Computadores de Coimbra
INESC - Coimbra
Rua Antero de Quental, 199; 3000-033 Coimbra; Portugal
`www.inescc.pt`

Hierarchical Multiobjective Routing in MPLS Networks with Two Service Classes - Report on a Pareto Archived Strategy*

Rita Girão-Silva^{a,c}, José Craveirinha^{a,c}, João Clímaco^{b,c}

^{a} Department of Electrical Engineering Science and Computers of the University of Coimbra
Pólo II, Pinhal de Marrocos; P-3030-290 Coimbra; Portugal
^{b} Faculty of Economics of the University of Coimbra
Av. Dias da Silva, 165; P-3004-512 Coimbra; Portugal
^{c} Institute of Computers and Systems Engineering of Coimbra (INESC-Coimbra)
R. Antero de Quental, 199; P-3000-033 Coimbra; Portugal

Tel.: +351 239 796252; Fax: +351 239 796247
`{rita,jcrav}@deec.uc.pt; jclimaco@fe.uc.pt`

Abstract

The report begins by reviewing a two-level hierarchical multicriteria routing model for MPLS networks with two service classes (QoS and Best Effort services) and alternative routing, as well as the foundations of a heuristic resolution approach, previously proposed by the authors. Afterwards a new variant of this heuristic approach, which includes a Pareto archive strategy (with the archive containing solutions found throughout the execution of the heuristics), is described. The application of the developed procedure to a benchmarking case study will show, by using analytic and simulation models that, in certain initial conditions, this approach provides improvements in the final results especially in more 'difficult' situations detected through sensitivity analysis.

Keywords Routing models, Multiobjective optimisation, Telecommunication networks, MPLS-Internet.

1 Introduction and Motivation

In modern multiservice networks, multiple and heterogeneous QoS (Quality of Service) routing requirements have to be taken into account. Therefore, the routing models designed to calculate and select one (or more) sequences of network resources (routes) have to satisfy certain QoS constraints and seek the optimisation of route related objectives. The formulation of important routing problems in these types of networks as multiple objective optimisation problems is potentially advantageous, as these multiple objective formulations allow the trade-offs among distinct performance metrics and other network cost function(s) (potentially conflicting) to be pursued in a consistent manner.

QoS issues have become increasingly relevant in the new technological platforms of multiservice networks, triggering an interest in the application of multicriteria approaches

---

[*] A shorter version was presented at the 8th International Conference on Decision Support for Telecommunications and Information Society (DSTIS 2009), Coimbra, Portugal, Sep. 4-7 2009.

to routing models in communication networks. In [27], an in-depth methodological analysis of issues raised by the use of multicriteria analysis in telecommunication network design and their relation with knowledge theory models is presented. The application of multicriteria models in telecommunication network design and routing problems are reviewed in [2, 4].

In particular, a significant number of multicriteria routing models has been proposed in the context of the emergent MPLS (Multiprotocol Label Switching) Internet networks (see [4]). The technical capabilities of MPLS networks allow for the implementation of multiple connection-oriented services with QoS (Quality of Service) requirements [1, 19]. Advanced QoS-based routing mechanisms are employed in the MPLS environment and "explicit routes" (i.e. routes completely determined at the originating node) for each traffic flow of a given service type may be devised. The authors have discussed key methodological and modelling issues associated with route calculation and selection in MPLS networks in [5].

A meta-model for hierarchical multiobjective network-wide routing optimisation in MPLS networks has also been presented in [5]. In this optimisation approach, two types of traffic flows are considered, QoS and BE (Best Effort) type flows. QoS flows are regarded as first priority flows and, when accepted by the network, have a guaranteed QoS level, related to the required bandwidth. As for BE flows, they are considered in the model as second priority flows, and are carried by the network in order to obtain the best possible QoS level. The traffic flows in the network are represented through an approximate stochastic model, based on the use of the concept of effective bandwidth for macro-flows and on a generalised Erlang model for estimating the blocking probabilities in the arcs, as in the model used in [23, 21].

In this hierarchical model, described in detail in [6, 7], the first priority objective functions concern network level objectives of QoS type flows, namely the total expected revenue and the maximal value of the mean blocking of all types of QoS traffic flows; the second priority objective functions are related to performance metrics for the different types of QoS services and the total expected revenue for the BE traffic flows.

An alternative routing principle is incorporated in the routing model, so that when a first choice route (corresponding to a loopless path) assigned to a given micro-flow[1], in a specific traffic flow (corresponding to a MPLS "traffic trunk") is blocked a second choice route may be attempted.

The theoretical foundations of a specialised heuristic strategy for finding "good" compromise solutions to the very complex bi-level routing optimisation problem, were presented in [6]. Note that in multiobjective optimisation [25] the concept of optimal solution is replaced by the concept of non-dominated (or Pareto optimal) solution. A non-dominated solution is a feasible solution such that, in minimisation problems, it is not possible to decrease the value of an objective function without increasing on, at least, the value of one of the other objective functions.

In [15], a heuristic approach (HMOR-S2 or Hierarchical Multiobjective Routing considering 2 classes of Service) devised to find "better" solutions (in the sense of multiobjective optimisation) to this hierarchical multiobjective routing optimisation problem, was proposed and applied to a test network used in a benchmarking case study, for various traffic matrices. In this dedicated heuristic, each new solution is obtained by 'processing' the current best solution. The specific paths which seem to be more adequate for

---

[1] A μ-flow corresponds in our model to a 'call', that is, a node to node connection request with certain traffic engineering features.

improvement are selected and a variation on these paths is performed, leaving the others unaltered. The new solution provided by this selection and by variation schemes on the current solution, is then analysed and its quality is evaluated.

In [14], sensitivity tests applied to the specialised heuristic were described. These sensitivity tests have shown that the heuristic is balanced in the treatment of the different objective functions. However, they have also shown that in some rare cases there was the potential for some improvement(s) in the first level objective functions, that is, the heuristic was not capable of finding a solution that slightly dominates the current solution. Therefore, new approaches have been devised to seek "better" solutions to the routing problem under analysis, seeking to make the most of the knowledge acquired with the problem by previous experimentation with the specialised heuristic HMOR-S2 and aiming to overcome possible limitations of this heuristic detected through the sensitivity analysis. Two new approaches consisting of the introduction of meta-heuristic techniques, namely a SA (Simulated Annealing) and a TS (Tabu Search) technique, in the structure of the basic heuristic were presented in [14]. The introduction of these techniques is advantageous in the search for further improvements of the final solution obtained with the basic heuristic. However, these variants add a greater complexity and computational cost to the basic heuristic.

This work presents a new resolution procedure for this model based on the introduction of a Pareto archive in the basic heuristic. Also computational experiments using an analytical model and stochastic simulation will be presented, in order to evaluate the performance of the proposed heuristic in a benchmarking case study. This heuristic strategy is inspired in one of the standard procedures used in Pareto archived evolutionary metaheuristics (see [18]). This procedure aims at finding even "better" solutions to the above hierarchical multiobjective routing optimisation problem, by incorporating an archive of non-dominated solutions obtained throughout the execution of the previously developed heuristic resolution approach. The addition and removal of solutions from the archive follow a certain set of rules, described in 3.2, which rely on a specific model of the system of preferences, to be implemented in an automated manner. This system of preferences relies on the definition of aspiration and reservation thresholds for the two network level objectives of QoS type flows, which leads to the definition of preference regions in the objective function space and allows us to compare all the calculated non-dominated solutions. Note that this technique is used as an auxiliary procedure, while the basic mechanisms of the dedicated heuristic are maintained. At the end of the algorithm, all the solutions in this archive are scrutinised and the "best" possible solution in the best possible preference region is chosen to be the actual solution to the routing problem, using a reference point-based procedure (see [26]) as the solution selection mechanism.

The report is organised as follows. The two-level hierarchical multiobjective alternative routing model with two service classes is reviewed in section 2, together with the basis of the dedicated heuristic. In the third section, the features of the variant of the basic heuristic, in particular an archive of non-dominated solutions, are presented. The results obtained with this procedure, by using analytic results and discrete-event simulations for a test network used in a benchmarking study, are revealed in the fourth section. Finally, conclusions are drawn and future work is outlined in the fifth section.

2 Review of the Multiobjective Routing Model

2.1 The Multiobjective Routing Model

The considered model is an application of the multiobjective modelling framework (or "meta-model") for MPLS networks proposed in [5], as previously mentioned. Two classes of services are considered: QoS, that is services with guaranteed QoS levels (when accepted by the network), and BE, corresponding to traffic flows that are routed with the best possible quality of service but not at the cost of deteriorating the QoS of the QoS traffic flows. Therefore, QoS flows are treated as first priority traffic flows. The different service types of each class are represented by the sets $\mathcal{S}_Q$ (for QoS service types) and $\mathcal{S}_B$ (for BE service types). The traffic flows of each service type $s \in \mathcal{S}_Q$ or $s \in \mathcal{S}_B$ may differ in important attributes, in particular the required bandwidth.

In the model the network is represented through a capacitated directed graph, where a capacity C_k is assigned to every arc (or 'link') l_k, and the traffic flows are represented in a stochastic form, as shown in [5]. A traffic flow is specified by $f_s = (v_i, v_j, \overline{\gamma}_s, \overline{\eta}_s)$ for $s \in \mathcal{S} = \mathcal{S}_Q \cup \mathcal{S}_B$ and a stochastic process is assigned to it, that is in general, a marked point process. The process describes the arrivals and basic requirements of micro-flows, originated at the MPLS ingress node v_i and destinated to the MPLS egress node v_j, using some LSP (Label Switched Path). The other features of the traffic flow are characterised by the vectors of traffic engineering attributes of flows of service type s, $\overline{\gamma}_s$, and by the vectors containing the description of mechanism(s) of admission control to all arcs l_k in the network by calls of flow f_s, $\overline{\eta}_s$. In particular these attributes include information on the required *effective bandwidth* d_s and the mean duration $h(f_s)$ of each μ-flow in f_s. The use of the concept of effective bandwidth (a concept developed in [16]) in the present context (MPLS networks with explicit routes) was earlier considered by [23] and in [21]. The effective bandwidth can be viewed as a stochastic measure of the utilisation of network resources allowing for an approximate, although effective, representation of the effects of the variability of the rates of traffic sources of different types, as well as the effects of statistical multiplexing of different traffic flows in a network.

The hierarchical multiobjective routing optimisation model considered here has two levels with several objective functions in each level. The first level includes the first priority objective functions (the total expected network revenue associated with QoS traffic flows, W_Q, and the worst average performance among QoS services, represented by the maximal average blocking probability among all QoS service types, $B_{Mm|Q}$), which are formulated at the network level for the QoS traffic. In the second level the objective functions are concerned with average performance metrics of the QoS traffic flows associated with the different types of QoS services (represented by the mean blocking probabilities for flows of type $s \in \mathcal{S}_Q$, $B_{ms|Q}$, and the maximal blocking probability $B_{Ms|Q}$, defined over all flows of type $s \in \mathcal{S}_Q$) as well as the total expected network revenue associated with BE traffic flows, W_B. These constitute the second priority objective functions. At the two levels of optimisation, 'fairness' objectives are explicitly considered in the form of min-max objectives.

Therefore the considered two-level hierarchical optimisation problem for two service classes P-M2-S2 ('**P**roblem - **M**ultiobjective with **2** optimisation hierarchical levels - with **2** **S**ervice classes') is:

Problem P-M2-S2

$$\begin{array}{|lll|}\hline
\text{1st level} & \left\{\begin{array}{l}\text{QoS: Network objectives}\\ \\ \end{array}\right. & \begin{array}{l}\min_{\overline{R}}\{-W_Q\}\\ \min_{\overline{R}}\{B_{Mm|Q}\}\end{array}\\
\text{2nd level} & \left\{\begin{array}{l}\text{QoS: Service objectives}\\ \\ \\ \text{BE: Network objectives}\end{array}\right. & \begin{array}{l}\min_{\overline{R}}\{B_{ms|Q}\}\\ \min_{\overline{R}}\{B_{Ms|Q}\}\\ \forall s\in\mathcal{S}_Q\\ \min_{\overline{R}}\{-W_B\}\end{array}\\
\multicolumn{3}{|c|}{\text{subject to equations of the underlying traffic model.}}\\ \hline
\end{array}$$

The decision variables $\overline{R} = \cup_{s=1}^{|\mathcal{S}|} R(s)$ represent the network routing plans, that is, the set of all the feasible routes (i.e. node to node loopless paths) for all traffic flows, with $R(s) = \cup_{f_s \in \mathcal{F}_s} R(f_s), s \in \mathcal{S}_Q \cup \mathcal{S}_B$ and $R(f_s) = (r^p(f_s)), p = 1, \cdots, M$ with $M = 2$ in our model. An alternative routing principle is used: for each flow f_s the first choice route $r^1(f_s)$ will be used; if it is blocked the routing method makes the connection request attempt the second choice route $r^2(f_s)$. A request will be blocked only if $r^2(f_s)$ is also blocked.

This routing optimisation approach is of network-wide[2] type, and it enables a full representation of the relations between the objective functions, taking into account the interactions between the multiple traffic flows associated with different services.

The very high complexity of the routing problem P-M2-S2 stems from two major factors: all objective functions are strongly interdependent (via the $\{B(f_s)\}$), and all the objective function parameters and (discrete) decision variables $\overline{R}$ (network route plans) are also interdependent. Note that all these interdependencies are defined explicitly or implicitly through the underlying traffic model. Also note that even in the simplest degenerated case (single service with single-criterion optimisation and no alternative routing) the problem is NP-complete in the strong sense, as proved in [11]. Having in mind the form of P-M2-S2, one may conclude on the great intractability of this problem. There are possible conflicts between the objective functions in P-M2-S2, because in many routing situations, the maximisation of W_Q leads to a deterioration on some $B(f_s), s \in \mathcal{S}_Q$, for certain traffic flows $A(f_s)$ with low intensity, and this tends to increase $B_{Ms|Q}$ and, consequently, $B_{Mm|Q}$. This is a major factor to justify the interest and potential advantage in using multiobjective approaches in this context.

The basic teletraffic sub-model allows for the blocking probabilities B_{ks}, for micro-flows of service type s in link l_k, to be given in the form

$$B_{ks} = \mathcal{B}_s \left(\overline{d_k}, \overline{\rho_k}, C_k\right)$$

where $\mathcal{B}_s$ represents the basic function (implicit in the teletraffic analytical model) that expresses the marginal blocking probabilities, B_{ks}, in terms of $\overline{d_k} = (d_{k1}, \cdots, d_{k|\mathcal{S}|})$ (vector of equivalent effective bandwidths for all service types), $\overline{\rho_k} = \left(\rho_{k1}, \cdots, \rho_{k|\mathcal{S}|}\right)$ (vector of reduced traffic loads ρ_{ks} offered by flows of type s to l_k) and the link capacity C_k. This type of approximation was suggested in [23] for off-line single-objective multiservice routing optimisation models and was also used in the multiobjective dynamic alternative routing model proposed in [21]. It enables the calculation of $\{B_{ks}\}$ through efficient and robust numerical algorithms, which are essential in a network-wide routing optimisation model of this type, for tractability reasons.

A teletraffic model, underlying the routing model, enables the calculation of node to node blocking probabilities $B(f_s)$ for all flows f_s of all service types, from which the

---

[2] This means in this context that the main objective functions of a given service class depend explicitly on all traffic flows in the network.

average blocking probability B_{ms}, for all traffic flows of type s, can be estimated for a given set of routes for all offered traffic flows. The maximal average blocking probability among all QoS service types, $B_{Mm|Q}$, is

$$B_{Mm|Q} = \max_{s \in \mathcal{S}_Q} \{B_{ms}\}$$

This will represent the fairness objective at the network level, as a first priority objective function.

The total expected network revenues, W_Q and W_B associated with QoS and BE traffic flows, respectively, are expressed in terms of the expected revenues $w(f_s)$ per call of flow f_s, and of the values of carried traffic A_s^c, for all service types:

$$W_{Q(B)} = \sum_{s \in \mathcal{S}_{Q(B)}} W_s = \sum_{s \in \mathcal{S}_{Q(B)}} A_s^c w_s$$

The usual simplification, $w(f_s) = w_s, \forall f_s \in \mathcal{F}_s$, where $\mathcal{F}_s$ is the set of traffic flows of type s, will be considered. Note that in the formulation of P-M2-S2 while W_Q is a first priority objective function (together with $B_{Mm|Q}$), W_B will be a second level objective function. This guarantees that the routing of BE traffic, in a quasi-stationary situation, will not be made at the cost of the decrease in revenue or at the expense of an increase in the maximal blocking probability of QoS traffic flows. Nevertheless, it is important to note that while QoS and BE traffic flows are treated separately in terms of objective functions so as to take into account their different priority in the routing optimisation, the interactions among all traffic flows are fully represented in the model. This is assured by the used traffic modelling approach, underlying the optimisation model, because the traffic model used to obtain the blocking probabilities $B(f_s)$ integrates the contributions of all traffic flows which may use every link of the network. This feature is a major difference in comparison with more common routing models that have been proposed for networks with two service classes, based on some form of decomposition of the network representation, corresponding to 'virtual networks', one for each service class.

Apart from the BE expected revenue, the second level of optimisation also includes $2|\mathcal{S}_Q|$ objective functions related to all QoS service types, the mean blocking probabilities for flows of type $s \in \mathcal{S}_Q$,

$$B_{ms|Q} = \frac{1}{A_s^o} \sum_{f_s \in \mathcal{F}_s} A(f_s) B(f_s)$$

where A_s^o is the total traffic offered by flows of type s and $A(f_s)$ is the mean traffic offered associated with f_s (in Erlang), and the maximal blocking probability $B_{Ms|Q}$, defined over all flows of type $s \in \mathcal{S}_Q$,

$$B_{Ms|Q} = \max_{f_s \in \mathcal{F}_s} \{B(f_s)\}$$

This function constitutes the fairness objective defined for every service type $s \in \mathcal{S}_Q$.

2.2 Basis of the Heuristic Approach

The resolution (in a multicriteria analysis sense) of the routing problem P-M2-S2 was earlier performed by a heuristic procedure in [15], which is briefly reviewed in this section. This heuristic uses the theoretical foundations described in [6] and it is based on the recurrent calculation of solutions to a constrained bi-objective shortest path problem, formulated for every end-to-end flow f_s:

$$\text{Problem } \mathcal{P}_{s2}^{(2)} : \min_{r(f_s)\in\mathcal{D}(f_s)} \left\{ m^n(r(f_s)) = \sum_{l_k\in r(f_s)} m_{ks}^n \right\}_{n=1;2} \tag{1}$$

The path metrics m^n to be minimised are the marginal implied costs $m_{ks}^1 = c_{ks}^{Q(B)}$ (the definition of which is reviewed in the following analysis) and the marginal blocking probabilities $m_{ks}^2 = -\log(1 - B_{ks})$; $\mathcal{D}(f_s)$ is the set of all feasible loopless paths for flow f_s, which satisfy specific traffic engineering constraints (other than the effective bandwidth) for flows of type s. A typical constraint is a maximal number of arcs per path depending on the class and type of service s. The logarithmic function is just used to transform the blocking probability into an additive metric. With these path metrics, the comparison of the efficiency of different candidate routes takes into account both the loss probabilities experienced along the candidate routes and the knock-on effects upon the other routes in the network, effects associated with the acceptance of a call on that given route. Also note that the minimisation of the metric blocking probability tends, at a network level, to minimise the maximal node-to-node blocking probabilities $B(f_s)$, while the minimisation of the metric implied cost tends to maximise the total average revenue W_T in a single class multiservice loss network (see [10, 22]).

The implied cost c_{ku} (resulting from the acceptance of a call of flow f_u in link l_k) is an important mathematical concept in routing optimisation in loss networks which was originally proposed in [17] for single-rate traffic networks and later extended to single route multirate traffic networks in [12, 23]. In [9], the definition of c_{ku} was adapted to multirate loss networks with alternative routing and in [6], it was further extended to a multi-rate network with alternative routing and two service classes. Therefore, the *marginal implied cost* for QoS(BE) traffic, $c_{ku}^{Q(B)}$, associated with the acceptance of a connection (or "call") of traffic f_u of any service type $u \in \mathcal{S}$ on a link l_k, as defined by the authors in [6], is the expected value of the traffic loss induced on all QoS(BE) traffic flows resulting from the capacity decrease in link l_k. These costs can be obtained by solving the system of implicit non-linear equations (3.2) in [15].

In the heuristic, the auxiliary constrained shortest path problem $\mathcal{P}_{s2}^{(2)}$ (1) is solved by the algorithm MMRA-S2 [6], an adaptation of MMRA-S (Modified Multiobjective Routing Algorithm for multiservice networks), described in [9, 21]. Generally, there is no feasible solution which minimises the two objective functions simultaneously. Hence, the resolution of this routing problem aims at finding a 'best' compromise path from the set of non-dominated solutions, according to a system of preferences embedded in the working of the algorithm MMRA-S2. This is implemented by defining preference regions in the objective function space obtained from aspiration and reservation levels (preference thresholds) defined for the two objective functions [10, 22]. Further details on this algorithmic approach can be seen in [21]. Note that the path computation and selection are fully automated.

Another important part of the addressed routing model is the underlying traffic model. This stochastic traffic model involves all the sub-models and associated numerical procedures, that are needed for obtaining all traffic related parameters, namely implied costs and blocking probabilities B_{ks} and $B(f_s)$, under certain simplifying assumptions. A description of the traffic modelling approach used in the routing model can be seen in [5].

Now let us review the basic features of the dedicated heuristic HMOR-S2, taken as the starting point and reference procedure in the present work.

In the heuristic, a basic searching strategy is to seek routing solutions $\overline{R}(s)$ for each service $s \in \mathcal{S}$, firstly for each QoS service and starting from the services with higher effective bandwidth (considering the numbering of s, $s = 1, \cdots, |\mathcal{S}_Q|$) and, secondly, for each BE service (also beginning by the higher bandwidth services, $s = |\mathcal{S}_Q| + 1, \cdots, |\mathcal{S}|$), solutions which dominate the current one, in terms of $B_{ms|Q}$ and $B_{Ms|Q}$ for $s \in \mathcal{S}_Q$ and in terms of W_B for $s \in \mathcal{S}_B$. These solutions will only be accepted if they do not lead to the worsening of any of the network functions W_Q and $B_{Mm|Q}$, therefore respecting the hierarchy of objective functions.

The general rules for the generation and selection of candidate solutions $(r^1(f_s), r^2(f_s))$ by MMRA-S2 for each f_s, are described in [6]. These rules take into account the network topology and the need to make a distinction between real time and non-real time QoS services, and BE services. In particular, for the candidate second choice routes $r^2(f_s)$ for QoS or BE traffic, a special procedure is used. In order to prevent performance degradation in overload conditions, some alternative routes should be eliminated in certain conditions. This is achieved through a mechanism designated as Alternative Path Removal (APR), an adaptation of the mechanism originally proposed in [22, 21].

The theoretical analysis of the model, confirmed by experimentation, showed that an instability phenomenon may arise in the path selection procedure, expressed by the fact that the route sets $\overline{R}$ (obtained by successive application of MMRA-S2 to every flow f_s) often tend to oscillate between certain solutions some of which may lead to poor global network performance under the prescribed metrics. Therefore, the successive application of MMRA-S2 to every traffic flow does not lead to an effective resolution approach to the network routing problem P-M2-S2. Detailed analysis and extensive experimentation with the heuristic led to the proposal of a criterion for choosing candidate paths for possible routing improvement by increasing order of a function $\xi(f_s)$ of the current $(r^1(f_s), r^2(f_s))$, given in [15]. The aim of $\xi(f_s)$ is to give preference (concerning the potential value in changing the routes when seeking to improve W_Q or W_B) to the flows for which the route $r^1(f_s)$ has a low implied cost and the route $r^2(f_s)$ has a high implied cost or to the flows which currently have worse end-to-end blocking probability. Note that a variable $nPaths$, representing the number of routes with smaller values of $\xi(f_s)$ that should possibly be changed by running MMRA-S2 once again, is specified in the heuristic.

The full description and formalisation of this heuristic as well as an application study are given in [15].

3 Variant of the Basic Heuristic Approach

The study of the heuristic approach HMOR-S2, the basis of which was reviewed in the previous section, was completed with a sensitivity analysis, which led to the consideration of variants of this heuristic.

3.1 Review of the Sensitivity Analysis and of the Variants of the Basic Heuristic Approach

In [13, 14], the sensitivity tests applied to the HMOR-S2 heuristic were described. The purpose of these sensitivity tests was to check whether the heuristic was treating the lower level objective functions in a balanced way (that is, to check whether better values of the second level objective functions could be obtained without worsening the values of the first level objective functions) and to check whether the value of an upper level objective function could be improved at the cost of worsening the value of the other upper level objective function. Generally speaking, the results of the sensitivity tests for the HMOR-S2 heuristic were as expected, allowing us to assume that the heuristic is balanced in the treatment of the different objective functions.

There were, however, a few results that were worth further analysis. In particular, there was one test where one of the first level objective functions improved and the other worsened. This result is not unexpected, as the two first level objective functions are conflicting in nature, but showed that there was at least one non-dominated solution that the basic heuristic had not been able to detect. Another test where both first level objective functions slightly improved suggests that, in some rare cases, the heuristic is not capable of finding a solution that slightly dominates the current selected solution.

In [14], two new approaches consisting of the introduction of meta-heuristic techniques, namely a SA technique and a TS technique, in the structure of the basic heuristic HMOR-S2 were described. The results show that the introduction of these meta-heuristic techniques in the specialised basic heuristic, does not necessarily lead to better results. However, the introduction of these techniques is advantageous in the search for improvements of the final solution obtained with the basic heuristic. In fact, a run of the basic heuristic HMOR-S2 followed by a further run of either the variants tends to provide improved results for the routing problem, especially in the case of the TS variant. Nonetheless, these variants have added a greater complexity to the basic heuristic. The computational burden of the resolution has also increased. These remain the major limitations of this type of routing method and prompt a further search for possible simplifications and improvements in the heuristic resolution approaches.

3.2 Application of a Pareto Archived Strategy to the Basic Heuristic Approach

The realisation that throughout the execution of the basic heuristic there were interesting solutions to the routing problem that were not further pursued due to the strict limitations imposed on the acceptance of a new solution[3] motivated the development of a new variant that could store these possibly interesting solutions and later go through them in order to try and find the "best" possible solution to the problem in hand, as described below.

The idea for this variant stems from PAES (Pareto Archived Evolution Strategy), an evolutionary algorithm applied to an offline routing problem, which is presented in [18]. The pseudocode for PAES, as described in this reference, is

```
I. Generate an initial random solution $c$;
II. Add $c$ to the archive;
III. Repeat
    Mutate $c$ to produce $m$;
    Evaluate $m$;
    1. If $c$ dominates $m$,
        (a) Discard $m$.
    2. Else
```

---

[3] As mentioned in 2.2, a new solution is accepted only if it has values better than the best found so far for the objective functions of interest: the network functions W_Q and $B_{Mm|Q}$, $\forall s \in \mathcal{S}$, and $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$, or W_B, $s \in \mathcal{S}_B$.

```
    (a) If $m$ dominates $c$,
        i. Replace $c$ with $m$;
        ii. Add $m$ to the archive.
    (b) Else (no solution $m$ or $c$ dominates the other)
        i. If $m$ is dominated by any member of the archive,
            • Discard $m$.
        ii. Else
            • Apply test($c, m$, archive) to determine which becomes the new current solution
              and whether to add $m$ to the archive.
Until a termination criterion has been reached.
```

There are significant differences between the PAES and the proposed variant of the basic heuristic implemented, the HMOR-S2$_{\text{PAS}}$ or HMOR-S2 with a Pareto Archived Strategy.

- The initial solution in HMOR-S2 and HMOR-S2$_{\text{PAS}}$ is not random, but obtained after the typical solution used in Internet routing conventional algorithms (see 4.2).

- The new solution m is not produced by a mutation of c. This is the evolutionary feature in PAES, which is not applicable here. In fact, the new solution in HMOR-S2 and HMOR-S2$_{\text{PAS}}$ stems from the current best solution in the specialised heuristic and is obtained by the recurrent calculation of solutions to the constrained bi-objective shortest path problem $\mathcal{P}_{s2}^{(2)}$, formulated for every end-to-end flow f_s. As explained in 2.2, the resolution of this auxiliary problem aims at finding a 'best' compromise path from the set of non-dominated solutions, according to a specific system of preferences.

- Under the conditions established in III.2(b)ii of PAES, a test function is used to determine which becomes the new current solution and whether to add m to the archive. In HMOR-S2$_{\text{PAS}}$, under conditions similar to these, that is, the new solution has some values better and others worse than the current best values of the objective functions of interest for the service under scrutiny (the first level objective functions and the second level objective functions $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$, or W_B, $s \in \mathcal{S}_B$) and is not dominated by any member of the archive with respect to the objective functions of interest for the service under scrutiny, the new solution does not replace the current solution. Plus, if the archive is not full, the new solution is always added to it; if it is full, further testing is required to decide if the solution should be added to the archive. This test is described next.

- In HMOR-S2$_{\text{PAS}}$, after the end of the outer cycle of the procedure, the solutions stored in the archive are analysed and the "best" possible solution to the problem in hand is finally found. The solution selection mechanism uses a reference point-based procedure.

The major features of the new heuristic, HMOR-S2$_{\text{PAS}}$, are described next and its formalisation is in Appendix A. Its numerical complexity is the same as the one for HMOR-S2 (see [13]). The differences between this heuristic and the basic heuristic are related to the management of the archive, that is, addition and removal of solutions from the archive, and the evaluation of the solutions stored in the archive after the end of the outer cycle of the algorithm, in order to choose the "best" possible solution to the problem under analysis.

3.2.1 Test to determine whether a new solution should be added to the archive

At the beginning of the algorithm (before the outer cycle) the initial solution is added to the archive. Afterwards, new solutions may be added to the archive, whenever:

1. the new solution $\overline{R}_a$ has value(s) better than the current best value(s) of the network functions W_Q and $B_{Mm|Q}$ and of the objective functions of interest for the service under scrutiny, $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$, or W_B, $s \in \mathcal{S}_B$. In this situation, the new solution is always added to the archive and if it is already full, one of the archived solutions must be removed prior to the addition of the new solution $\overline{R}_a$.

2. the new solution $\overline{R}_a$ has some value(s) better and other(s) worse than the current best values of the network functions W_Q and $B_{Mm|Q}$ and of the objective functions of interest for the service under scrutiny, $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$, or W_B, $s \in \mathcal{S}_B$. In this situation, the new solution is added to the archive if it is not dominated by any of the solutions in the archive (i.e., there is no solution in the archive with better values for the first level objective functions and better values for the objective functions of interest for the service under scrutiny) and

   (a) if the archive is not full, or

   (b) if the evaluation of the new solution and the solutions in the archive shows that one of the archived solutions should be removed so as to allow for the addition of $\overline{R}_a$ to the archive, in certain conditions described below.

3.2.2 Test to determine which solution should be removed from the archive

As mentioned earlier, it may be necessary to remove an archived solution to make room for the new solution. To select the solution to be removed, preference thresholds defined in the objective function space are employed. The main features of this approach are: i) the representation of QoS requirements through *requested* (or aspirational) and *acceptable* (or reservation) thresholds for each network function W_Q and $B_{Mm|Q}$; ii) the consideration of this type of thresholds defines priority regions in the bidimensional objective function space in which non-dominated solutions may be searched for. As an example of the definition of priority regions in the bidimensional objective function space of the solutions in the archive, see figure 1.

The ideal optimum is represented by $O*$ and is obtained when both first level objective functions W_Q and $B_{Mm|Q}$ are optimised. The point $(B^{log}_{Max}; W_{min})$ is the Nadir point, that is, the point with the worst values for each objective function when the other objective function is optimised. The first priority region A is defined by the points for which the requested levels are satisfied for both objective functions. The second priority regions B_1 and B_2 are those for which only one of the requested values is satisfied and an acceptable value is guaranteed for the other metric. Although regions B_1 and B_2 have the same priority in theory, in practice the maximisation of the QoS traffic revenue is considered more important than the minimisation of $B_{Mm|Q}$. Therefore, B_2 will be considered preferable to B_1. A third priority region C, where only acceptable values are guaranteed for both metrics, is defined. Beyond the acceptable values, lies the least priority region D, that defines worst values W_{min} and B^{log}_{Max} for both metrics.

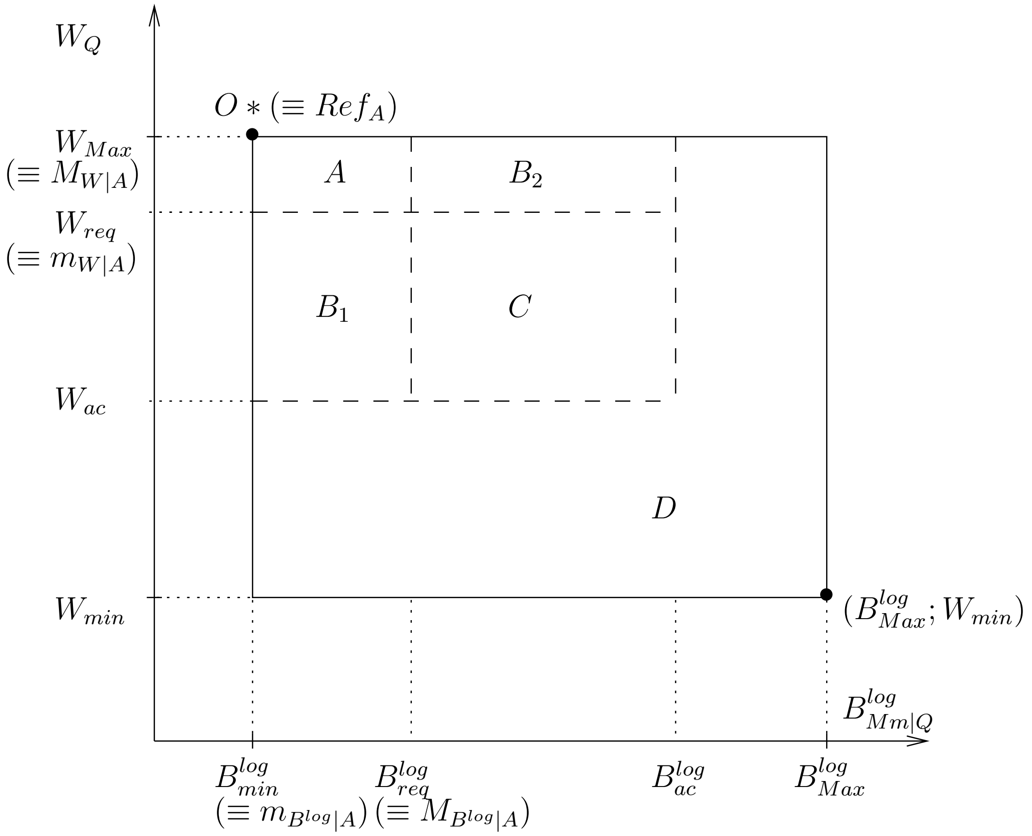

Figure 1: QoS requirements used to define priority regions in the bidimensional objective function space

The preference thresholds used to define the priority regions are given by

$$W_{req} = W_{av} + \Delta W = \frac{W_{av} + W_{Max}}{2} \tag{2}$$

$$W_{ac} = W_{av} - \Delta W = \frac{3W_{av} - W_{Max}}{2} \tag{3}$$

$$B^{log}_{req} = -\ln\left(1 - (B_{av} - \Delta B)\right) = -\ln\left(1 - \frac{B_{av} + B_{min}}{2}\right) \tag{4}$$

$$B^{log}_{ac} = -\ln\left(1 - (B_{av} + \Delta B)\right) = -\ln\left(1 - \frac{3B_{av} - B_{min}}{2}\right) \tag{5}$$

where

$$W_{min} = \min_{X \in \text{ archive}} W_Q(X) \tag{6}$$

$$W_{Max} = \max_{X \in \text{ archive}} W_Q(X) \tag{7}$$

$$W_{av} = \frac{\sum_{X \in \text{ archive}} W_Q(X)}{\#\{X\}} \tag{8}$$

$$\Delta W = \frac{W_{Max} - W_{av}}{2} \tag{9}$$

$$B^{log}_{Mm|Q} = -\ln\left(1 - B_{Mm|Q}\right) \tag{10}$$

$$B_{min} = \min_{X \in \text{ archive}} B_{Mm|Q}(X) \tag{11}$$

$$B^{log}_{min} = -\ln\left(1 - B_{min}\right) \tag{12}$$

$$
\begin{aligned}
B_{Max} &= \max_{X \in \text{ archive}} B_{Mm|Q}(X) && (13)\\
B_{Max}^{log} &= -\ln\left(1 - B_{Max}\right) && (14)\\
B_{av} &= \frac{\sum_{X \in \text{ archive}} B_{Mm|Q}(X)}{\#\{X\}} && (15)\\
\Delta B &= \frac{B_{av} - B_{min}}{2} && (16)
\end{aligned}
$$

and where X represents any solution in the archive.

When an archived solution has to be removed to make room for the new solution, the priority regions in the bidimensional objective function space are re-evaluated and the first solution found in the last priority region is selected. After its removal, the new solution can be added to the archive.

As mentioned in point 2b, if $\overline{R}_a$ has some values better and others worse than the current best values of the network QoS functions and of the objective functions of interest for the service under scrutiny, and $\overline{R}_a$ is not dominated by any of the solutions in the archive and the archive is full, an analysis is performed to decide whether $\overline{R}_a$ should be added to the archive. In this situation, the concept of preference thresholds is employed again, but this time not only the solutions in the archive but also the new solutions are considered. Therefore, for the calculation of (2)-(5), the parameters in (6)-(8), (11), (13) and (15) are calculated as follows:

$$
\begin{aligned}
W_{min} &= \min\left\{\min_{X \in \text{ archive}} W_Q(X); W_Q(\overline{R}_a)\right\}\\
W_{Max} &= \max\left\{\max_{X \in \text{ archive}} W_Q(X); W_Q(\overline{R}_a)\right\}\\
W_{av} &= \frac{\sum_{X \in \text{ archive}} W_Q(X) + W_Q(\overline{R}_a)}{\#\{X\} + 1}\\
B_{min} &= \min\left\{\min_{X \in \text{ archive}} B_{Mm|Q}(X); B_{Mm|Q}(\overline{R}_a)\right\}\\
B_{Max} &= \max\left\{\max_{X \in \text{ archive}} B_{Mm|Q}(X); B_{Mm|Q}(\overline{R}_a)\right\}\\
B_{av} &= \frac{\sum_{X \in \text{ archive}} B_{Mm|Q}(X) + B_{Mm|Q}(\overline{R}_a)}{\#\{X\} + 1}
\end{aligned}
$$

If the new solution $\overline{R}_a$ is in the last priority region, then it will not be added to the archive. Otherwise, the first archived solution in the worst priority region is selected for removal, after which the new solution can be added to the archive.

3.2.3 Test to determine the final solution

At the end of the algorithm (after the outer cycle), the solutions stored in the archive are analysed. By employing again the concept of preference thresholds, the priority regions in the bidimensional objective function space are re-evaluated so as to select the final solution of the algorithm.

The approach chosen to select the "best" solution in the best possible priority region is based on the calculation of a Chebyshev distance to a reference point, as described in [3]. Note that this operation is more time-consuming than simply choosing the first solution

Table 1: Reference points and weights for the Chebyshev metric for each priority region in the objective function space

Priority region $\mathcal{R}$	Reference point $\left(\mathcal{C}^*_{1\vert\mathcal{R}};\mathcal{C}^*_{2\vert\mathcal{R}}\right)$	Weights for the Chebyshev metric $m_{B^{log}\vert\mathcal{R}}$	$M_{B^{log}\vert\mathcal{R}}$	$m_{W\vert\mathcal{R}}$	$M_{W\vert\mathcal{R}}$
A	$\left(B^{log}_{min};W_{Max}\right)$	B^{log}_{min}	B^{log}_{req}	W_{req}	W_{Max}
B_2	$\left(B^{log}_{req};W_{Max}\right)$	B^{log}_{req}	B^{log}_{ac}	W_{req}	W_{Max}
B_1	$\left(B^{log}_{min};W_{req}\right)$	B^{log}_{min}	B^{log}_{req}	W_{ac}	W_{req}
C	$\left(B^{log}_{req};W_{req}\right)$	B^{log}_{req}	B^{log}_{ac}	W_{ac}	W_{req}
D	$\left(B^{log}_{min};W_{Max}\right)$	B^{log}_{min}	B^{log}_{Max}	W_{min}	W_{Max}

found in the best possible priority region. However, this operation is methodologically more correct and as it is performed only once, the amount of time it takes is not of primary importance.

As described in [26], reference point type approaches minimise the distance of the solutions in the objective function space to a selected point (often considered as the ideal one) according to a specific metric, which in our procedure is the weighted Chebyshev distance.

Let $\mathcal{R}$ be the best possible priority region in the objective function space where at least one solution ϱ can be found. In our approach, only this region $\mathcal{R}$ will be swept. A specific reference point is chosen in region $\mathcal{R}$ as the ideal point in that region, $\left(\mathcal{C}^*_{1|\mathcal{R}};\mathcal{C}^*_{2|\mathcal{R}}\right)$. The two metrics in the region are related to the upper level objective functions of the problem P-M2-S2, $B^{log}_{Mm|Q}$ (which has to be minimised) and W_Q (which has to be maximised). That is, the ideal point in each rectangular region is the top left corner of that region. See the example in figure 1 where the reference point for region A (Ref_A) is displayed. For a non-rectangular region such as D the ideal point of the whole objective function space $O*$ is the reference point. The reference points for all the regions are in table 1.

Another set of parameters that must be defined is the minimum $m_{i|\mathcal{R}}$ and maximum $M_{i|\mathcal{R}}$ of each metric i for region $\mathcal{R}$. See the example in figure 1 where the minimum and maximum for both metrics in region A, are displayed. These parameters for all the regions are in table 1. Notice that the reference point for each region can be written as $\left(\mathcal{C}^*_{1|\mathcal{R}};\mathcal{C}^*_{2|\mathcal{R}}\right) = \left(m_{B^{log}|\mathcal{R}};M_{W|\mathcal{R}}\right)$, because it is the point in the region where the $B^{log}_{Mm|Q}$ metric is minimised and the W_Q metric is maximised.

The problem to be solved is

$$\min_{\varrho\in\mathcal{R}}\max_{i=1,2}\left\{w_{i|\mathcal{R}}\left|\mathcal{C}_i(\varrho)-\mathcal{C}^*_{i|\mathcal{R}}\right|\right\}$$

where the metrics for solution ϱ are $\mathcal{C}_1(\varrho) = B^{log}_{Mm|Q}(\varrho)$ and $\mathcal{C}_2(\varrho) = W_Q(\varrho)$. The weights in the weighted Chebyshev distance are $w_{i|\mathcal{R}} = \frac{1}{M_{i|\mathcal{R}}-m_{i|\mathcal{R}}}$, which allow the Chebyshev metrics $\left\{w_{i|\mathcal{R}}\left|\mathcal{C}_i(\varrho)-\mathcal{C}^*_{i|\mathcal{R}}\right|\right\}$ to be dimension free and proportional to the size of the rectangular region. These weights also account for the difference in range of the parameters $B^{log}_{Mm|Q}$ and W_Q.

4 Experimental Results

In this section, the analytical and simulation results obtained with the HMOR-S2$_{\text{PAS}}$ heuristic in a network case study analogous to the one in [24] are presented.

In [24] a model for traffic routing optimisation and admission control in multiservice networks supporting traffic with different QoS requirements, was proposed. This model for MPLS networks with two service classes uses a lexicographic optimisation formulation, including admission control for BE traffic, based on a deterministic MCF (Multicommodity Flow) model, with the expected revenues associated with QoS and BE traffic as objective functions. It will be used as a benchmarking study for the present work concerning upper bounds for the optimal value of the QoS traffic revenue W_Q. For a brief summary of this application model, see [15, 14].

4.1 Application of the Model to a Network Case Study

The routing model in [24] and our routing model were applied to the test network depicted in figure 2. It has $N = 8$ nodes, with 10 pairs of nodes linked by a direct arc and a total of $|\mathcal{L}| = 20$ unidirectional arcs. The bandwidth of each arc C'_k [Mbps] is shown in figure 2. The number of channels C_k is $C_k = \left\lceil \frac{C'_k}{u_0} \right\rceil$, with basic unit capacity $u_0 = 16$ kbps. There are $|\mathcal{S}| = 4$ service types with the features displayed in table 2. The values of the required effective bandwidths $d_s = \frac{d'_s}{u_0}$ [channels] $\forall s \in \mathcal{S}$ are also in the table (where d'_s is the required bandwidth in kbps). The expected revenue for a call of type s is assumed to be $w_s = d_s, \forall s \in \mathcal{S}$. The average duration of a type s call is h_s and D_s represents the maximum number of arcs for a type s call.

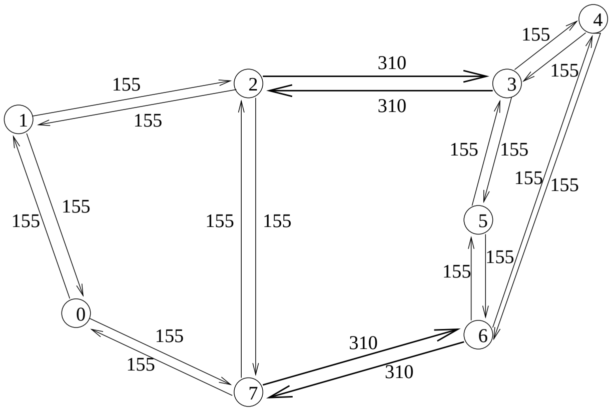

Figure 2: Test network $\mathcal{M}$ [24], with the indication of the bandwidth of each arc C'_k, in Mbps

A base matrix $T = [T_{ij}]$ with offered total bandwidth values from node i to node j [Mbps] is provided in [24]. From these data all the parameters needed by our traffic model can be obtained as shown in [6].

In this application example, results for the QoS flows revenue W_Q are presented for three values of a compensation parameter α: $\alpha = 0.0$ corresponds to a deterministic situation; $\alpha = 0.5$ is the compensation parameter when calls arrive according to a Poisson process, service times follow an exponential distribution and the network is critically loaded; and $\alpha = 1.0$ is used for traffic flows with higher 'variability'.

Table 2: Service features on the test network $\mathcal{M}$

Service	Class	d'_s [kbps]	d_s [channels]	w_s	h_s [s]	D_s [arcs]	m_s
1 - video	QoS	640	40	40	600	3	0.1
2 - Premium data	QoS	384	24	24	300	4	0.25
3 - voice	QoS	16	1	1	60	3	0.4
4 - data	BE	384	24	24	300	7	0.25

For further details on the application of this traffic model to the network case study under analysis, see [6].

4.2 Analytical Results

In the analytical study, the heuristic HMOR-S2$_{\text{PAS}}$ was run only once. For the archived routing plans obtained at the end of this single run, values for all the objective functions are computed and the "best" possible solution in the best possible preference region is chosen to be the final solution of the algorithm, using a reference point-based procedure as the solution selection mechanism.

Two different sets of tests were conducted:

- (i) tests: the initial solution is the same as the one used in the basic heuristic HMOR-S2 runs, a solution which is typical of Internet routing conventional algorithms. In this initial solution, only one path for each flow (i.e. without an alternative path) is considered. The initial solution is the same for all the services $s \in \mathcal{S}$ and the paths are symmetrical. The path for every flow f_s is the shortest one (that is, the one with minimum number of arcs); if there is more than one shortest path, the one with maximal bottleneck bandwidth (i.e. the minimal capacity of its arcs) is chosen; if there is more than one shortest path with equal bottleneck bandwidth, the choice is arbitrary.
- (f) tests: the initial solution of the HMOR-S2$_{\text{PAS}}$ heuristic is the routing plan obtained at the end of the basic heuristic runs for each specific α. The aim is to check whether this heuristic variant can improve the quality of the final solutions obtained with HMOR-S2 as an alternative to the direct use of the heuristic variant (as in the case of the (i) tests).

The analytical results concerning W_Q were compared with results obtained with the previous heuristic HMOR-S2 [15] and with the model proposed in [24].

The experiences with the HMOR-S2$_{\text{PAS}}$ were conducted with an archive of size 5. The results displayed in table 3 were obtained in approximately 47s on average in a Linux environment on a Pentium 4 processor with 3GHz CPU and 1GB of RAM.

In table 3, two different comparative analysis can be performed. For HMOR-S2$_{\text{PAS}}$(i), the initial solution is the same as the one used in the corresponding basic heuristic so the table allows for a comparison of the final results obtained with HMOR-S2 and HMOR-S2$_{\text{PAS}}$. As for the PAS(f) version, the initial solution has the objective function values displayed in the table under HMOR-S2 (Basis) so that a comparison of the initial and the final results with HMOR-S2$_{\text{PAS}}$ can be performed. The best values for the objective functions are in bold. The table also shows the results obtained for W_Q as a percentage of the upper bound optimal value given in [24].

Table 3: Objective function values for the final solution for HMOR-S2, HMOR-S2$_{\mathrm{PAS}}$(i) and HMOR-S2$_{\mathrm{PAS}}$(f), for different traffic matrices

Objective Functions	HMOR-S2 (Basis)	HMOR-S2$_{\mathrm{PAS}}$	
		(i)	(f)
	$\alpha = 0.0$		
W_Q	64731.51*	64848.17⋄	**64905.26**⋆
$B_{Mm\|Q}$	0.0898	0.0803	**0.0752**
$B_{m1\|Q}$	0.0898	0.0803	**0.0752**
$B_{m2\|Q}$	0.0199	0.0189	**0.0184**
$B_{m3\|Q}$	0.00216	0.00190	**0.00184**
$B_{M1\|Q}$	**0.691**	0.706	0.708
$B_{M2\|Q}$	**0.0723**	0.101	0.103
$B_{M3\|Q}$	**0.0287**	0.0299	0.0301
W_B	17007.15	17018.80	**17039.20**
	$\alpha = 0.5$		
W_Q	60569.09†	60694.00•	**60739.76**⊙
$B_{Mm\|Q}$	0.0424	0.0311	**0.0278**
$B_{m1\|Q}$	0.0424	0.0311	**0.0278**
$B_{m2\|Q}$	0.00534	0.00347	**0.00230**
$B_{m3\|Q}$	0.00119	0.000867	**0.000857**
$B_{M1\|Q}$	**0.628**	0.629	0.629
$B_{M2\|Q}$	0.0432	0.0206	**0.00959**
$B_{M3\|Q}$	**0.0243**	0.0244	0.0244
W_B	**16904.99**	16898.77	16685.60
	$\alpha = 1.0$		
W_Q	56100.60‡	**56106.78**□	56106.51⊗
$B_{Mm\|Q}$	0.0263	0.0257	**0.0256**
$B_{m1\|Q}$	0.0263	0.0257	**0.0256**
$B_{m2\|Q}$	0.00515	**0.00495**	0.00499
$B_{m3\|Q}$	**0.000560**	0.000564	0.000567
$B_{M1\|Q}$	**0.544**	0.556	0.556
$B_{M2\|Q}$	0.0185	**0.0178**	0.0186
$B_{M3\|Q}$	**0.0193**	0.0200	0.0200
W_B	**16479.60**	16464.68	16465.58

HMOR-S2: *)99.35%; †)99.57%; ‡)99.58% of $W_Q^{\max}$ (the optimal revenue in [24]);
HMOR-S2$_{\mathrm{PAS}}$(i): ⋄)99.53%; •)99.78%; □)99.59% of $W_Q^{\max}$;
HMOR-S2$_{\mathrm{PAS}}$(f): ⋆)99.62%; ⊙)99.85%; ⊗)99.59% of $W_Q^{\max}$.

For both versions of the heuristic HMOR-S2_{PAS}, the final results for the upper level objective functions show an improvement on the ones obtained with the basic heuristic, for all the values of α. As the PAS variant takes practically the same time to run as the basic heuristic and provides better results for W_Q and $B_{Mm|Q}$, it can be considered a better approach for solving the routing problem. Plus, its use on a second stage of the resolution of the routing problem (after the basic heuristic has been used on a first stage) tends to provide even more interesting results. In fact, a run of the basic heuristic HMOR-S2 followed by a run of the PAS variant ((f) version) provides improved results for the first level functions for the routing problem under analysis for $\alpha = 0.0$ and $\alpha = 0.5$, which correspond to higher overload situations. For $\alpha = 1.0$, the results of the (i) and the (f) versions are two non-dominated solutions in terms of the values of the first level functions, and they are practically the same and very similar to the ones of the basic heuristic. Of course more thorough studies could be conducted, in particular with different values of the tunable parameters for the variant. Also note that the best results obtained with the heuristic variant are more than 99.5% of the optimal value W_Q.

These results can also be compared with those obtained with the meta-heuristics HMOR-S2_{SA} and HMOR-S2_{TS} [14], which are reviewed in table 5 (see Appendix B). No bold or italic styles are used in this table.

The first level objective function values obtained with the SA variant of the basic heuristic are worse than the ones obtained with HMOR-S2_{PAS}, both for the (i) and the (f) version. Plus, the HMOR-S2_{SA} takes a lot longer to run. Therefore, the PAS variant can be clearly considered better than the SA variant.

A comparative analysis of the first level objective function values obtained with the TS and the PAS variants of the basic heuristic can also be performed. For the (i) version with $\alpha = 0.0$ and $\alpha = 0.5$, the results for W_Q and $B_{Mm|Q}$ are better with the PAS variant. In fact, in this situation the results with the HMOR-S2_{PAS} are an improvement on the result values of the basic heuristic, while the results with the HMOR-S2_{TS} are worse than the ones obtained with the basic heuristic. For the (i) version with $\alpha = 1.0$ and for the (f) version, the results for W_Q and $B_{Mm|Q}$ are slightly better with the TS variant. However, as the HMOR-S2_{TS} takes a lot longer to run than HMOR-S2_{PAS}, then the PAS variant can be considered globally more advantageous than the TS variant.

Globally speaking, we consider HMOR-S2_{PAS} and especially the (f) version to be a procedure better than the basic heuristic and the two meta-heuristic variants, for solving the very complex routing problem P-M2-S2.

4.3 Simulation Results

After the analytical experiences were performed, simulation experiences, with a static routing method using the solution provided by the heuristic, were also carried out. This simulation study enables the validation of the routing model results and the evaluation of the errors intrinsic to the analytical model which provides the estimates for the objective functions.

A discrete-event stochastic simulation platform was used with the static routing model, where the routing plan is the final solution obtained after the (i) or the (f) version for the PAS variant was run. This routing plan does not change throughout the simulation regardless of the random variations of traffic offered to the network. The simulation starts with an initialisation phase that lasts for a time $t_{warm-up}$, that should be long enough to guarantee that the system state at the end of the initialisation phase is representative of the steady state behaviour of the system. After this time, information on the number of offered calls and carried calls in the network for each flow $f_s, s \in \mathcal{S}$, is gathered, until the end of the simulation. With this information, $B(f_s), \forall s \in \mathcal{S}$ and subsequently, the values of the upper and lower level objective functions related to blocking probabilities can be estimated. The calculation of the expected revenues is based on the number of carried calls in the network.

In table 4, the analytical values of each objective function are displayed, together with the simulation results (average value ± half length of a 95% confidence interval, computed by the independent replications method [20]) for these functions. The bold style indicates that the statistical estimate of an objective function value obtained with one of the versions of the PAS variant is the same or better than the corresponding value obtained with the basic heuristic. The italic style indicates that a simulation result is better than the corresponding analytical value. The revenue values have 2 decimal places and the blocking probability values have 3 significant figures. The results displayed in the table were obtained with a total simulated time $t_{total} = 48$h and a warm-up time $t_{warm-up} = 8$h. It took almost 2h to get these results in the computer mentioned earlier.

The analytical results and the corresponding static routing model simulation results are of similar magnitude, but the analytical results tend to be better, as expected. This is especially noticeable in situations of lower traffic loads (which correspond to higher values of α in this routing problem application example). In fact, only for the simulations with $\alpha = 0.0$ did we get a result where an upper level objective function analytical value was in the corresponding confidence interval and had a value worse than the corresponding static routing model simulation result. The differences between the simulation and analytic results are mainly due to the imprecisions/inaccuracies intrinsic to the analytic/numerical resolution, in particular those associated with the simplifications of the traffic model, and the associated error propagation. As the overflow traffic is treated as Poisson traffic, the analytical model is actually a simplification which tends to underestimate the blocking probabilities in the network (and to overestimate the revenues). The errors resulting from this simplification propagate throughout the complex and lengthy numerical calculations associated with the resolution, for a great number of times, of the large systems of implicit non-linear equations used to calculate B_{ks} and $c_{ks}^{Q(B)}$. Further simplifications were assumed in the stochastic model for the traffic in the links, namely a superposition of independent Poisson flows and independent occupations of the links. If a more accurate and realistic representation of the traffic flows was used, better estimates of the blocking probabilities would be achieved (see [8], for example). Nonetheless, the approximations in our model can be considered appropriate in this context for practical reasons. In fact, if more complex models were used to represent the traffic and to calculate the blockings in overflow conditions, the computational burden would be too heavy since the analytical model has to be numerically solved many times during the execution of the heuristic and the routing method would become intractable. Moreover, these errors do not compromise the inequality relations between the objective function values, as the aim of the routing optimisation procedure is just the comparison of routing solutions in terms of the values of the objective functions. That is, the focus is on the relative value of the results of the traffic model rather than on the absolute accuracy of such values.

A comparison of the results obtained with the basic heuristic and with both versions of the PAS variant shows that the analytical and simulation results are coherent, in the sense that whenever the analytical value of an objective function is better for the (f) version than for the (i) version, the same tends to happen with the average values obtained with the static routing model simulation.

Table 4: Average objective function values with 95% confidence intervals, for simulations with the routing plan obtained with the HMOR-S2$_{\mathrm{PAS}}$(i) and the HMOR-S2$_{\mathrm{PAS}}$(f)

Objective Functions	HMOR-S2 Analytical	 Static routing model	HMOR-S2$_{\mathrm{PAS}}$(i) Analytical	 Static routing model	HMOR-S2$_{\mathrm{PAS}}$(f) Analytical	 Static routing model
	Results for $\alpha = 0.0$					
W_Q	64731.51	64642.53±64.17(0.10%)	64848.17	**64733.71**±63.18(0.10%)	64905.26	**64774.12**±68.28(0.11%)
$B_{Mm\mid Q}$	0.0898	*0.0887*±0.00336(3.79%)	0.0803	**0.0811**±0.00299(3.69%)	0.0752	**0.0773**±0.00356(4.61%)
$B_{m1\mid Q}$	0.0898	*0.0887*±0.00336(3.79%)	0.0803	**0.0811**±0.00299(3.69%)	0.0752	**0.0773**±0.00356(4.61%)
$B_{m2\mid Q}$	0.0199	0.0246±0.000647(2.63%)	0.0189	**0.0238**±0.000634(2.66%)	0.0184	**0.0236**±0.000576(2.44%)
$B_{m3\mid Q}$	0.00216	0.00226±0.0000663(2.93%)	0.00190	**0.00205**±0.0000491(2.40%)	0.00184	**0.00200**±0.0000499(2.50%)
$B_{M1\mid Q}$	0.691	*0.684*±0.00802(1.17%)	0.706	*0.703*±0.00982(1.40%)	0.708	*0.706*±0.00912(1.29%)
$B_{M2\mid Q}$	0.0723	0.0843±0.00242(2.87%)	0.101	0.114±0.0124(10.90%)	0.103	0.110±0.00600(5.45%)
$B_{M3\mid Q}$	0.0287	0.0291±0.000206(0.71%)	0.0299	0.0302±0.000274(0.91%)	0.0301	0.0303±0.000146(0.48%)
W_B	17007.15	16982.33±37.02(0.22%)	17018.80	**16992.82**±39.09(0.23%)	17039.20	**17017.10**±39.32(0.23%)
	Results for $\alpha = 0.5$					
W_Q	60569.09	60491.22±50.79(0.08%)	60694.00	**60606.56**±57.00(0.09%)	60739.76	**60676.12**±61.43(0.10%)
$B_{Mm\mid Q}$	0.0424	0.0460±0.00163(3.54%)	0.0311	**0.0356**±0.00145(4.08%)	0.0278	**0.0306**±0.00145(4.73%)
$B_{m1\mid Q}$	0.0424	0.0460±0.00163(3.54%)	0.0311	**0.0356**±0.00145(4.08%)	0.0278	**0.0306**±0.00145(4.73%)
$B_{m2\mid Q}$	0.00534	0.00809±0.000328(4.06%)	0.00347	**0.00637**±0.000289(4.53%)	0.00230	**0.00463**±0.000355(7.67%)
$B_{m3\mid Q}$	0.00119	0.00126±0.0000403(3.20%)	0.000867	**0.000947**±0.0000223(2.36%)	0.000857	**0.000922**±0.0000167(1.81%)
$B_{M1\mid Q}$	0.628	0.631±0.0151(2.40%)	0.629	0.632±0.0153(2.42%)	0.629	***0.626***±0.0196(3.14%)
$B_{M2\mid Q}$	0.0432	0.0503±0.00266(5.29%)	0.0206	**0.0263**±0.00237(9.00%)	0.00959	**0.0158**±0.00216(13.67%)
$B_{M3\mid Q}$	0.0243	0.0245±0.000196(0.80%)	0.0244	**0.0245**±0.000213(0.87%)	0.0244	**0.0245**±0.000261(1.07%)
W_B	16904.99	16899.02±38.69(0.23%)	16898.77	16896.50±38.25(0.23%)	16685.60	*16696.08*±40.87(0.24%)
	Results for $\alpha = 1.0$					
W_Q	56100.60	56027.72±46.92(0.08%)	56106.78	**56035.09**±47.22(0.08%)	56106.51	**56036.04**±45.53(0.08%)
$B_{Mm\mid Q}$	0.0263	0.0281±0.00126(4.48%)	0.0257	**0.0276**±0.000989(3.58%)	0.0256	**0.0274**±0.00174(6.36%)
$B_{m1\mid Q}$	0.0263	0.0281±0.00126(4.48%)	0.0257	**0.0276**±0.000989(3.58%)	0.0256	**0.0274**±0.00174(6.36%)
$B_{m2\mid Q}$	0.00515	0.00832±0.000685(8.23%)	0.00495	**0.00804**±0.000662(8.24%)	0.00499	**0.00805**±0.000619(7.70%)
$B_{m3\mid Q}$	0.000560	0.000637±0.0000154(2.42%)	0.000564	0.000638±0.0000153(2.41%)	0.000567	0.000643±0.0000157(2.45%)
$B_{M1\mid Q}$	0.544	0.547±0.0281(5.13%)	0.556	0.557±0.0304(5.46%)	0.556	*0.552*±0.0304(5.50%)
$B_{M2\mid Q}$	0.0185	0.0325±0.00353(10.88%)	0.0178	**0.0307**±0.00251(8.17%)	0.0186	**0.0310**±0.00318(10.28%)
$B_{M3\mid Q}$	0.0193	0.0195±0.000167(0.86%)	0.0200	0.0201±0.000183(0.91%)	0.0200	0.0201±0.000295(1.47%)
W_B	16479.60	16453.09±17.05(0.10%)	16464.68	16437.83±16.40(0.10%)	16465.58	16436.45±17.45(0.11%)

5 Conclusions and Further Work

In this work a hierarchical bi-level multiobjective routing model in MPLS networks with alternative routing, with two classes of service (with different priorities in the optimisation model) and different types of traffic flows in each class, was reviewed. A specialised heuristic strategy, HMOR-S2, for finding "good" compromise solutions to this very complex routing optimisation problem, was also reviewed. Sensitivity tests performed on HMOR-S2 showed that in some cases there were "better" solutions to the routing problem that the basic heuristic was unable to deliver as the final result. Therefore, new variants that could possibly find solutions "better" than the ones obtained with the HMOR-S2 basic heuristic were devised. In this work, a variant of the previous heuristic, HMOR-S2 with a Pareto Archived Strategy, was put forward. This new procedure maintains the resolution framework of HMOR-S2 but introduces and treats in a special manner an archive of possible good solutions found throughout the execution of the heuristic.

The analytical results obtained with the new variant were compared with the optimal values for the QoS service expected revenue in the benchmarking case study [24] and with the values obtained with the basic heuristic HMOR-S2 [15]. The results show that the HMOR-$\mathrm{S2_{PAS}}$(i), where an archive of non-dominated solutions found throughout the execution of the specialised basic heuristic HMOR-S2 is introduced, leads to better results for the upper level objective functions. Moreover, the version HMOR-$\mathrm{S2_{PAS}}$(f) can be even more advantageous in the search for improvements of the final solution obtained with the basic heuristic. In this version, a run of the basic heuristic HMOR-S2 followed by a run of the variant tends to provide improved results for the routing problem.

A more exact evaluation of the results of the heuristic was accomplished with a discrete-event simulation platform, in a stochastic environment closer to real network working conditions. In most cases, the analytical results obtained with the HMOR-S2 or the HMOR-$\mathrm{S2_{PAS}}$(i)/(f) are not inside the 95% confidence interval of the static routing model simulation results, although they are of similar magnitude, due to the inaccuracies intrinsic to the analytic/numerical resolution, namely those associated with the simplifications of the traffic model, and the associated error propagation.

It is important to note that this variant has not added a greater complexity to the basic heuristic. Nevertheless, the computational burden of either resolution approach is still heavy. This remains the major limitation of this type of routing method and restrains its potential practical application, at present, to networks with a limited number of nodes, such as the core and intermediate (metro-core) level networks of low dimension.

Further work on this model will focus on the search for possible simplifications and improvements in the heuristic resolution approaches. Also the extension of the model to broader routing principles such as probabilistic load sharing or traffic splitting might be studied and tested.

References

[1] D. Awduche, A. Chiu, A. Elwalid, I. Widjaja, and X. Xiao. Overview and principles of Internet traffic engineering. RFC 3272, Network Working Group, May 2002.

[2] J. Clímaco and J. Craveirinha. Multicriteria analysis in telecommunication network planning and design – Problems and issues. In J. Figueira, S. Greco, and M. Ehrgott, editors, *Multiple Criteria Decision Analysis – State of the Art Surveys*, volume 78 of

International Series in Operations Research & Management Science, pages 899–951. Springer, 2005.

[3] J. C. N. Clímaco, J. M. F. Craveirinha, and M. M. B. Pascoal. An automated reference point-like approach for multicriteria shortest path problems. *Journal of Systems Science and Systems Engineering*, 15(3):314–329, Sep. 2006.

[4] J. C. N. Clímaco, J. M. F. Craveirinha, and M. M. B. Pascoal. Multicriteria routing models in telecommunication networks – Overview and a case study. In Y. Shi, D. L. Olson, and A. Stam, editors, *Advances in Multiple Criteria Decision Making and Human Systems Management: Knowledge and Wisdom*, pages 17–46. IOS Press, 2007.

[5] J. Craveirinha, R. Girão-Silva, and J. Clímaco. A meta-model for multiobjective routing in MPLS networks. *Central European Journal of Operations Research*, 16(1):79–105, Mar. 2008.

[6] J. Craveirinha, R. Girão-Silva, J. Clímaco, and L. Martins. A hierarchical multiobjective routing model for MPLS networks with two service classes – Analysis and resolution approach. Research Report 5/2007 (ISSN 1645-2631), INESC-Coimbra (`www.inescc.pt`), Oct. 2007.

[7] J. Craveirinha, R. Girão-Silva, J. Clímaco, and L. Martins. A hierarchical multiobjective routing model for MPLS networks with two service classes. In A. Korytowski, K. Malanowski, W. Mitkowski, and M. Szymkat, editors, *Revised Selected Papers of the 23rd IFIP TC7 Conference on System Modeling and Optimization, Cracow, Poland, July 23-27, 2007*, volume 312 of *IFIP Advances in Information and Communication Technology*, pages 196–219. Springer, 2010.

[8] J. Craveirinha, T. Gomes, S. Esteves, and L. Martins. A method for calculating marginal variances in teletraffic networks with multiple overflows. In J. Janssen and S. Osaki, editors, *Proceedings of the First Euro-Japanese Workshop on Stochastic Risk Modelling for Finance, Insurance, Production and Reliability*, volume II, Bruxelles, Belgique, Sep. 1998.

[9] J. Craveirinha, L. Martins, and J. N. Clímaco. Dealing with complexity in a multiobjective dynamic routing model for multiservice networks – A heuristic approach. In *Electronic proceedings of the 15th Mini-EURO Conference on Managing Uncertainty in Decision Support Models (MUDSM 2004)*, Coimbra, Portugal, Sep. 22-24 2004.

[10] J. Craveirinha, L. Martins, T. Gomes, C. H. Antunes, and J. N. Clímaco. A new multiple objective dynamic routing method using implied costs. *Journal of Telecommunications and Information Technology*, 3:50–59, 2003.

[11] H. M. Elsayed, M. S. Mahmoud, A. Y. Bilal, and J. Bernussou. Adaptive alternate-routing in telephone networks: Optimal and equilibrium solutions. *Information and Decision Technologies*, 14:65–74, 1988.

[12] A. Faragó, S. Blaabjerg, L. Ast, G. Gordos, and T. Henk. A new degree of freedom in ATM network dimensioning: Optimizing the logical configuration. *IEEE Journal on Selected Areas in Communications*, 13(7):1199–1206, Sep. 1995.

[13] R. Girão-Silva, J. Craveirinha, and J. Clímaco. Hierarchical multiobjective routing in MPLS networks with two service classes – Heuristic resolution and sensitivity analysis. Research Report 8/2008 (ISSN 1645-2631), INESC-Coimbra (`www.inescc.pt`), Jul. 2008.

[14] R. Girão-Silva, J. Craveirinha, and J. Clímaco. Hierarchical multiobjective routing in MPLS networks with two service classes - A meta-heuristic solution. *Journal of Telecommunications and Information Technology*, 3:20–37, 2009.

[15] R. Girão-Silva, J. Craveirinha, and J. Clímaco. Hierarchical multiobjective routing in Multiprotocol Label Switching networks with two service classes – A heuristic solution. *International Transactions in Operational Research*, 16(3):275–305, May 2009.

[16] F. Kelly. Notes on effective bandwidths. In F. P. Kelly, S. Zachary, and I. Ziedins, editors, *Stochastic Networks: Theory and Applications*, volume 4 of *Royal Statistical Society Lecture Notes Series*, pages 141–168. Oxford University Press, 1996.

[17] F. P. Kelly. Routing in circuit-switched networks: Optimization, shadow prices and decentralization. *Advances in Applied Probability*, 20(1):112–144, Mar. 1988.

[18] J. Knowles, M. Oates, and D. Corne. Advanced multi-objective evolutionary algorithms applied to two problems in telecommunications. *BT Technology Journal*, 18(4):51–65, Oct. 2000.

[19] F. Kuipers, P. Van Mieghem, T. Korkmaz, and M. Krunz. An overview of constraint-based path selection algorithms for QoS routing. *IEEE Communications Magazine*, 40(12):50–55, Dec. 2002.

[20] A. M. Law and W. D. Kelton. *Simulation Modeling and Analysis.* Industrial Engineering and Management Science. McGraw-Hill, Inc., 2nd edition, 1991.

[21] L. Martins, J. Craveirinha, and J. Clímaco. A new multiobjective dynamic routing method for multiservice networks: Modelling and performance. *Computational Management Science*, 3(3):225–244, July 2006. (Also Research Report 2/2005, INESC-Coimbra)

[22] L. Martins, J. Craveirinha, J. N. Clímaco, and T. Gomes. Implementation and performance of a new multiple objective dynamic routing method for multiexchange networks. *Journal of Telecommunications and Information Technology*, 3:60–66, 2003. (Also Research Report ET-N8-5; 11/2002, INESC-Coimbra)

[23] D. Mitra, J. A. Morrison, and K. G. Ramakrishnan. Optimization and design of network routing using refined asymptotic approximations. *Performance Evaluation*, 36-37:267–288, 1999.

[24] D. Mitra and K. G. Ramakrishnan. Techniques for traffic engineering of multiservice, multipriority networks. *Bell Labs Technical Journal*, 6(1):139–151, Jan. 2001.

[25] R. E. Steuer. *Multiple Criteria Optimization: Theory, Computation and Application.* Probability and Mathematical Statistics. John Wiley & Sons, 1986.

[26] A. P. Wierzbicki. Reference point methods in vector optimization and decision support. Interim Report IR-98-017, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria, Apr. 1998.

[27] A. P. Wierzbicki. Telecommunications, multiple criteria analysis and knowledge theory. *Journal of Telecommunications and Information Technology*, 3:3–13, 2005.

A Formalisation of the Heuristic HMOR-S2$_{\mathbf{PAS}}$

```
I. $\overline{R}_a \leftarrow \overline{R}_o$
II. Compute $\overline{B}$ and $W_Q, B_{Mm|Q}$ for $\overline{R}_a$
III. $W_Q^o \leftarrow W_Q, B_{Mm|Q}^o \leftarrow B_{Mm|Q}$
IV. $\overline{R}_* \leftarrow \overline{R}_a$
V. Compute $\overline{B}$ for $\overline{R}_a$
   Compute $W_Q, B_{Mm|Q}, B_{ms|Q}, B_{Ms|Q}(\forall s \in \mathcal{S}_Q), W_B$ for $\overline{R}_a$
VI. $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
   $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}(\forall s \in \mathcal{S}_Q)$ and $\max\{W_B\} \leftarrow W_B$
VII. Add $\overline{R}_a$ to the archive
VIII. For $nPaths = |\overline{\mathcal{F}}|$ to $nPaths = 1$
   1. For $ape = 0$ to $ape = 1$
      (a) If $ape = 0$, $z_{APR} \leftarrow 1.0$
          Else, $z_{APR} \leftarrow 0.01 \cdot nPaths$
      (b) For $s = 1$ to $s = |\mathcal{S}|$
          i. For $nCycles = 1$ to $nCycles = 0$
             A. Compute $\overline{B}$ and $\overline{c}^Q, s \in \mathcal{S}_Q$ or $\overline{c}^B, s \in \mathcal{S}_B$ for $\overline{R}_a$
             B. Compute and order the values of the function $\xi(f_s)$, with $\xi(f_s) = F_L(f_s)$ if $nCycles = 1$ and $\xi(f_s) = F_C^{Q(B)}(f_s)$ if $nCycles = 0$
             C. Find the $nPaths$ flows with lower value of $\xi(f_s)$
             D. Compute with MMRA-S2 new candidate paths for the corresponding O-D pairs and define a new set of first and second choice paths for the service $s$, $\overline{R}_a(s)$, according to the rules established for each service
             E. Compute $\overline{B}$ for $\overline{R}_a$
                Compute $B_{ms|Q}, B_{Ms|Q}$ if $s \in \mathcal{S}_Q$ or $W_B$ if $s \in \mathcal{S}_B$ for $\overline{R}_a$
                Compute $W_Q, B_{Mm|Q}$
             F. If $s \in \mathcal{S}_Q$ then
                • If $[(B_{ms|Q} < \min\{B_{ms|Q}\}$ and $B_{Ms|Q} < \min\{B_{Ms|Q}\})$ and $(W_Q > \max\{W_Q\}$ and $B_{Mm|Q} < \min\{B_{Mm|Q}\})]$ then
                  – $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}$
                  – $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
                  – $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$
                  – Add $\overline{R}_a$ to the archive (If it is already full, the priority regions of the solutions in the archive must be evaluated and the first solution found in the worst region of the archive should be removed first.)
```

```
    • Else,
      – If $[(B_{ms|Q} > \min\{B_{ms|Q}\}$ and $B_{Ms|Q} > \min\{B_{Ms|Q}\})$ and $(W_Q < \max\{W_Q\}$ and $B_{Mm|Q} > \min\{B_{Mm|Q}\})]$
        * (Discard $\overline{R}_a$)
      – Else,
        * If there is at least one solution $X$ in the archive for which $[(B_{ms|Q} > B_{ms|Q}(X)$ and $B_{Ms|Q} > B_{Ms|Q}(X))$ and $(W_Q < W_Q(X)$ and $B_{Mm|Q} > B_{Mm|Q}(X))]$, i.e. $X$ dominates $\overline{R}_a$ in terms of the objective functions of interest,
            † (Discard $\overline{R}_a$)
        * Else ($\overline{R}_a$ and the solutions in the archive are non-dominated)
            † If the archive is not full,
                ‡ Add $\overline{R}_a$ to the archive
            † Else
                ‡ Evaluate the priority regions of $\overline{R}_a$ and the solutions in the archive;
                ‡ If $\overline{R}_a$ is in the worst priority region,
                    ○ (Discard $\overline{R}_a$)
                ‡ Else,
                    ○ Remove the first solution found in the worst region of the archive;
                    ○ Add $\overline{R}_a$ to the archive
G. Else ($s \in \mathcal{S}_B$)
    • If $[(W_B > \max\{W_B\})$ and $(W_Q > \max\{W_Q\}$ and $B_{Mm|Q} < \min\{B_{Mm|Q}\})]$ then
      – $\max\{W_B\} \leftarrow W_B$
      – $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
      – $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$
      – Add $\overline{R}_a$ to the archive (If it is already full, the priority regions of the solutions in the archive must be evaluated and the first solution found in the worst region of the archive should be removed first.)
    • Else,
      – If $[(W_B < \max\{W_B\})$ and $(W_Q < \max\{W_Q\}$ and $B_{Mm|Q} > \min\{B_{Mm|Q}\})]$
        * (Discard $\overline{R}_a$)
      – Else,
        * If there is at least one solution $X$ in the archive for which $[(W_B < W_B(X))$ and $(W_Q < W_Q(X)$ and $B_{Mm|Q} > B_{Mm|Q}(X))]$, i.e. $X$ dominates $\overline{R}_a$ in terms of the objective functions of interest,
            † (Discard $\overline{R}_a$)
        * Else ($\overline{R}_a$ and the solutions in the archive are non-dominated)
            † If the archive is not full,
                ‡ Add $\overline{R}_a$ to the archive
            † Else
                ‡ Evaluate the priority regions of $\overline{R}_a$ and the solutions in the archive;
                ‡ If $\overline{R}_a$ is in the worst priority region,
```

```
                    ∘ (Discard $\overline{R}_a$)
                  ‡ Else,
                    ∘ Remove the first solution found in the worst region
                      of the archive;
                    ∘ Add $\overline{R}_a$ to the archive
          H. $\overline{R}_a(s) \leftarrow \overline{R}_*(s)$
          End of the cycle For ($nCycles$)
        End of the cycle For ($s$)
    End of the cycle For ($ape$)
End of the cycle For ($nPaths$)
IX. If $W_Q^o > \max\{W_Q\}$ or $B_{Mm|Q}^o < \min\{B_{Mm|Q}\}$ then
    1. The best solution is $\overline{R}_o$.
X. Else,
    1. Evaluate the priority regions of the solutions in the archive;
    2. The final solution is found in the best region of the archive, using a reference point-
       based procedure.
XI. Compute the objective function values for the final solution.
```

B Results Obtained with the Meta-Heuristics HMOR-S2$_{\mathbf{SA}}$ and HMOR-S2$_{\mathbf{TS}}$

Table 5: Analytical objective function values obtained with the HMOR-S2$_{\mathrm{SA}}$(i) and (f) and the HMOR-S2$_{\mathrm{TS}}$(i) and (f), and average objective function values with 95% confidence intervals, for simulations with the routing plan obtained with the HMOR-S2$_{\mathrm{SA}}$(f) and the HMOR-S2$_{\mathrm{TS}}$(f)

	HMOR-S2$_{\mathrm{SA}}$			HMOR-S2$_{\mathrm{TS}}$		
Objective	(i) version	(f) version		(i) version	(f) version	
Functions	Analytical	Analytical	Static routing model	Analytical	Analytical	Static routing model
	Results for $\alpha = 0.0$					
W_Q	64517.97	64795.66	64704.03±72.85(0.11%)	64619.61	64915.35	64781.55±67.82(0.10%)
$B_{Mm\mid Q}$	0.107	0.0843	0.0830±0.00389(4.68%)	0.116	0.0731	0.0749±0.00316(4.22%)
$B_{m1\mid Q}$	0.107	0.0843	0.0830±0.00389(4.68%)	0.116	0.0731	0.0749±0.00316(4.22%)
$B_{m2\mid Q}$	0.0218	0.0194	0.0242±0.000551(2.27%)	0.0105	0.0189	0.0243±0.000609(2.51%)
$B_{m3\mid Q}$	0.00283	0.00206	0.00216±0.0000624(2.89%)	0.00480	0.00179	0.00196±0.0000485(2.47%)
$B_{M1\mid Q}$	0.673	0.700	0.687±0.0119(1.74%)	0.854	0.721	0.714±0.0180(2.52%)
$B_{M2\mid Q}$	0.115	0.0811	0.0923±0.00377(4.09%)	0.0434	0.0953	0.106±0.0107(10.05%)
$B_{M3\mid Q}$	0.0274	0.0295	0.0298±0.000171(0.57%)	0.0467	0.0312	0.0315±0.000236(0.75%)
W_B	17662.81	17121.51	17102.41±40.75(0.24%)	17489.36	17163.01	17137.81±49.80(0.29%)
	Results for $\alpha = 0.5$					
W_Q	60569.09	60724.32	60655.12±60.57(0.10%)	60162.90	60751.77	60655.33±57.72(0.10%)
$B_{Mm\mid Q}$	0.0424	0.0289	0.0320±0.00162(5.08%)	0.0805	0.0258	0.0308±0.00104(3.39%)
$B_{m1\mid Q}$	0.0424	0.0289	0.0320±0.00162(5.08%)	0.0805	0.0258	0.0308±0.00104(3.39%)
$B_{m2\mid Q}$	0.00534	0.00270	0.00521±0.000329(6.32%)	0.0104	0.00259	0.00577±0.000269(4.66%)
$B_{m3\mid Q}$	0.00119	0.000854	0.000927±0.0000182(1.96%)	0.00254	0.000744	0.000838±0.0000167(2.00%)
$B_{M1\mid Q}$	0.628	0.619	0.615±0.0210(3.41%)	0.742	0.634	0.637±0.0157(2.46%)
$B_{M2\mid Q}$	0.0432	0.0108	0.0179±0.00201(11.27%)	0.0385	0.00769	0.0139±0.000742(5.35%)
$B_{M3\mid Q}$	0.0243	0.0237	0.0239±0.000117(0.49%)	0.0330	0.0246	0.0248±0.000278(1.12%)
W_B	16904.99	16738.50	16752.53±39.75(0.24%)	17664.88	16905.73	16905.09±39.59(0.23%)
	Results for $\alpha = 1.0$					
W_Q	56100.60	56100.60	56027.72±46.92(0.08%)	56191.34	56109.97	56038.54±47.33(0.08%)
$B_{Mm\mid Q}$	0.0263	0.0263	0.0281±0.00126(4.48%)	0.0179	0.0252	0.0269±0.00126(4.70%)
$B_{m1\mid Q}$	0.0263	0.0263	0.0281±0.00126(4.48%)	0.0179	0.0252	0.0269±0.00126(4.70%)
$B_{m2\mid Q}$	0.00515	0.00515	0.00832±0.000685(8.23%)	0.00266	0.00494	0.00806±0.000648(8.04%)
$B_{m3\mid Q}$	0.000560	0.000560	0.000637±0.0000154(2.42%)	0.000430	0.000555	0.000633±0.0000168(2.65%)
$B_{M1\mid Q}$	0.544	0.544	0.547±0.0281(5.13%)	0.489	0.556	0.558±0.0192(3.45%)
$B_{M2\mid Q}$	0.0185	0.0185	0.0325±0.00353(10.88%)	0.00955	0.0177	0.0312±0.00331(10.60%)
$B_{M3\mid Q}$	0.0193	0.0193	0.0195±0.000167(0.86%)	0.0165	0.0200	0.0202±0.000307(1.52%)
W_B	16479.60	16479.60	16453.09±17.05(0.10%)	16288.89	16464.83	16438.45±18.54(0.11%)