Dynamic Location Problems with Discrete Expansion and Reduction Sizes of Available Capacities

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Abstract: In this paper a dynamic location problem is formulated that considers the possibility of expanding or reducing the maximum available capacity at any given location during the planning horizon. The expansion (or reduction) of available capacity at a given location is achieved through the opening (or closure) of one or more facilities with different discrete capacities. The linear mixed-integer model developed considers fixed costs for opening the first facility at any location, plus additional fixed costs for every open facility in a location with already existing facilities. It is possible to open, close and reopen any facility at any location more than once during the planning horizon. It is also possible to consider different assignment costs depending on the size of the facility that is assigned to each client. This is important, because, in general, smaller facilities have smaller fixed costs but greater unitary operating costs. A primal-dual heuristic is developed that is able to find primal feasible solutions to the problem here described.

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1 Introduction

Capacitated location problems have been widely studied in the literature (see, for instance, Guignard and Spielberg, 1979, Jacobsen, 1983; Christofides and Beasley, 1983; Van Roy, 1986; Beasley, 1988; Cornuejols *et al*, 1991; Sridharan, 1995). Dynamic location problems have also been studied (see, for instance, Van Roy and Erlenkotter, 1982; Saldanha da Gama and Captivo, 2002). It is interesting to note that a capacitated dynamic location problem is, in essence, a capacity expansion problem: facilities are open in different time periods, increasing the total available capacity, in order to serve a (generally) increasing demand. In this research report, the authors study a problem where the expansion of capacity is explicitly considered and is achieved not only through the location of facilities at new sites but also through the location of facilities that will increase the already existing capacity at a given site (as in Shulman, 1991). Each facility capacity has to be chosen from a finite (small) set of feasible capacities, similar to what is described in Lee (1991) and Mazzola and Neebe (1999). Lee extends the classical capacitated location problem and considers a multiproduct capacitated facility location problem in which each facility capacity has to be chosen from a given set of admissible capacities. The author solves the problem using an algorithm based on a Benders' decomposition. Sridharan (1991) studies the problem of locating and choosing the size of the facility, by solving a capacitated location problem with side constraints (guaranteeing that at most one facility is located at each site). The problem is solved using a lagrangean heuristic. Mazzola and Neebe (1999) study a similar problem and develop a branch and bound algorithm. Ghiani *et al* (2002) study the problem of locating capacitated facilities, allowing several identical facilities to be located at the same site. The problem was motivated by a polling station location problem in an Italian Municipality. The authors solve the problem using a Lagrangean Heuristic.

As far as we know, in most of the references dealing explicitly with capacity expansions, they can be continuously incremented.

Hinomoto (1965) studies the problem of capacity expansion of a productive system, assuming the capacity can be expanded by the addition of facilities in discrete steps, and the size of a facility can be treated as a continuous factor. Erlenkotter (1975) develops two approaches to deal with capacity planning for large multilocation systems: an approximate approach based on an equivalent cost measure and an incomplete dynamic programming approach to systematically improve the approximate solution. Application to a real problem is described (India's nitrogenous fertilizer industry). Fong and Srinivasan (1981a) formulate the problem of continuous capacity expansion as a dynamic discrete time location mixed integer programming problem. The authors develop a heuristic to tackle the problem. In the sequel of this paper (Fong and Srinivasan, 1981b), the authors extend the problem considering a fixed cost if a capacity expansion takes place at a given location plus a cost proportional to the size of the expansion. Freindenfelds (1981a) considers the capacity expansion problem in which there are two types of demand and two types of facilities. The author considers that the capacity can be increased in a continuum of sizes, at a cost that does not depend either on time or on previous expansion decisions. In his book (Freindenfelds, 1981b), the author introduces a series of capacity expansion analytical models and applications, emphasizing the real capital investment decisions involved in establishment of new productive capacity. Smith (1981) generalizes the work of Manne[1], presenting an efficient algorithm that solves the deterministic capacity problem considering a finite planning horizon[2].

In 1982, Luss publishes a survey of the existing literature on capacity expansion problems. The author calls the readers attention to the lack of existing literature dealing with dynamic capacity expansion problems. In this survey the author considers both single and multi-facility location problems, with finite or infinite horizon time planning[2].

Min (1988) studies the problem of dynamic expansion and relocation of capacitated public facilities, considering multiple objectives. The author formulates the problem as a mixed integer goal programming model in a fuzzy decision environment. The problem is illustrated by considering the real case of expanding and relocating public libraries in the Columbus metropolitan public library system.

Shulman (1991) formulates the problem of dynamically locating and expanding the capacity of facilities. The author considers a small set of feasible expansion sizes (capacity expansion is achieved by dynamically locating more than one facility at a given location), and develops two algorithms: one deals with the more general problem that allows several facilities of different capacities to be located at the same site; the other solves the special case where it is only possible to locate facilities of the same size at one location.

The problem studied in this research report considers the situation where capacity expansion is achieved by locating additional facilities and the reduction of capacity is

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[1] A. S. Manne (1961), *Capacity Expansion and Probabilistic Growth*, Econometrics, 29

[2] In the paper's context, planning horizon refers to the time period when the additional capacity can be used. It can be interpreted as the lifetime of a facility. In this research report, the planning horizon is interpreted as the time interval, explicitly considered, in which it is possible to change the configuration of facility locations.

achieved by closing existing facilities. There is a finite (small) set of feasible capacities for the facilities to be located. The major differences between the problem here presented and the problems studied in the literature are the possibility of reducing the capacity at any time period (most of the problems studied consider only the possibility of increasing the capacity), the possibility of locating several facilities of different sizes in the same location and also the possibility of a facility being open, closed and reopen more than once during the planning horizon. Canel *et al* (2001) consider the possibility of a service being open, closed and reopen more than once. Nevertheless, the authors do not differentiate between open and reopen fixed costs (which, in most cases, are clearly different), and present a non-linear objective function. In the model here presented it is also possible to differentiate the operating costs of the different facilities.

According to Luss (1982) the major decisions in capacity expansion problems are: expansion sizes, expansion times and expansion locations. In this problem, one can say that the major decisions in capacity expansion and reduction are: expansion and reduction sizes, times and locations. The problem is formulated as a mixed-integer linear problem, and a primal-dual heuristic is described that can find primal feasible solutions. This work was motivated by the problem of locating transfer stations in a solid waste treatment system (see, for instance, Wirasinghe and Waters, 1983). Most transfer stations are composed by one or more equipments that can take one of a small set of different sizes. Each equipment has fixed and operating costs that are, usually, directly and inversely proportional (respectively) to its capacity.

In the next section the model developed is presented. In section 3, the dual problem of its linear relaxation is formulated. In section 4 the primal-dual heuristic (based on the work of Erlenkotter (1978) and Van Roy and Erlenkotter (1982)) is described. In section 5 some conclusions and future work directions are drawn.

2 The Proposed Mathematical Model

Consider the following definitions:

$J = \{1,...,n\}$ set of indexes corresponding to the clients' locations;

$I = \{1,...,m\}$ set of indexes corresponding to facilities' possible locations;

$S = \{1,...,q\}$ set of indexes corresponding to facilities' possible dimensions, ordered by ascending order of the corresponding capacities;

T = number of time periods considered in the planning horizon;

c_{ijs}^{t} = cost of fully assigning client j to a facility of dimension s located at i in period t;

FA_{ist}^{ξ} = fixed cost of opening a facility of dimension s at i at the beginning of period t, and closing the facility at the end of period ξ (the facility will be in operation from the beginning of t to the end of ξ), knowing that this is the first facility located at i;

FR_{ist}^{ξ} = unitary fixed cost of locating one facility of dimension s at i at the beginning of period t, and closing it at the end of period ξ (the facility will be in operation from the beginning of t to the end of ξ), knowing that this facility is not the first to be located at i.[3]

d_j^t = demand of client j at period t;

Q_s = maximum capacity of a facility of dimension s;

$Nmax$ = maximum number of facilities that can be operational at one location at the same time.

Let us define the variables:

$$a_{ist}^{\xi} = \begin{cases} 1 & \text{if a facility of dimension } s \text{ located at } i \text{ is open at the beginning of period } t \\ & \text{and stays open until the end of period } \xi, \text{knowing that this is the first} \\ & \text{facility to be located at } i \\ 0 & \text{otherwise} \end{cases}$$

r_{ist}^{ξ} = number of facilities of dimension s located at i at the beginning of period t and staying open until the end of period ξ, knowing that this is not the first facility to be located at i.

x_{ijs}^{t} = fraction of customer j's demand that is served by a facility of dimension s located at i during period t.

The first facility to be located at i will be called *i-first* facility. All the other facilities that are located at i will be called *i-follow* facilities.

The dynamic location problem of expansion and reduction of available capacities, considering that it is possible to reconfigure one location more than once during the planning horizon can be formulated as:

[3] The fixed cost FA_{ist}^{ξ} should be equal to FR_{ist}^{ξ} plus the additional cost of installing for the first time a facility at location i. This additional cost may represent costs of land acquisition, development of infra-structures, etc. Let us define this additional cost as f_i^t. Then $FA_{ist}^{\xi} = FR_{ist}^{\xi} + f_i^t, \forall i,s,t,\xi \geq t$.

DLPER

$$Min\sum_{t}\sum_{i}\sum_{j}\sum_{s}c_{ijs}^{t}x_{ijs}^{t}+\sum_{t}\sum_{i}\sum_{s}\sum_{\xi=t}^{T}FA_{ist}^{\xi}a_{ist}^{\xi}+\sum_{t}\sum_{i}\sum_{s}\sum_{\xi=t}^{T}FR_{ist}^{\xi}r_{ist}^{\xi} \quad (1)$$

subject to:

$$\sum_{i}\sum_{s}x_{ijs}^{t}=1, \qquad \forall\ j,t \quad (2)$$

$$\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{is\tau}^{\xi}+r_{is\tau}^{\xi}\right)-x_{ijs}^{t}\geq 0, \qquad \forall\ i,j,s,t \quad (3)$$

$$Nmax\sum_{s'}\sum_{\tau=1}^{t}\sum_{\psi=\tau}^{T}a_{is'\tau}^{\psi}-r_{ist}^{\xi}\geq 0, \qquad \forall\ i,s,t,\xi\geq t \quad (4)$$

$$\sum_{s}\sum_{t=1}^{T}\sum_{\xi=t}^{T}a_{ist}^{\xi}\leq 1, \qquad \forall\ i \quad (5)$$

$$\sum_{s}\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{is\tau}^{\xi}+r_{is\tau}^{\xi}\right)\leq Nmax, \qquad \forall\ i,t \quad (6)$$

$$Q_s\sum_{\tau=1}^{t}\sum_{\xi=t}^{T}\left(a_{is\tau}^{\xi}+r_{is\tau}^{\xi}\right)-\sum_{j}d_{j}^{t}x_{ijs}^{t}\geq 0, \qquad \forall i,s,t \quad (7)$$

$$\begin{aligned} &a_{ist}^{\xi}\in\{0,1\}, && \forall i,s,t,\xi\geq t \\ &r_{ist}^{\xi}\geq 0 \text{ and integer}, && \forall i,s,t,\xi\geq t \end{aligned} \quad (8)$$

These constraints guarantee that:

(2): Each client’s demand will be fully satisfied in each time period;

(3): A client will be assigned to open facilities only;

(4): A facility of dimension *s* that is open at *i* at the beginning of period *t* can be considered as an *i-follow* facility only if there is an *i-first* facility that has been open at the beginning of a time period *t*’≤ *t* (an *i-follow* facility and the *i-first* facility can be located simultaneously);

(5): For each location *i*, there can be at most one *i-first* facility during the whole planning horizon;

(6): There is an upper limit to the number of operating facilities at location i, in each time period;

(7): the facilities' maximum capacity will not be exceeded in any time period.

The proposed model allows that, in each time period and in each location, every mix of facilities of different or equal dimensions is feasible.

3 The Dual Problem and Complementary Conditions

3.1 Formulation of the Dual Problem

Consider constraints (5') and (6') that are equivalent to (5) and (6), respectively:

$$\sum_{s}\sum_{t=1}^{T}\sum_{\xi=t}^{T} -a_{ist}^{\xi} \geq -1, \qquad \forall\ i \qquad (5')$$

$$\sum_{s}\sum_{\tau=1}^{t}\sum_{\xi=\tau}^{T} -\left(a_{is\tau}^{\xi} + r_{is\tau}^{\xi}\right) \geq -Nmax, \qquad \forall\ i,t \qquad (6')$$

Associating dual variables v_j^t with constraints (2), dual variables w_{ijs}^{t} with constraints (3), dual variables u_{ist}^{ξ} with constraints (4), dual variables ρ_i with constraints (5'), dual variables π_i^t with constraints (6'), and dual variables λ_{is}^{t} with constraints (7), the dual problem of DLPER can be formulated as D-DLPER:

D-DLPER

$$Max\sum_{t}\sum_{j} v_j^t - \sum_{i}\rho_i \ - Nmax\sum_{t}\sum_{i}\pi_i^t \qquad (9)$$

subject to:

$$v_j^t - w_{ijs}^t - d_j^t\lambda_{is}^t \leq c_{ijs}^t, \qquad \forall i,j,s,t \qquad (10)$$

$$\sum_{j}\sum_{\tau=t}^{\xi} w_{ijs}^{\tau} + Nmax\sum_{s'}\sum_{\tau=t}^{T}\sum_{\psi=\tau}^{T} u_{is'\tau}^{\psi} - \rho_i - \sum_{\tau=t}^{\xi}\pi_i^{\tau} + Q_s\sum_{\tau=t}^{\xi}\lambda_{is}^{\tau} \leq FA_{ist}^{\xi},$$

$$\forall i,s,t,\xi = t,\cdots,T \qquad (11)$$

$$\sum_{j}\sum_{\tau=t}^{\xi} w_{ijs}^{\tau} - u_{ist}^{\xi} - \sum_{\tau=t}^{\xi}\pi_i^{\tau} + Q_s\sum_{\tau=t}^{\xi}\lambda_{is}^{\tau} \le FR_{ist}^{\xi}, \qquad \forall i,s,t,\xi = t,\cdots,T \tag{12}$$

$$w_{ijs}^{t}, u_{ist}^{\xi}, \rho_i, \pi_i^{t}, \lambda_{is}^{t} \ge 0, \qquad \forall i,j,s,t,\xi = t,\cdots,T$$

An equivalent condensed formulation is obtained by considering $w_{ijs}^{t} = max\{0, v_j^{t} - c_{ijs}^{t} - d_j^{t}\lambda_{is}^{t}\}$.

CD-DLPER

$$Max\sum_{t}\sum_{j} v_j^{t} - \sum_{i}\rho_i - Nmax\sum_{t}\sum_{i}\pi_i^{t}$$

subject to:

$$\sum_{j}\sum_{\tau=t}^{\xi} max\{0, v_j^{\tau} - c_{ijs}^{\tau} - d_j^{\tau}\lambda_{is}^{\tau}\} \le FA_{ist}^{\xi} - Nmax\sum_{s}\sum_{\tau=t}^{T}\sum_{\psi=\tau}^{T} u_{is\tau}^{\psi} + \rho_i + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - Q_s\sum_{\tau=t}^{\xi}\lambda_{is}^{\tau},$$

$$\forall i,s,t,\xi = t,\cdots,T \tag{13}$$

$$\sum_{j}\sum_{\tau=t}^{\xi} max\{0, v_j^{\tau} - c_{ijs}^{\tau} - d_j^{\tau}\lambda_{is}^{\tau}\} \le FR_{ist}^{\xi} + u_{ist}^{\xi} + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - Q_s\sum_{\tau=t}^{\xi}\lambda_{is}^{\tau},$$

$$\forall i,s,t,\xi = t,\cdots,T \tag{14}$$

$$u_{ist}^{\xi}, \rho_i, \pi_i^{t}, \lambda_{is}^{t} \ge 0, \qquad \forall i,j,s,t,\xi = t,\cdots,T$$

3.2 Complementary Conditions

Let us define:

$$SA_{ist}^{\xi} = FA_{ist}^{\xi} - N\,max\sum_{s'}\sum_{\tau=t}^{T}\sum_{\psi=\tau}^{T} u_{is'\tau}^{\psi} + \rho_i + \sum_{\tau=t}^{\xi}\pi_i^{\tau} - \sum_{j}\sum_{\tau=t}^{\xi} max\{0, v_j^{\tau} - c_{ijs}^{\tau} - d_j^{\tau}\lambda_{is}^{\tau}\} - Q_s\sum_{\tau=t}^{\xi}\lambda_{is}^{\tau},$$

$$\forall i,s,t,\xi = t,\cdots,T \tag{15}$$

$$SR_{ist}^{\xi} = FR_{ist}^{\xi} + u_{ist}^{\xi} + \sum_{\tau=t}^{\xi} \pi_i^{\tau} - \sum_j \sum_{\tau=t}^{\xi} max\left\{0, v_j^{\tau} - c_{ijs}^{\tau} - d_j^{\tau} \lambda_{is}^{\tau}\right\} - Q_s \sum_{\tau=t}^{\xi} \lambda_{is}^{\tau} ,$$

$$\forall i, s, t, \xi = t, \cdots, T \qquad (16)$$

$$S_{ist}^{\xi} = min\left\{SA_{ist}^{\xi}, SR_{ist}^{\xi}\right\}, \qquad \forall i, s, t, \xi = t, \cdots, T \qquad (17)$$

The following complementary conditions hold if in presence of optimal primal and dual solutions to the respective problems (when there is no duality gap).

$$\left(\sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{is\tau}^{\xi} + r_{is\tau}^{\xi} \right) - x_{ijs}^{t} \right) w_{ijs}^{t} = 0, \qquad \forall i, j, t \qquad (18)$$

$$\left(Nmax \sum_{s'} \sum_{\tau=1}^{t} \sum_{\psi=\tau}^{T} a_{is'\tau}^{\psi} - r_{ist}^{\xi} \right) u_{ist}^{\xi} = 0, \qquad \forall\ i,s,t,\xi \geq t \qquad (19)$$

$$\left(\sum_s \sum_{t=1}^{T} \sum_{\xi=t}^{T} a_{ist}^{\xi} - 1 \right) \rho_i = 0, \qquad \forall i \qquad (20)$$

$$\left(\sum_s \sum_{\tau=1}^{t} \sum_{\xi=\tau}^{T} \left(a_{is\tau}^{\xi} + r_{is\tau}^{\xi} \right) - N\, max \right) \pi_i^t = 0, \qquad \forall i, t \qquad (21)$$

$$SA_{ist}^{\xi} \cdot a_{ist}^{\xi} = 0, \qquad \forall i, s, t, \xi = t, \cdots, T \qquad (22)$$

$$SR_{ist}^{\xi} \cdot r_{ist}^{\xi} = 0, \qquad \forall i, s, t, \xi = t, \cdots, T \qquad (23)$$

$$\left(Q_s \sum_{\tau=1}^{t} \sum_{\xi=t}^{T} \left(a_{is\tau}^{\xi} + r_{is\tau}^{\xi} \right) - \sum_j d_j^t x_{ijs}^t \right) \lambda_{is}^t = 0 , \qquad \forall i,s,t \qquad (24)$$

4 Primal-Dual Heuristic

The primal-dual heuristic here proposed follows the work of Erlenkotter (1978), Guignard and Spielberg (1979) and Van Roy and Erlenkotter (1982).

The heuristic builds primal admissible solutions based on dual admissible solutions, trying to force the satisfaction of the complementary conditions. If a pair of primal and dual admissible solutions is found that satisfies conditions (18) – (24), then the optimal solution has been calculated. When this is not achieved, the best dual solution it was possible to obtain gives a valid lower bound for the optimal solution and may be used to assess the quality of the best admissible primal solution found.

The heuristic operating scheme is the following:

1. Initialisation of dual variables;
2. Dual Ascent Procedure for dual variables v_j^t;
3. Primal Procedure;
4. Dual Adjustment Procedure for dual variables ρ_i. If the dual solution is changed go to 2;
5. Repeat the Primal-dual Adjustment Procedure for variables v_j^t until there is no improvement in the dual objective function value;
6. Dual Adjustment Procedure for dual variables ρ_i. If the dual solution is changed go to 2;
7. Dual Ascent Procedure for dual variables u_{ist}^{ξ}. If the dual solution is changed go to 2;
8. Dual Descent Procedure for dual variables u_{ist}^{ξ}. If the dual solution is changed go to 2;
9. If *Nmax* =1, then execute the Dual Adjustment Procedure for variables π_i^t. If the dual solution is changed go to 2.
10. Dual Ascent Procedure for dual variables λ_{is}^t. If the dual solution is changed go to 2;
11. Dual Descent Procedure for dual variables λ_{is}^t. If the dual solution is changed go to 2;

The heuristic will stop when the optimal primal solution is found or when there are no improvements in either the primal or the dual objective function values.

The Dual Ascent Procedure for dual variables v_j^t, the Primal-dual Adjustment Procedure for variables v_j^t, the Dual Adjustment Procedures for variables π_i^t and ρ_i are exactly the same as those developed by us for the resolution of the Dynamic Location Problem with Opening, Closure and Reopening of Facilities - DLPOCR (Dias *et al*, 2004a). Instead of considering the set I of possible locations, it should be considered the set $I \times S$. In the Dual Ascent Procedure for dual variables v_j^t, and in the Primal-dual Adjustment Procedure the assignment costs should be considered equal to $c_{ijs}^t + d_j^t \lambda_{is}^t$. For this reason, these procedures will not be described here. The Dual Adjustment Procedure for variables π_i^t is executed only when *Nmax* equals 1. The computational experiments put in evidence that, in every other situation, the change of this dual variable does not increase the value of the dual objective function due to the variable's dual objective function coefficient $-Nmax$.

The Dual Ascent and Descent Procedures for variables λ_{is}^t are also similar to the ones already developed by us for the DLPOCR with maximum capacity restrictions (Dias *et al*, 2004b). It is sufficient to consider facilities (i,s) with maximum capacities equal to Q_s, instead of facilities i with maximum capacities equal to Q_i. The two procedures referred will not be repeated here.

4.1 Initialization of dual variables

The dual variables are initialised as follows:

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1. $v_j^t \leftarrow \min_{i,s}\left\{c_{ijs}^t\right\}, \forall j,t;\ \pi_i^t \leftarrow 0, \forall i,t;$

2. If $u_{ist}^{\xi} \leftarrow \max\left\{0, -FR_{ist}^{\xi}\right\}, \forall i,s,t,\xi = t,\cdots,T$

3. $\rho_i \leftarrow \max\left\{0, -\min_{\substack{t,s\\ \xi \geq t}}\left(FA_{ist}^{\xi} - Nmax\sum_{s}\sum_{\tau=t}^{T}\sum_{\psi=\tau}^{T} u_{is\tau}^{\psi}\right)\right\}, \forall i$

4.2 Primal Procedure

Consider the following definitions:

$I^* = \{ (i,s,\tau,\xi): S_{is\tau}^{\xi} = 0\}$,

$I_t^* = \{(i,s) : (i,s,\tau,\xi) \in I^* \text{ and } \tau \leq t \leq \xi\}$,

$I_t^+ = \{(i,s)$: at least one facility of dimension s is open at i during period $t\}$,

$I^+ = \{ (i,s,\tau,\xi) : a_{is\tau}^{\xi} = 1 \text{ or } r_{is\tau}^{\xi} > 0\}$,

F_{is}^t = smallest cost incurred by opening a facility of dimension s at i during period t,

$Num[i,s,t]$ = total number of open facilities at location i of dimension s during period t[4],

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[4] Represents the total number of elements $(i,s,\tau,\xi) \in I^+$ such that $\tau \leq t \leq \xi$.

$N[i,t]$ = total number of open facilities at location i during period t[5].

Primal Procedure

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1. $I^+ \leftarrow \emptyset$. $I_t^+ \leftarrow \emptyset, \forall t$. Build sets I^* and I_t^*. $Num[i,s,t] \leftarrow 0$, $\forall i,s,t$. $Num[i,t] \leftarrow 0$, $\forall i,t$.
2. For t=1 until T, include in set I_t^+ all pairs $(i,s)\in I_t^*$ such that $\exists j : v_j^t \geq c_{ijs}^t$ and $v_j^t < c_{i's'j}^t, \forall (i',s') \neq (i,s)$ (essential facilities as in Van Roy and Erlenkotter (1982) and Dias et al (2004a)).
3. For each client j such that $v_j^t < c_{ijs}^t, \forall (i,s) \in I_t^+$, include in set I_t^+ the pair (i,s) such that $c_{ijs}^t = \min_{v_j^t \geq c_{i'js'}^t} c_{i'js'}^t$.
4. Build set I^+. Update $I_t^+, \forall$t.
5. $t \leftarrow 1$;
6. $N[i,t] \leftarrow \sum_s Num[i,s,t], \forall i \in I$.
7. $D \leftarrow \sum_j d_j^t$; $C \leftarrow \sum_{(i,s)\in I_t^+} (Q_s \cdot Num[i,s,t])$. If $D \leq C$ then go to 17.
8. Calculate $F_{is}^t, \forall\ i \in I, s \in S$.
9. If $F_{is}^t = +\infty$, $\forall\ i \in I, s \in S$, then go to 13.
10. Calculate $F'^t_{is} = \frac{F_{is}^t}{Q_s}\left\lceil \frac{\phi_s}{Q_s} \right\rceil, \forall\ i \in I, s \in S$, where $\phi_s = \begin{cases} D-C, & \text{if } C+Q_s < D \\ Q_s, & \text{otherwise} \end{cases}$.
11. Consider the pair (i',s') such that $F'^t_{i's'} = \min_{i\in I, s \in S} \left\{ F'^t_{is} \right\}$;
12. $I_t^+ \leftarrow I_t^+ \cup \{(i',s')\}$; Rebuild sets I^+ and $I_t^+, \forall t$; $C \leftarrow C + Q_s$; $Num[i,s,t] \leftarrow Num[i,s,t]+1$; $N[i,t] \leftarrow N[i,t]+1$. If $D \leq C$ then go to 17. Else go to 8.
13. If $s = q$ for every $(i,s,\tau,\xi) \in I^+$ with $\tau \leq t \leq \xi$, then Stop. The procedure cannot find a feasible solution. Else go to 14.
14. If $D>C$ then, for every $(i,s,\tau,\xi) \in I^+$ with $s<q$ and $\tau \leq t \leq \xi$, calculate $H_{is\tau}^{\xi} = \left(FR_{i(s+1)\tau}^{\xi} - FR_{is\tau}^{\xi} \right) \cdot \left\lceil \frac{D-C}{Q_{s+1}-Q_s} \right\rceil$.

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[5] Represents the total number of elements $(i,s,\tau,\xi) \in I^+$ such that $\tau \leq t \leq \xi$, $\forall\ s \in S$.

15. Choose $(i,s,\tau,\xi) \in I^+$ with $s<q$ and $\tau \le t \le \xi$ that corresponds to the smallest $H^{\xi}_{is\tau}$.

$I^+ \leftarrow I^+ \setminus (i,s,\tau,\xi)$; $I^+ \leftarrow I^+ \cup \{(i,s+1,\tau,\xi)\}$; $Num[i,s,t] \leftarrow Num[i,s,t] - 1$;

$Num[i,s+1,t] \leftarrow Num[i,s+1,t] + 1$; $C \leftarrow C + Q_{s+1} - Q_s$.

16. If $D \le C$ then go to 17, else go to 13.

17. $t \leftarrow t + 1$; If $t \le T$ go to 6. Else go to 18.

18. $t \leftarrow 1$;

19. Solve one transportation problem considering as sources the set J of clients (with supplies d_j^t), as destinations all pairs $(i,s) \in I_t^+$ (with demands $Q_s Num[i,s,t]$), and transportation costs (per unit) given by $\dfrac{c^t_{ijs}}{d^t_j}$.

20. $t \leftarrow t + 1$; If $t \le T$ go to 19. Else go to 21.

21. Calculate the values of primal variables $a^{\xi}_{is\tau}$ and $r^{\xi}_{is\tau}$.

22. Execute a local exchange search procedure.

There are several steps in this primal procedure that deserve further explanations.

Step 4 of the primal procedure (building set I^+) can be described as follows:

Step 4 of the Primal Procedure

1. $i \leftarrow 1$.
2. $s \leftarrow 1$.
3. If $\exists\, t$: $(i,s) \in I_t^+$, go to 4; else go to 9.
4. $t_1 \leftarrow min\{\tau : (i,s) \in I_\tau^+\}$; $t_2 \leftarrow max\{\tau : (i,s) \in I_\tau^+\}$.
5. Calculate $Num[i,s,t]$ and $N[i,t]$, $\forall t$. $I1^+ \leftarrow I^+$. Execute Procedure 1.
6. Calculate $Num[i,s,t]$ and $N[i,t]$, $\forall t$. $I2^+ \leftarrow I^+$. Execute Procedure 2.
7. $sum1 \leftarrow \sum_{(i,s,\tau,\xi)\in I1^+} FR^{\xi}_{is\tau}$; $sum2 \leftarrow \sum_{(i,s,\tau,\xi)\in I2^+} FR^{\xi}_{is\tau}$.
8. If $(sum1 < sum2)$ $I^+ \leftarrow I1^+$; else $I^+ \leftarrow I2^+$. Calculate $Num[i,s,t]$, $\forall t$.
9. $s \leftarrow s + 1$; If $s > q$ then go to 10. Else go to 3.
10. $i \leftarrow i + 1$; if $i > m$ stop. Else go to 2.

After the execution of step 4, set I^{+} has as many (i,s,τ,ξ) elements as the number of facilities of dimension s that are operating at i from the beginning of time period τ to the end of time period ξ.

Procedures 1 and 2 are based on similar procedures described in Dias *et al*, 2002. The main differences are due to the fact that in DLPOCR an admissible solution has, at most, one facility open in each location during each time period. In DLPER, it is admissible to have more than one facility simultaneously open. As described in Dias *et al*, 2002, procedure 1 builds a solution from period t_1 forward, while procedure 2 builds a solution from period t_2 backwards.

Step 7 of the primal procedure tests the admissibility of the primal solution constructed in terms of total available capacity.

In step 8 of the primal procedure, the calculation of F_{is}^{t} accounts for all the hypotheses of having a facility of dimension s open at i during time period t. There are two possibilities: a new facility is placed or the operating upper and/or lower time limits of an already existing facility are changed. The calculation of F_{is}^{t} tries to find the best choice in terms of fixed costs incurred.

Let us define:

$$F_i^{\tau} = \begin{cases} FA_{is\tau}^{\xi} - FR_{is\tau}^{\xi}, \forall s, \xi, & if\ \nexists (i,s') \in I_t^{+} : t < \tau, \forall s' \in S \\ 0, & \text{otherwise} \end{cases}.$$

This value represents the fixed cost incurred if the first equipment is placed in i at the beginning of time period τ.

Consider that facility i is not open during period t but is open in time periods before and after t, as depicted in figure 1.

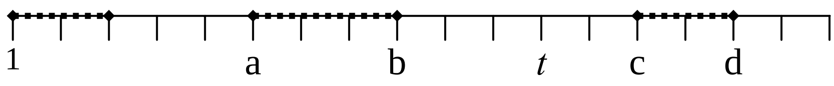

Figure 1: ◆•••••••◆ represents facility i operating time periods

Time periods a, b, c, d can be defined formally as:

$$b = max\left\{0, \max_{t'<t}\left\{t' : i \in I_{t'}^{+}\right\}\right\};\ a = \left\{t' : (i,t',b) \in I_A^{+} \cup I_R^{+}\right\};$$

$$c = min\left\{T+1, \min_{t'>t}\left\{t' : i \in I_{t'}^{+}\right\}\right\};\ d = \left\{t' : (i,c,t') \in I_A^{+} \cup I_R^{+}\right\};$$

Procedure 1:

```
begin ← 1;time ← t1;
WHILE time ≤ t2
        IF N[i,time]=Nmax THEN time ← time +1; CONTINUE; ENDIF
        IF (i,s) ∈ I+time THEN
                τ ← begin; ξ ← T; t ← time; stop ← false;
                WHILE τ ≤ t and not stop
                        WHILE ξ ≥ t and not stop
                                IF ∃ (i, s,τ, ξ ) ∈ I* THEN
                                        (i, s, τ, ξ )→ I1+
                                        FOR k = τ TO k = ξ DO
                                                Num[i,s,k] ← Num[i,s,k] + 1
                                                N[i,k] ← N[i,k] + 1
                                                IF N[i,k]=Nmax and begin ≤ k THEN
                                                        begin ← k +1
                                                ENDIF
                                        ENDFOR
                                        time ← ξ +1
                                        stop ← true
                                ENDIF
                                ELSE ξ ← ξ -1 ENDELSE
                        ENDWHILE
                        τ ← τ +1; ξ ← T
                      ENDWHILE
               IF not stop THEN
                        ξ ← t2
                        FOR k = time TO k = t2 DO
                                IF N[i,k] = Nmax THEN
                                        ξ ← k-1
                                        BREAK
                                ENDIF
                        ENDFOR
                        IF (time ≤ ξ ) THEN
                                (i, s, time, ξ )→ I1+
                                FOR k = time TO k = ξ DO
                                        N[i,k] ← N[i,k]+1; Num[i,s,k] ← Num[i,s,k] +1
                                        IF N[i,k]=Nmax and begin ≤ k THEN
                                                begin ← k +1
                                        ENDIF
                                ENDFOR
                                     time ← ξ +1
                        ENDIF
                        ELSE time ← time +1; begin ← ξ +1; ENDELSE
               ENDIF
        ENDIF
        ELSE time ← time +1 ENDELSE
ENDWHILE
```

Procedure 2

```
end ← T; time ← t2 ;
WHILE time ≥ t1
        IF N[i,time]=Nmax THEN time ← time – 1; CONTINUE; ENDIF
        IF (i,s) ∈ I+time THEN
                τ ← 1; ξ ← end; t ← time; stop ← false;
                WHILE ξ ≥ t and not stop
                        WHILE τ ≤ t and not stop
                                IF ∃ (i, s, τ ,ξ ) ∈ I* THEN
                                        (i, s, τ,ξ )→ I2+
                                        FOR k = τ TO k = ξ DO
                                                N[i,k] ← N[i,k]+1; Num[i,s,k] ← Num[i,s,k] +1
                                                IF N[i,k]=Nmax and end ≥ k THEN
                                                        end ← k –1
                                                ENDIF
                                        ENDFOR
                                        time ← τ -1
                                        stop ← true
                                ENDIF
                                ELSE τ ← τ +1 ENDELSE
                        ENDWHILE
                        ξ ← ξ -1; τ ←1
                ENDWHILE
                IF not stop THEN
                        τ ← t1
                        FOR k = time DOWNTO k = τ DO
                                IF N[i,k] = Nmax THEN
                                        τ ← k +1
                                        BREAK
                                ENDIF
                        ENDFOR
                        IF (time ≥ τ ) THEN
                                (i, s, τ, time)→ I2+
                                FOR k = τ TO k = time DO
                                        N[i,k] ← N[i,k]+1; Num[i,s,k] ← Num[i,s,k] +1
                                        IF N[i,k]=Nmax and end ≥ k THEN
                                                end ← k –1
                                        ENDIF
                                ENDFOR
                                        time ← τ –1
                        ENDIF
                        ELSE time ← time –1; end ← τ –1; ENDELSE
                ENDIF
        ENDIF
        ELSE time ← time –1; ENDELSE
ENDWHILE
```

Time period b represents the time period before and nearest to t such that facility i is operating. Time period c represents the time period after and nearest to t such that facility i is operating.

Let us also define F_i' as the fixed cost of installing for the first time a facility at i, considering the set I^+:

$$F_i' = \begin{cases} F_i^\tau & \text{if } \exists (i,s') \in I_\tau^+, s' \in S \wedge \nexists (i,s') \in I_t^+ : t < \tau, \forall s' \in S \\ 0, & \text{otherwise} \end{cases}$$

Calculation of F_{is}^t

1. $F_{is}^t \leftarrow +\infty$. If $N[i,t] = Nmax$ then stop, else go to 2.

2. $$F_{is}^t \leftarrow min\left\{ F_{is}^t, \begin{cases} min\left\{ FA_{is\tau}^\xi - F_i' : \tau \le t \le \xi \ \ and \ \ N[i,k] < Nmax, k \in [\tau,\xi] \right\}, & if \ \exists (i,s') \in I_{t'}^+, t' \le t, s' \in S \\ min\left\{ FR_{is\tau}^\xi : \tau \le t \le \xi \ \ and \ \ N[i,k] < Nmax, k \in [\tau,\xi] \right\}, & otherwise \end{cases} \right\}$$ [6]

3. Calculate $b = max\left\{ 0, \max_{t'<t}\left\{ t' : (i,s) \in I_{t'}^+ \right\} \right\}$; $a = \left\{ t' : (i,s,t',b) \in I^+ \right\}$;

   $c = min\left\{ T+1, \min_{t'>t}\left\{ t' : (i,s) \in I_{t'}^+ \right\} \right\}$; $d = \left\{ t' : (i,s,c,t') \in I^+ \right\}$.

4. If $b = 0$ and $c \le T$ then

   $F_{is}^t \leftarrow min\left\{ F_{is}^t, \min_\tau \left\{ FR_{is\tau}^d + F_i^\tau - FR_{isc}^d - F_i^c : \tau \le t, N[i,k] < Nmax, k \in [\tau, c[\right\} \right\}$ [7], stop. Else go to 5.

5. If $c = T + 1$ and $b > 0$ then

---

[6]If there is any facility operating in i during a time period less than or equal to t, then the procedure considers only reopening fixed costs. Otherwise, it considers the cost of opening a facility for the first time, but discounts the fixed cost of installing for the first time a facility at time period $t' > \tau$ (if that is the case).

[7] At this step the procedure tries to calculate the changes in fixed costs by merging two time intervals: (τ,ξ) and (c,d). If period c corresponds to the period of the first installation of a facility in i then $\nexists (i,s') \in I_t^+, t < \tau, \forall s'$, so F_i^τ corresponds to the fixed open costs of installing the first facility in i. If the facility was open for the first time before c then $F_i^c = 0$. The same holds for $F_i^\tau = 0$.

$$F_{is}^{t} \leftarrow min\left\{F_{is}^{t}, \min_{\xi}\left\{FR_{isa}^{\xi} - FR_{isa}^{b} : \xi \geq t, N[i,k] < Nmax, k \in]b,\xi]\right\}\right\}$$, stop. Else go to 6.

6. If $c \leq T$ and $b > 0$ and $N[i,k] < Nmax$, $k \in]b,c[$ then

$$F_{is}^{t} \leftarrow min\left\{F_{is}^{t}, FR_{isa}^{d} - FR_{isa}^{b} - FR_{isc}^{d}\right\}.$$

Step 10 penalizes the facilities whose dimension is not sufficient to cover the difference between the clients' total demand and the total available capacity (in this case it would be necessary to open more than one facility). The set I^{+} is updated, at step 12, considering the upper and lower limits that correspond to the smallest F'^{t}_{is} value. It is important to note that, in set I^{+}, it is possible to have several identical elements (i,s,τ,ξ). This corresponds to the situation where several identical facilities are installed at the same location and are open and closed in the same time periods.

If it is not possible to find a feasible solution by opening more facilities during period t, steps (13)-(16) try to build a feasible solution by changing the dimension of the already located facilities. If all the facilities installed are of dimension q, then the procedure will not be able to build a feasible solution. Otherwise, for every $(i,s,\tau,\xi)\in I^{+}$ with $s<q$ and $\tau\leq t\leq\xi$, the procedure calculates the cost of changing dimension s to s+1 (penalizing those changes that are not sufficient to guarantee primal admissibility).

Steps (18)-(20) solve the allocation problem, through the resolution of T transportation problems.

Step 21 calculates the values of the primal variables $a_{is\tau}^{\xi}$ and $r_{is\tau}^{\xi}$. This is done in a straightforward manner. As costs $FA_{is\tau}^{\xi}$ are equal to $FR_{is\tau}^{\xi}$ plus the additional cost of opening a facility in i for the first time (see footnote 1), the following procedure is used for each location i:

1. If $N[i,t] = 0$, $\forall t$, then stop.
2. Choose arbitrarily one element $(i,s,\tau,\xi) \in I^{+}$, such that $\tau = min\{t: \exists\, (i,s) \in I_t^{+}\}$.
3. Set $a_{is\tau}^{\xi} = 1$. Eliminate (i,s,τ,ξ) from set I^{+}.
4. All variables $r_{is\tau}^{\xi}$ are set to the number of elements $(i,s,\tau,\xi) \in I^{+}$.

In step 22, the procedure tries to improve the feasible primal solution found executing a very simple local exchange heuristic. The local exchange heuristic tries to change variable

$a^{\xi}_{is\tau}$ (and $r^{\xi}_{is\tau}$), to $a^{\xi}_{is'\tau}$ (and $r^{\xi}_{is'\tau}$), $\forall$ $s' \neq s$ and $s' \in S$. It chooses the admissible change that corresponds to the greatest improve in the primal objective function. The process is repeated until there are no improvements in the primal objective function value.

4.3 Dual Ascent Procedure for dual variables u^{ξ}_{ist}

As can be seen by expressions (15) and (16), the increase in the dual variable u^{ξ}_{ist} can increase slacks SR^{ξ}_{ist}, but decreases slacks $SA^{\xi}_{is'\tau}$, $\tau \leq t$, $\forall s' \in S$. It is only worth trying to increase slacks SR^{ξ}_{ist} such that $SR^{\xi}_{ist} = 0$ and $SA^{\xi}_{ist} > 0$, otherwise the value S^{ξ}_{ist} would not be changed and wouldn't be possible to increase the value of dual variables v_j^t.

Dual Ascent Procedure for variables u^{ξ}_{ist}

1. $i \leftarrow 1$.
2. $s \leftarrow 1$.
3. $t \leftarrow 1$.
4. $\xi \leftarrow t$.
5. $\Delta u^{\xi}_{ist} \leftarrow 0$.
6. If $SR^{\xi}_{ist} = 0$ and $SA^{\xi}_{ist} > 0$, then $\Delta u^{\xi}_{ist} \leftarrow \dfrac{SA^{\xi}_{ist}}{2}$. Else go to 10.
7. $\Delta u^{\xi}_{ist} \leftarrow min\left\{ \Delta u^{\xi}_{ist}, \min_{\substack{s' \in S \\ \tau \leq t \\ \psi \geq \tau}} SA^{\psi}_{is'\tau} \right\}$. $\Delta u^{\xi}_{is\tau} \leftarrow \dfrac{\Delta u^{\xi}_{is\tau}}{Nmax}$. If $\Delta u^{\xi}_{ist} = 0$ then go to 10. Else go to 8.
8. $SR^{\xi}_{ist} \leftarrow SR^{\xi}_{ist} + \Delta u^{\xi}_{ist}$; $SA^{\xi}_{is'\tau} \leftarrow SA^{\xi}_{is'\tau} - \Delta u^{\xi}_{ist} \cdot Nmax$, $\tau \leq t$, $\forall s' \in S$; $u^{\xi}_{ist} \leftarrow u^{\xi}_{ist} + \Delta u^{\xi}_{ist}$.
9. Execute the Dual Ascent Procedure for Variables v_j^t.
10. $\xi \leftarrow \xi + 1$. If $\xi > T$ then $t \leftarrow t + 1$ and go to 11. Else go to 5.
11. If $t > T$ then $s \leftarrow s + 1$ and go to 12. Else go to 4.
12. If $s > q$ then $i \leftarrow i + 1$ and go to 13. Else go to 3.
13. If $i > m$ then stop. Else go to 2.

In step 6 of this procedure, Δu_{ist}^{ξ} takes the value $\frac{SA_{ist}^{\xi}}{2}$ because if it would take the value SA_{ist}^{ξ} this slack could become equal to zero in step 8 of this procedure, decreasing the possibilities of improving the dual objective function value. If $SA_{ist}^{\xi} = \min_{\substack{s' \in S \\ \tau \leq' t \\ \psi \geq \tau}} \left\{ SA_{is'\tau}^{\psi} \right\}$, then SA_{ist}^{ξ} and SR_{ist}^{ξ} will end up with the same value in step 8 of the procedure.

4.4 Dual Descent Procedure for dual variables u_{ist}^{ξ}

Decreasing the value of the dual variable u_{ist}^{ξ} will decrease the value of slack SR_{ist}^{ξ}, but will increase the value of all slacks $SA_{is'\tau}^{\xi}$, $\tau \leq t$, $\forall s' \in S$. If the procedure increases the value of a slack that was blocking dual variables v_j^t, it is possible to increase the dual objective function value.

Dual Descent Procedure for variables u_{ist}^{ξ}

1. $i \leftarrow 1$.
2. $s \leftarrow 1$.
3. $t \leftarrow 1$.
4. $\xi \leftarrow t$.
5. $\Delta u_{ist}^{\xi} \leftarrow min\{ u_{ist}^{\xi}, \frac{SR_{ist}^{\xi}}{2} \}$. If $\Delta u_{ist}^{\xi} = 0$, go to 8.
6. $SR_{ist}^{\xi} \leftarrow SR_{ist}^{\xi} - \Delta u_{ist}^{\xi}$; $SA_{is'\tau}^{\psi} \leftarrow SA_{is'\tau}^{\psi} + \Delta u_{ist}^{\xi} \cdot Nmax$, $\tau \leq t$ and $\psi \geq \tau$, $\forall s' \in S$; $u_{ist}^{\xi} \leftarrow u_{ist}^{\xi} - \Delta u_{ist}^{\xi}$.
7. Execute the Dual Ascent Procedure for Variables v_j^t.
8. $\xi \leftarrow \xi + 1$. If $\xi > T$ then $t \leftarrow t + 1$ and go to 11. Else go to 5.
9. If $t > T$ then $s \leftarrow s + 1$ and go to 12. Else go to 4.
10. If $s > q$ then $i \leftarrow i + 1$ and go to 13. Else go to 3.
11. If $i > m$ then stop. Else go to 2.

In step 5 of the procedure Δu_{ist}^{ξ} takes the minimum value between u_{ist}^{ξ} and $\frac{SR_{ist}^{\xi}}{2}$ for the same reasons already pointed out for the dual ascent procedure.

4.5 Example

Consider the following problem with two time periods, four clients, two facilities, two possible dimensions (with maximum capacities equal to 10 and 16) and *Nmax* equal to one. Table 1 shows the clients' demands; table 2 the assignment costs for period one, table 3 the assignment costs for period 2 and table 4 the fixed (re) opening costs.

t \ j	1	2	3	4
1	1	4	9	4
2	1	4	6	3

Table 1

(i,s) \ j	1	2	3	4
(1,1)	4	28	18	12
(1,2)	6	20	18	32
(2,1)	2	12	18	8
(2,2)	7	8	9	8

Table 2

(i,s) \ j	1	2	3	4
(1,1)	4	24	6	6
(1,2)	6	20	36	21
(2,1)	2	8	12	27
(2,2)	1	28	6	12

Table 3

(i, s)	FA_{is1}^{1}	FA_{is1}^{2}	FA_{is2}^{2}	FR_{is1}^{1}	FR_{is1}^{2}	FR_{is2}^{2}
(1,1)	104	104	74	53	53	8
(1,2)	68	74	112	17	23	46
(2,1)	49	49	62	41	41	16
(2,2)	68	76	90	60	68	44

Table 4

After the dual ascent procedure, variables v_j^t take the values depicted in table 5. Slacks are change and their values after the dual ascent procedure for variables v_j^t are showed in table 6. The dual objective function value is equal to 111.

t \ j	1	2	3	4
1	7	28	18	12
2	4	22	8	12

Table 5

(i, s)	SA_{is1}^1	SA_{is1}^2	SA_{is2}^2	SR_{is1}^1	SR_{is1}^2	SR_{is2}^2
(1,1)	101	93	66	50	42	0
(1,2)	59	63	110	8	12	44
(2,1)	24	8	46	16	0	0
(2,2)	35	38	85	27	30	39

Table 6

During the primal procedure, $I^* = \{(1,1,2,2),(2,1,1,2),(2,1,2,2)\}$ and step 4 calculates $I_1^* = \{(2,1)\}; I_2^* = \{(2,1),(1,1)\}$. $I^+ = \{(1,1,2,2),(2,1,1,2)\}$; $I_1^+ = \{(2,1)\}; I_2^+ = \{(2,1),(1,1)\}$. In step 6, for *t* equal to one, the procedure calculates $N[1,1] = 0$ and $N[2,1] = 1$. In step 7, it calculates *D* = 18 and *C* = 10. As *D* > *C*, the procedure calculates the F_{is}^1 values as shown in table 7.

i	1	1	2	2
s	1	2	1	2
F_{is}^1	30	2	+∞	+∞

Table 7

F_{21}^1 and F_{22}^1 are equal to +∞ because *N*[2,1] is equal to *Nmax*. F_1^1 is equal to 51, F_1^2 is equal to 66, and F_1' is equal to 66. $F_{11}^1 = min\{+\infty, 104-66\} = 38$ and *b* = 0, *c* = 2, *d* = 2. $F_{11}^1 = min\{38, FR_{111}^2 + F_1^1 - FR_{112}^2 - F_1^2\} = min\{38, 53+51-8-66\} = 30$.

$F_{12}^1 = \min\{+\infty, 68-66, 74-66\} = 2$; *b* = 0 and *c* = *T* +1.

At step 10, the procedure calculates $F'^1_{11} = \frac{F_{11}^1}{10}\left\lceil\frac{10}{10}\right\rceil = 3$ and $F'^1_{12} = \frac{F_{12}^1}{16}\left\lceil\frac{16}{16}\right\rceil = 0.13$.

At step 11 the procedure chooses pair (1,2) and sets I_1^+ and I^+ are changed accordingly: $I_1^+ = I_1^+ \cup \{(1,2)\}, I^+ = I^+ \cup \{(1,2,1,1)\}$. $C = C + 16 = 26$, $Num[1,2,1]=1$, $N[1,1]=1$. As $D \leq C$, t is increased by 1.

For period t equal to 2, $D = 14$ and $C = 20$. At step 19, the procedure solves the transportation problems for t equal to one and two (with optimal objective function values equal to 40 and 22). At step 21 the primal solution is calculated as $a_{211}^2 = 1$, $r_{112}^2 = 1$ and $a_{121}^1 = 1$. This is an admissible solution and has an objective function value equal to 187.

The dual solution is changed again by the dual ascent procedure for variables u_{ist}^{ξ}.

As SR_{211}^2 is equal to zero and SA_{211}^2 is equal to 8, then $\Delta u_{211}^2 = 4$. Slacks are changed according to table 8 and the dual variable is increased to 4.

(i, s)	SA_{is1}^1	SA_{is1}^2	SA_{is2}^2	SR_{is1}^1	SR_{is1}^2	SR_{is2}^2
(1,1)	101	93	66	50	42	0
(1,2)	59	63	110	8	12	44
(2,1)	20	4	46	16	4	0
(2,2)	31	34	85	27	30	39

Table 8

The dual ascent procedure for variables v_j^t is able to increase variable v_1^1 to 11, and the dual objective function value is increased to 115. Slacks' values after the execution of the dual ascent procedure for variables v_j^t are presented in table 9.

(i, s)	SA_{is1}^1	SA_{is1}^2	SA_{is2}^2	SR_{is1}^1	SR_{is1}^2	SR_{is2}^2
(1,1)	97	89	66	46	38	0
(1,2)	55	59	110	4	8	44
(2,1)	16	0	46	12	0	0
(2,2)	27	30	85	23	26	39

Table 9

As SR_{112}^2 is equal to zero and SA_{112}^2 is equal to 66, then $\Delta u_{112}^2 = 33$. Slacks are changed according to table 10 and the dual variable is increased to 33.

(i, s)	SA_{is1}^1	SA_{is1}^2	SA_{is2}^2	SR_{is1}^1	SR_{is1}^2	SR_{is2}^2
(1,1)	64	56	33	46	38	33
(1,2)	22	26	77	4	8	44
(2,1)	16	0	46	12	0	0
(2,2)	27	30	85	23	26	39

Table 10

The dual ascent procedure for variables v_j^t is able to increase variables v_3^2 to 12 and v_4^2 to 27, and the dual objective function value is increased to 134. Slacks' values after the execution of the dual ascent procedure for variables v_j^t are presented in table 11.

(i, s)	SA_{is1}^1	SA_{is1}^2	SA_{is2}^2	SR_{is1}^1	SR_{is1}^2	SR_{is2}^2
(1,1)	64	37	14	46	19	14
(1,2)	22	20	71	4	2	38
(2,1)	16	0	46	12	0	0
(2,2)	27	11	66	23	7	20

Table 11

The dual ascent procedure for variables u_{ist}^{ξ} tries to increase variable u_{212}^2, calculating Δu_{212}^2 equal to 23. It will not be possible to increase this dual variable because $SA_{211}^2 = 0$.

The dual descent procedure for variables u_{ist}^{ξ} tries to decrease variable u_{211}^2, but $\Delta u_{211}^2 = min\{4,0\}$. The variable is not decreased. Variable u_{112}^2 is decreased by $\Delta u_{112}^2 = min\{33,7\}$, but the dual ascent procedure for variables v_j^t cannot increase the dual objective function value.

After several iterations, the heuristic ends with the best primal objective function value equal to 187 ($a_{121}^1 = 1; r_{112}^2 = 1; a_{211}^2 = 1$) and the best dual objective function value equal to 134.

5 Conclusions and Future Work Directions

The algorithm described in this research report has already been implemented in C language and tested with some randomly generated problems. Nevertheless, the computational tests performed are not yet sufficient to draw justified conclusions. The results are not going to be as good as those obtained with the DLPOCR and with the DLPOCR with maximum capacity restrictions. The lower bound obtained by the best dual solution found seems to be far from the optimal primal solution, so it probably will not be useful to assess the quality of the primal feasible solutions found by the heuristic. Despite the fact that the primal procedure presented cannot guarantee the calculation of a feasible solution, this has not proved to be a problem in the tests already performed.

After the execution of the primal-dual heuristic, it is possible to improve the primal solution found by executing a local search procedure. This local search procedure will consider changes to the best primal solution calculated that consist in adding or removing one or more operating time periods to facilities of dimension s in i.

The quality of the lower bounds calculated by the primal-dual heuristic described in Dias *et al*, 2002, for the resolution of DLPOCR motivates the use of lagrangean relaxation with subgradient optimisation for the resolution of DLPER. If the capacity constraints (7) are relaxed, a problem very similar to the DLPOCR is obtained. A primal feasible solution to DLPER can be obtained using the same procedures described in this research report.

The good results that have been obtained with primal-dual heuristics in the resolution of dynamic location problems motivated the study of more complicated problems, namely, hierarchical location problems. We are presently working in the application of primal-dual heuristics to uncapacitated and capacitated dynamic hierarchical location models.

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