

Instituto de Engenharia de Sistemas e Computadores
de Coimbra
INESC - Coimbra

Rita Girão-Silva, José Craveirinha, João Clímaco

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Hierarchical Multiobjective Routing in MPLS Networks with Two Service Classes - A Meta-Heuristic Resolution Approach*

Rita Girão-Silva^{a,c}, José Craveirinha^{a,c}, João Clímaco^{b,c}

^a Department of Electrical Engineering Science and Computers of the University of Coimbra
Pólo II, Pinhal de Marrocos; P-3030-290 Coimbra; Portugal

^b Faculty of Economics of the University of Coimbra
Av. Dias da Silva, 165; P-3004-512 Coimbra; Portugal

^c Institute of Computers and Systems Engineering of Coimbra (INESC-Coimbra)
R. Antero de Quental, 199; P-3000-033 Coimbra; Portugal

Tel.: +351 239 796252; Fax: +351 239 796247
{rita,jcrav}@deec.uc.pt; jclimaco@inescc.pt

Abstract

The paper begins by reviewing a two-level hierarchical multicriteria routing model for MPLS networks with two service classes (QoS and Best Effort services) and alternative routing, as well as the foundations of a heuristic resolution approach, previously proposed by the authors. Afterwards a new approach, of meta-heuristic nature, based on the introduction of Simulated Annealing and Tabu Search techniques, in the structure of the dedicated heuristic, is described. The application of the developed procedures to a benchmarking case study will show that, in certain initial conditions, this approach provides improvements in the final results especially in more ‘difficult’ situations detected through sensitivity analysis.

Keywords Routing models, Multiobjective optimisation, MPLS-Internet, Simulated Annealing, Tabu Search.

1 Introduction and Motivation

Modern multiservice network routing functionalities have to deal with multiple and heterogeneous QoS (Quality of Service) requirements. This led to routing models designed to calculate and select one (or more) sequences of network resources (routes), satisfying certain QoS constraints and seeking the optimisation of route related objectives. There are potential advantages in formulating important routing problems in these types of networks as multiple objective optimisation problems, as these multiple objective formulations enable the trade-offs among distinct performance metrics and other network cost function(s) to be pursued in a consistent manner. It is important to note that the specification of the objective functions and constraints of the model depends strongly on the nature of the used routing principles, the type of network technology and the essential features of the offered services and associated traffic flows.

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The interest in the application of multicriteria approaches to routing models in communication networks has been fostered mainly by the increasing relevance of QoS issues in the new technological platforms of multiservice networks.

An in-depth methodological analysis of issues raised by the use of multicriteria analysis in telecommunication network design and their relation with knowledge theory models is given in [46]. A review on multicriteria models in telecommunication network design problems including a section on routing models is in [2]. A recent overview on multicriteria routing models in telecommunication networks with a case study is presented in [3].

In particular, a significant number of routing models of multicriteria nature has been proposed in the context of the emergent MPLS (Multiprotocol Label Switching) Internet networks (see [3]). This has to do mainly with the capability of implementing multiple connection-oriented services with QoS (Quality of Service) requirements. This technology is based on the introduction of LSRs (Label Switching Routers) in the MPLS network that forward the packets (grouped in FECs, Forward Equivalence Classes), through the so-called LSPs (Label Switched Paths) by using a specific packet label switching technique. As a result of this and other technical capabilities of MPLS, advanced QoS-based routing mechanisms can be implemented, in particular involving “explicit routes” (i.e. routes completely determined at the originating node) for each traffic flow of a given service type.

A discussion on key methodological and modelling issues associated with route calculation and selection in MPLS networks and the proposal of a meta-model for hierarchical multiobjective network-wide routing in MPLS networks, were presented in [5]. This meta-model is associated with a network-wide multiobjective routing optimisation approach of a new type. Two types of traffic flows are considered: firstly QoS type flows (first priority flows) such that, when accepted by the network, have a guaranteed QoS level, related to the required bandwidth; secondly BE (Best Effort) flows, that are considered in the model as second priority flows, and are carried by the network in order to obtain the best possible QoS level. The routing model incorporates an alternative routing principle: when a first choice route (corresponding to a loopless path) assigned to a given micro-flow¹, in a specific traffic flow (corresponding to a MPLS “traffic trunk”) is blocked a second choice route may be attempted. An important feature of the multicriteria modelling approach model is the use of hierarchical optimisation with up to three optimisation levels, including fairness objectives, and the possibility of considering a two-level stochastic representation of traffic in the network: macro-level (corresponding to the model ‘traffic flows’) and micro-level (corresponding to μ -flows, in terms of streams of packets, for a given connection).

In the present model, described in detail in [6], the first priority objective functions concern network level objectives of QoS type flows, namely the total expected revenue and the maximal value of the mean blocking of all types of QoS traffic flows; the second priority objective functions are related to performance metrics for the different types of QoS services and the total expected revenue for the BE traffic flows. The traffic flows in the network are represented in an approximate stochastic form, based on the use of the concept of effective bandwidth for macro-flows and on a generalised Erlang model for estimating the blocking probabilities in the arcs, as in the model used in [33, 30].

The theoretical foundations of a specialised heuristic strategy for finding “good” compromise solutions to the very complex bi-level routing optimisation problem, were also presented in [6]. In [16], a heuristic approach (HMOR-S2) devised to find “better” solu-

¹A μ -flow corresponds in our model to a ‘call’, that is, a connection request with certain features.

tions to this hierarchical multiobjective routing optimisation problem, was proposed and applied to a test network used in a benchmarking case study, for various traffic matrices.

This work presents a new approach, of meta-heuristic nature, that aims at finding even “better” solutions to the above hierarchical multiobjective routing optimisation problem namely in very specific situations where sensitivity analysis showed that there was the potential for some improvement(s) in the first level objective functions. The basis of the approach is the following: beginning with the analytic results obtained after one run of the HMOR-S2 heuristic, a further run is executed, this time by using a new algorithm that includes a meta-heuristic strategy, namely, a SA (Simulated Annealing) or a TS (Tabu Search) strategy (see e.g. [24, 19]).

The developed meta-heuristic procedures seek to make the most of the knowledge acquired with the problem by previous experimentation with the specialised heuristic HMOR-S2 and aim to overcome possible limitations of this heuristic detected through sensitivity analysis. We can say that the essence of the motivation underlying this work was to make the most of the previously developed substantive or core model (in the sense defined in the theory on model-based decision support [47]) on hierarchical multicriteria network-wide routing optimisation, described in [5, 6], by incorporating new OR tools (namely SA and TS) in the previously developed heuristic resolution approach. That is, we tried to make the most of a synthesis of knowledge about a given automated routing decision model, acquired through theoretical analysis and extensive experimentation. It will be shown that some of the meta-heuristic techniques, especially TS, introduced in the context of the previous heuristic approach, provide some improvements on the results in certain situations.

The paper is organised as follows. The two-level hierarchical multiobjective alternative routing model with two service classes is reviewed in section 2, together with the basis of the dedicated heuristic. In the third section, the features of the application of the two meta-heuristic techniques SA and TS, in the context of the heuristic approach, are presented. The formal description of the proposed specialised meta-heuristics applied to the routing problem are also described in the third section. The results obtained with these procedures, by using analytic and discrete-event simulation experiments for a test network used in a benchmarking study, are revealed in the fourth section. Finally, conclusions are drawn and future work is outlined in the fifth section.

2 Review of the Multiobjective Routing Model

2.1 The Multiobjective Routing Model

As previously mentioned the considered model is an application of the multiobjective modelling framework for MPLS networks proposed in [5]. This framework (or “meta-model”) in [5] considers hierarchical optimisation with up to three optimisation levels. In the first priority objective functions, global network performance metrics are considered; the second priority objective functions are concerned with performance metrics for the different types of services in the network; the third priority functions refer to performance metrics for packet streams micro-flows of the carried traffic flows and are related to average delays. Traffic flows in the network are represented in a stochastic form, considering two levels of representation: ‘macro’ level or traffic flow level, and ‘micro’ level (corresponding to packet streams in a traffic flow). Two classes of services are considered: QoS, that is services with guaranteed QoS levels (when accepted by the network), and BE,

corresponding to traffic flows that are routed having in mind to obtain the best possible quality of service but not at the cost of deteriorating the QoS of the QoS traffic flows. This implies that QoS flows are treated as first priority traffic flows. The different service types of each class are represented through the sets \mathcal{S}_Q (for QoS service types) and \mathcal{S}_B (for BE service types). Note that the traffic flows of each service type $s \in \mathcal{S}_Q$ or $s \in \mathcal{S}_B$ may differ in important attributes, in particular the required bandwidth.

The model now reviewed is a simplification of the general model for QoS and BE service classes outlined in [5, sec.3.3] where only the macro level traffic stochastic representation was considered. In this simplification, the additional complexity which would result from the inclusion of a third optimisation level in the routing model, as well as the corresponding additional computational burden associated with the stochastic model for calculating average delays, can be avoided. Therefore, the hierarchical multiobjective routing optimisation model has two levels with several objective functions in each level. The first level (first priority) includes objective functions formulated at the network level for the QoS traffic, namely the expected revenue and the worst average performance among QoS services. In the second level the objective functions are concerned with average performance metrics of the QoS traffic flows associated with the different types of QoS services as well as the expected revenue of the BE traffic.

This is a network-wide² routing optimisation approach, which takes into account the nature of the formulated objectives, enabling a full representation of the relations between the objective functions, taking into account the interactions between the multiple traffic flows associated with different services.

A detailed discussion on the methodological foundations of this type of approach and its comparison with other types of approaches can be seen in [5].

Also note that in this model, ‘fairness’ objectives are explicitly considered at the two levels of optimisation, in the form of min-max objectives. These objective functions seek to make the most of the proposed multiobjective formulation. In currently proposed formulations of routing models for these networks, such aims concerning ‘fairness’ are usually not considered in any form or just represented through constraints on certain performance metrics.

In the model the network is represented through a capacitated directed graph, where a capacity C_k is assigned to every arc (or ‘link’) l_k , and the traffic flows are represented in a stochastic form, as shown in [5]. A traffic flow is specified by $f_s = (v_i, v_j, \bar{\gamma}_s, \bar{\eta}_s)$ for $s \in \mathcal{S} = \mathcal{S}_Q \cup \mathcal{S}_B$ and a stochastic process is assigned to it, that is in general, a marked point process. The process describes the arrivals and basic requirements of micro-flows, originated at the MPLS ingress node v_i and destined to the MPLS egress node v_j , using some LSP. The other features of the traffic flow are characterised by the vectors of ‘attributes’ $\bar{\gamma}_s$ and $\bar{\eta}_s$, for service type s . The vector $\bar{\gamma}_s$ represents the traffic engineering attributes of flows of service type s and the vector $\bar{\eta}_s$ enables the description of mechanism(s) of admission control to all arcs l_k in the network by calls of flow f_s . In particular these attributes include information on the required *effective bandwidth* d_s and the mean duration $h(f_s)$ of each μ -flow in f_s . The use of the concept of effective bandwidth (a concept developed in [22]) in the present context (MPLS networks with explicit routes) was earlier considered by [33] and in [29, 30]. The effective bandwidth can be viewed as a stochastic measure of the utilisation of network resources allowing for an approximate, although effective, representation of the effects of the variability of the

²This means in this context that the main objective functions of a given service class depend explicitly on all traffic flows in the network.

rates of traffic sources of different types, as well as the effects of statistical multiplexing of different traffic flows in a network.

A teletraffic model, that underlies the routing model, enables the calculation of node to node blocking probabilities $B(f_s)$ for all flows f_s of all service types, from which the average blocking probability B_{ms} , for all traffic flows of type s , can be estimated for a given set of routes for all offered traffic flows. The maximal average blocking probability among all QoS service types, $B_{Mm|Q}$, is

$$B_{Mm|Q} = \max_{s \in \mathcal{S}_Q} \{B_{ms}\} \quad (2.1)$$

This will represent the fairness objective at the network level, as a first priority objective function.

The total expected network revenues, W_Q and W_B associated with QoS and BE traffic flows, respectively, are expressed in terms of the expected revenues $w(f_s)$ per call³ of flow f_s , and of the values of carried traffic A_s^c , for all service types:

$$W_{Q(B)} = \sum_{s \in \mathcal{S}_{Q(B)}} W_s = \sum_{s \in \mathcal{S}_{Q(B)}} A_s^c w_s$$

The usual simplification, $w(f_s) = w_s, \forall f_s \in \mathcal{F}_s$, where \mathcal{F}_s is the set of traffic flows of type s , will be considered. The total expected revenue for the traffic flows of QoS type W_Q is a first priority objective function together with the maximal blocking probability for all QoS service types, $B_{Mm|Q}$, given in (2.1), while the total expected revenue for the BE traffic flows, W_B , will be a second level objective function. Therefore, the routing of BE traffic, in a quasi-stationary situation, will not be made at the cost of the decrease in revenue or at the expense of an increase in the maximal blocking probability of QoS traffic flows. Nevertheless, it is important to note that while QoS and BE traffic flows are treated separately in terms of objective functions so as to take into account their different priority in the routing optimisation, the interactions among all traffic flows are fully represented in the model. This is guaranteed by the used traffic modelling approach, underlying the optimisation model, because the traffic model used to obtain the blocking probabilities $B(f_s)$ integrates the contributions of all traffic flows which may use every link of the network. This feature is a major difference in comparison with more common routing models that have been proposed for networks with two service classes, based on some form of decomposition of the network representation, corresponding to ‘virtual networks’, one for each service class.

The second level of optimisation includes the BE expected revenue, and $2|\mathcal{S}_Q|$ objective functions related to all QoS service types, the mean blocking probabilities for flows of type $s \in \mathcal{S}_Q$,

$$B_{ms|Q} = \frac{1}{A_s^o} \sum_{f_s \in \mathcal{F}_s} A(f_s) B(f_s)$$

where A_s^o is the total traffic offered by flows of type s and $A(f_s)$ is the mean traffic offered associated with f_s (in Erlang), and the maximal blocking probability $B_{Ms|Q}$, defined over all flows of type $s \in \mathcal{S}_Q$,

$$B_{Ms|Q} = \max_{f_s \in \mathcal{F}_s} \{B(f_s)\}$$

This function constitutes the fairness objective defined for every service type $s \in \mathcal{S}_Q$.

Therefore the considered two-level hierarchical optimisation problem for two service classes is:

³The term ‘call’ means a node to node connection request with certain traffic engineering features.

Problem P-M2-S2

1st level	{	QoS - Network objectives: $\min_{\overline{R}}\{-W_Q\}$ $\min_{\overline{R}}\{B_{Mm Q}\}$
2nd level	{	QoS - Service objectives: $\min_{\overline{R}}\{B_{ms Q}\}$ $\min_{\overline{R}}\{B_{Ms Q}\}$ $\forall s \in \mathcal{S}_Q$
		BE - Network objective: $\min_{\overline{R}}\{-W_B\}$
subject to the equations of the underlying traffic model.		

The decision variables \overline{R} represent the network routing plans, that is, the set of all the feasible routes (i.e. node to node loopless paths) for all traffic flows. The acronym P-M2-S2 stands for ‘**P**roblem - **M**ultiobjective with **2** optimisation hierarchical levels - with **2** Service classes’.

The basic teletraffic sub-model allows for the blocking probabilities B_{ks} , for micro-flows of service type s in link l_k , to be given in the form

$$B_{ks} = \mathcal{L}_s(\overline{d}_k, \overline{\rho}_k, C_k) \quad (2.2)$$

Here \mathcal{L}_s represents the basic function (implicit in the teletraffic analytical model) that expresses the marginal blocking probabilities, B_{ks} , in terms of $\overline{d}_k = (d_{k1}, \dots, d_{k|S|})$ (vector of equivalent effective bandwidths for all service types), $\overline{\rho}_k = (\rho_{k1}, \dots, \rho_{k|S|})$ (vector of reduced traffic loads ρ_{ks} offered by flows of type s to l_k) and the link capacity C_k .

This type of approximation was suggested in [33] for off-line single-objective multi-service routing optimisation models and was also used in the multiobjective dynamic alternative routing model proposed in [30]. It enables the calculation of $\{B_{ks}\}$ through efficient numerical algorithms. We should stress that very efficient and robust approximations have to be used in a network-wide routing optimisation model of the type associated with P-M2-S2, for tractability reasons.

2.2 Basis of the Heuristic Approach

The dedicated heuristic resolution approach that is the starting point for the meta-heuristics analysed in this paper uses the theoretical foundations described by the authors in the report [6], which will now be reviewed.

In the hierarchical multiobjective routing problem P-M2-S2 an alternative routing principle is used. This means that the network routing plans $\overline{R} = \cup_{s=1}^{|\mathcal{S}|} R(s)$ (decision variables) for all the network services, where $R(s) = \cup_{f_s \in \mathcal{F}_s} R(f_s)$, $s \in \mathcal{S}_Q \cup \mathcal{S}_B$ are such that $R(f_s) = (r^p(f_s))$, $p = 1, \dots, M$ with $M = 2$ in our model. It is assumed that for each flow f_s the first choice route $r^1(f_s)$ will be used unless it is blocked because one of its links l_k does not have the required available bandwidth d_s (or a call is not accepted according to the probabilistic availability function ψ_{ks}). If $r^1(f_s)$ is blocked the routing method makes the current connection request attempt the second choice route $r^2(f_s)$. This request will be blocked only if $r^2(f_s)$ is also blocked. If $M > 2$, routes $r^3(f_s), \dots, r^M(f_s)$ would be attempted in this order.

The high ‘complexity’ of the routing problem P-M2-S2 stems from two major factors: all objective functions are strongly interdependent (via the $\{B(f_s)\}$), and all the objective function parameters and (discrete) decision variables \overline{R} (network route plans) are also interdependent. Note that all these interdependencies are defined explicitly or implicitly through the underlying traffic model. Regarding computational complexity, it must be remarked that the simplest, ‘degenerated’ single objective version of the problem, that

is, concerning a model with a single objective function W_Q , one single service and no alternative routing ($M = 1$) is NP-complete in the strong sense, as shown in [13]. The addressed problem may be viewed as a bi-level, multiobjective extension of this type of problem.

Concerning the possible conflict between the objective functions in P-M2-S2, it should be observed that in many routing situations, the maximisation of W_Q leads to a deterioration on some $B(f_s), s \in \mathcal{S}_Q$, for certain traffic flows $A(f_s)$ with low intensity, and this tends to increase $B_{M_s|Q}$ and, consequently, $B_{M_m|Q}$. In single-objective routing models this aspect is usually addressed by imposing upper bounds on the values $B(f_s)$. This is a major factor to justify the interest and potential advantage in using multiobjective approaches when dealing with this type of routing methods.

The resolution (in a multicriteria analysis sense) of the routing problem P-M2-S2 was earlier performed by a heuristic procedure in [16], which is briefly reviewed in this section. This heuristic is an improved version of the heuristic approach described in [6] and it is based on the recurrent calculation of solutions to a constrained bi-objective shortest path problem, formulated for every end-to-end flow f_s :

$$\text{Problem } \mathcal{P}_{s2}^{(2)} : \min_{r(f_s) \in \mathcal{D}(f_s)} \left\{ m^n(r(f_s)) = \sum_{l_k \in r(f_s)} m_{ks}^n \right\}_{n=1;2} \quad (2.3)$$

The path metrics m^n to be minimised are the marginal implied costs $m_{ks}^1 = c_{ks}^{Q(B)}$ (the definition of which is reviewed in the following analysis) and the marginal blocking probabilities $m_{ks}^2 = -\log(1 - B_{ks})$; $\mathcal{D}(f_s)$ is the set of all feasible loopless paths for flow f_s , which satisfy specific traffic engineering constraints (other than the effective bandwidth) for flows of type s . A typical constraint is a maximal number of arcs per path depending on the class and type of service s . The logarithmic function is just used to transform the blocking probability into an additive metric. The link cost coefficients $m_{ks}^1 = c_{ks}^{Q(B)}$ are then used in problems of form (2.3) when candidate solutions have to be obtained to seek the improvement of the revenue of the QoS(BE) traffic, in different steps of the heuristic procedure. According to this approach, the comparison of the efficiency of different candidate routes in the context of a multicriteria routing framework of this type should take into account both the loss probabilities experienced along the candidate routes and the knock-on effects upon the other routes in the network, effects associated with the acceptance of a call on that given route. Such effects can be measured exactly through the implied costs.

It is important to note that this auxiliary constrained bi-objective shortest path problem was used as a basis of the heuristic approach having in mind that the consideration of the metric blocking probability tends, at a network level, to minimise the maximal node-to-node blocking probabilities $B(f_s)$, while the metric implied cost tends to maximise the total average revenue W_T in a single class multiservice loss network (see [9, 32]). However we must draw attention to the exact meaning of the statement that the minimisation of path implied cost ‘tends’ to maximise W_T . This would be, in rigour, only valid if the choice of such ‘optimal’ path, for a given f_s , would not change in any form the network working conditions concerning all the remaining traffic flows, an assumption obviously not true, due to the interdependencies among $\{c_{ks}\}, \{B_{ks}\}$ and \bar{R} . Naturally this is yet another source of difficulty in devising a heuristic of this nature for this complex routing problem (P-M2-S2).

Concerning the implied cost c_{ku} (resulting from the acceptance of a call of flow f_u in link l_k) this is an important mathematical concept in routing optimisation in loss networks

which was originally proposed by Kelly [23] for single-rate traffic networks. The definition was later extended to single route multirate traffic networks in [14, 33]. The implied cost can be viewed as the expected value of the loss of revenue in all traffic flows which may use link l_k , resulting from the acceptance of a connection request from f_u stemming from the decrease in the capacity of this link. Therefore we can say that the implied cost measures in a probabilistic manner the knock-on effects on all network routes (of all traffic flows) associated with the acceptance of a call from f_u in a link l_k . In [8], the definition of c_{ku} was adapted to multirate loss networks with alternative routing by extending the model for single-service networks given in [23]. The extension of this definition to a multi-rate network with alternative routing and two service classes was proposed in the report [6]. For this purpose the following definition of marginal implied costs associated with QoS (BE) traffic was put forward [6]. The *marginal implied cost* for QoS(BE) traffic, $c_{ku}^{Q(B)}$, associated with the acceptance of a connection (or “call”) of traffic f_u of any service type $u \in \mathcal{S}$ on a link l_k is defined as the expected value of the traffic loss induced on all QoS(BE) traffic flows resulting from the capacity decrease in link l_k .

In [6], a conjecture was presented, implying the marginal implied costs for QoS (BE) traffic can be obtained by solving a system of equations

$$c_{ku}^{Q(B)} = \sum_{s \in \mathcal{S}_{Q(B)}} \frac{1}{1 - B_{ks}} \zeta_{kus} \left[\sum_{f_s \in \mathcal{F}_s : l_k \in r^1(f_s)} \lambda_{r^1(f_s)} \left(s_{r^1(f_s)}^{Q(B)} + c_{ks}^{Q(B)} \right) + \sum_{f_s \in \mathcal{F}_s : l_k \in r^2(f_s)} \lambda_{r^2(f_s)} \left(s_{r^2(f_s)}^{Q(B)} + c_{ks}^{Q(B)} \right) \right] \quad (2.4)$$

with

$$\begin{aligned} s_{r^2(f_s)}^{Q(B)} &= w^{Q(B)}(f_s) - \sum_{l_j \in r^2(f_s)} c_{js}^{Q(B)} \\ s_{r^1(f_s)}^{Q(B)} &= w^{Q(B)}(f_s) - \sum_{l_j \in r^1(f_s)} c_{js}^{Q(B)} - (1 - L_{r^2(f_s)}) s_{r^2(f_s)}^{Q(B)} \\ \zeta_{kus} &= \mathcal{L}_s(\bar{d}_k, \bar{\rho}_k, C_k - d_{ku}) - \mathcal{L}_s(\bar{d}_k, \bar{\rho}_k, C_k) \end{aligned}$$

where $s_{r^p(f_s)}^{Q(B)}$ denotes the surplus value of a call on route $r^p(f_s)$, $\lambda_{r^p(f_s)}$ is the marginal traffic carried on $r^p(f_s)$ by flow f_s , $L_{r^p(f_s)}$ represents the blocking probability for calls of f_s on route $r^p(f_s)$ ($p = 1; 2$) (considering that $r^1(f_s)$ and $r^2(f_s)$ are arc-disjoint paths) and ζ_{kus} is the increase in call blocking probability for type s calls on link l_k resulting from a decrease in the capacity of l_k associated with the acceptance of a type u call. The coefficients $w^{Q(B)}(f_s)$ are the marginal expected revenues per call of f_s , such that

$$w^Q(f_s) + w^B(f_s) = w(f_s)$$

and can be written:

$$w^{Q(B)}(f_s) = \alpha^{Q(B)} w(f_s)$$

in terms of the coefficients $\alpha^{Q(B)} \in]0.0; 1.0[$ which satisfy the normalisation condition

$$\alpha^Q + \alpha^B = 1.0$$

These marginal revenues may be interpreted as the part of the expected revenue $w(f_s)$ associated with a connection of f_s that is accepted by the network (for a given choice of the pair of routes ($r^1(f_s), r^2(f_s)$)) that corresponds to the calculation of the sensitivity of

the revenue from the point of view of traffic losses induced either in the QoS traffic flows or in the BE flows.

A system of implicit non-linear equations can be defined in order to calculate the B_{ks} in terms of link capacities (matrix $\overline{C} = [C_k]$), the offered traffic matrix $\overline{A} = [A(f_s)]$, and the current network routing solution \overline{R} ,

$$B_{ks} = \beta_{ks}(\overline{B}, \overline{C}, \overline{A}, \overline{R}) \quad (2.5)$$

with $k = 1, \dots, |\mathcal{L}|$; $s = 1, \dots, |\mathcal{S}|$ and $\overline{B} = [B_{ks}]$. Concerning the calculation of $c_{ks}^{Q(B)}$ through (2.4), it implies the resolution of a system of equations of the general form:

$$c_{ks}^{Q(B)} = \kappa_{ks}^{Q(B)}(\overline{c}, \overline{B}, \overline{C}, \overline{A}, \overline{R}) \quad (2.6)$$

where $\overline{c} = [c_{ks}^{Q(B)}]$. The numerical resolution of these two systems of equations in B_{ks} and $c_{ks}^{Q(B)}$ is performed by fixed point iterators, given the matrices \overline{C} , \overline{A} and \overline{R} .

In the heuristic, the auxiliary constrained shortest path problem $\mathcal{P}_{s2}^{(2)}$ (2.3) is solved by the algorithm MMRA-S2 [6], an adaptation of a previously developed algorithmic approach, MMRA-S (Modified Multiobjective Routing Algorithm for multiservice networks), described in [8, 30]. Generally, there is no feasible solution which minimises the two objective functions simultaneously. Hence, the resolution of this routing problem aims at finding a ‘best’ compromise path from the set of non-dominated solutions, according to some system of preferences. In this context, path computation and selection have to be fully automated. Therefore the system of preferences is embedded in the working of the algorithm MMRA-S2. This is implemented by defining preference regions in the objective function space obtained from aspiration and reservation levels (preference thresholds) defined for the two objective functions [9, 32].

As for the algorithm MMRA-S2, it is a variant of the algorithm proposed in [1], with the necessary adaptations resulting from the requirements and specifics of the described routing model. Note that the algorithmic approach in [1] was inspired by the one given in [41, 4], which is an algorithm designed to search interactively non-dominated paths. With MMRA-S2, non-dominated paths can be calculated and selected in the framework of a routing control mechanism for multiservice networks with alternative routing. This procedure uses the MPS, a very efficient k -shortest paths algorithm described in [28], to calculate non-dominated solutions of the problem. The other main features of the algorithm are: i) QoS requirements are made to correspond to requested and acceptable thresholds for each QoS metric; ii) these thresholds enable the definition of priority regions in the objective function space where non-dominated solutions are searched for, leading to the choice of a pair of candidate routes $(r^1(f_s), r^2(f_s))$; iii) the candidate non-dominated paths are calculated with the MPS algorithm, in the improved version adapted to paths with a maximum number of arcs, given in [20]; iv) the preference thresholds are dynamic and adapt to the network working conditions.

Further details on this algorithmic approach can be seen in [30].

Another important part of the addressed routing model is the underlying traffic model. This stochastic traffic model involves all the sub-models and associated numerical procedures, that are needed for obtaining all traffic related parameters, namely implied costs and blocking probabilities B_{ks} and $B(f_s)$, under certain simplifying assumptions.

A description of the traffic modelling approach used in the routing model can be seen in [5].

Now let us review the basic features of the dedicated heuristic HMOR-S2 (see Appendix A), taken as the starting point and reference procedure in the present work.

In the heuristic, a basic searching strategy is to seek for routing solutions $\overline{R}(s)$ for each service $s \in \mathcal{S}$, in order to achieve a better performance in terms of W_B , $B_{ms|Q}$ and $B_{Ms|Q}$, $s \in \mathcal{S}_Q$ while respecting the hierarchy of objective functions. This also means that network resources are left available for traffic flows of other services so that the solutions selected at each step of the procedure may improve the first priority objective functions W_Q and $B_{Mm|Q}$. The heuristic was designed in order to seek, firstly for each QoS service and starting from the services with higher effective bandwidth (considering the numbering of s , $s = 1, \dots, |\mathcal{S}_Q|$) and, secondly, for each BE service (also beginning by the higher bandwidth services, $s = |\mathcal{S}_Q| + 1, \dots, |\mathcal{S}|$), solutions which dominate the current one, in terms of $B_{ms|Q}$ and $B_{Ms|Q}$ for QoS services and in terms of W_B for BE services. These solutions will only be accepted if they do not lead to the worsening of any of the network functions W_Q and $B_{Mm|Q}$.

Another basic idea of the heuristic is the generation of candidate solutions ($r^1(f_s)$, $r^2(f_s)$) for each f_s , using the mentioned algorithm MMRA-S2, and their possible selection through specific criteria, to be ‘tuned’ throughout the execution of the heuristic. A maximal number of arcs D_s per route for each service type s is previously defined and a feasible route set $\mathcal{D}(f_s)$ is obtained for each f_s . For example, for real time QoS services, D_s is equal to the network diameter; for the non-real time QoS services, D_s is the network diameter plus 1, while for the BE services, no limits are imposed on D_s .

Note that special rules had to be constructed for the selection of candidate first choice routes $r^1(f_s)$ taking into account the network topology and the need to make a distinction between real time QoS services (typically video and voice services) and non-real time QoS services (for example ‘premium data’ service). These rules are described in [6].

Concerning the calculation of candidate second choice routes $r^2(f_s)$ for QoS or BE traffic, the MMRA-S2 procedure is used. Having in mind to prevent performance degradation in overload conditions, these alternative routes may be eliminated through a mechanism designated as Alternative Path Removal (APR), an adaptation of the mechanism originally proposed in [31, 30].

The theoretical analysis of the model, confirmed by experimentation, showed that successive application of MMRA-S2 to every traffic flow does not lead to an effective resolution approach to the network routing problem P-M2-S2. This results from an instability phenomenon that arises in such path selection procedure, expressed by the fact that the route sets \overline{R} often tend to oscillate between certain solutions some of which may lead to poor global network performance under the prescribed metrics. This instability phenomenon is associated with the complexity and interdependencies in the addressed problem P-M2-S2, namely the interdependencies between $\{c_{ks}^{Q(B)}\}$ and $\{B_{ks}\}$ and between these two sets and the current network route set \overline{R} .

Therefore, another core idea of the heuristic approach (similarly to MODR-S [30]) is the search for the subset of the path set $\overline{R}^a = \cup_{s=1}^{|\mathcal{S}|} \overline{R}^a(s) : \overline{R}^a(s) = \{(r^1(f_s), r^2(f_s)), f_s \in \mathcal{F}_s\}$ the elements of which should be possibly changed in the next route improvement cycle. Detailed analysis and extensive experimentation with the heuristic led to the proposal of a criterion for choosing candidate paths for possible routing improvement by increasing order of a function $\xi(f_s)$ of the current $(r^1(f_s), r^2(f_s))$, given in [16]. The use of this criterion considers two search cycles, where $\xi(f_s) = F_L(f_s)$ in the first cycle and $\xi(f_s) =$

$F_C^{Q(B)}(f_s)$ in the second cycle, if the effect over QoS(BE) traffic is being considered, with:

$$\begin{aligned}
F_C^{Q(B)}(f_s) &= (n_2 - n_1)c_1'^{Q(B)} + c_{r^1(f_s)}^{Q(B)} - c_{r^2(f_s)}^{Q(B)} \\
c_{r(f_s)}^{Q(B)} &= \sum_{l_k \in r(f_s)} c_{ks}^{Q(B)} \\
c_1'^{Q(B)} &= \frac{1}{n_1} \sum_{l_k \in r^1(f_s)} c_{ks}^{Q(B)} = \frac{1}{n_1} c_{r^1(f_s)}^{Q(B)} \\
F_L(f_s) &= 1 - L_{r^1(f_s)} L_{r^2(f_s)}
\end{aligned}$$

The aim of $F_C^{Q(B)}(f_s)$ is to give preference (concerning the potential value in changing the second choice route when seeking to improve W_Q or W_B) to the flows for which the route $r^1(f_s)$ has a low implied cost and the route $r^2(f_s)$ has a high implied cost. The factor $(n_2 - n_1)$ was introduced for normalisation purposes, considering that $r^1(f_s)$ has n_1 arcs and $r^2(f_s)$ has n_2 arcs. The aim of $F_L(f_s)$ is to give preference to the choice of the flows which currently have worse end-to-end blocking probability given by $L_{r^1(f_s)} L_{r^2(f_s)}$.

Another key point tackled by the heuristic is the specification of a variable $nPaths$, which represents the number of routes with smaller values of $\xi(f_s)$ that should possibly be changed by running MMRA-S2 once again. In order to do so, the effect of each candidate route on the relevant objective functions is anticipated by solving the corresponding analytical model.

The full description and formalisation of this heuristic as well as an application study are given in [16].

3 Developed Meta-Heuristics

The study of the heuristic approach HMOR-S2, the basis of which was reviewed in the previous section, was completed with a sensitivity analysis, which led to the consideration of variants of this heuristic. In the report [17], two variants to the HMOR-S2 were described, firstly the HMOR-S2_R where a floating relaxation was imposed on one of the first level objective function values, and secondly the HMOR-S2_B where a floating barrier was imposed on one of the first level objective function values. Extensive experimental analysis was carried out for those variants and a simulation study was also conducted. In this paper, two new variants to the basic heuristic approach are described, where meta-heuristic techniques are introduced, namely a SA (Simulated Annealing) technique and a TS (Tabu Search) technique.

In this section, a brief description of the sensitivity tests which gave the motivation for this work, is presented. Next, after a generic review of the features of SA and TS, a global perspective as well as the specifics of the application of these techniques to the basic heuristic, are presented.

3.1 Sensitivity Analysis

The purpose of the sensitivity tests applied to the HMOR-S2 heuristic was to check whether the heuristic was treating the lower level objective functions in a balanced way (that is, to check whether better values of the second level objective functions could be obtained without worsening the values of the first level objective functions) and to check whether the value of an upper level objective function could be improved at the cost of worsening the value of the other upper level objective function.

In the first set of tests, either an upper bound was imposed on one of the blocking probability functions B_{ms} or B_{Ms} , $s \in \mathcal{S}_Q$, or a lower bound was imposed on the BE traffic revenue W_B , $s \in \mathcal{S}_B$. These bounds constitute barriers, in the sense that they are more demanding than the corresponding values obtained at the end of the HMOR-S2 run.

In the second set of tests (relaxation tests), the focus was on the first level objective functions. In one of the tests, the blocking function $B_{Mm|Q}$ is no longer treated as an objective function and an upper bound on its value is imposed. This upper bound is less demanding than the corresponding value $[B_{Mm|Q}]_{\text{basis}}$ obtained at the end of the HMOR-S2 run. The purpose of this test is to check whether the QoS traffic revenue can still be improved by relaxing the value of the other main objective function. In the other test, the QoS services revenue W_Q is no longer treated as an objective function and a lower bound on its value is imposed. This lower bound is less demanding than the corresponding value $[W_Q]_{\text{basis}}$ obtained at the end of the HMOR-S2 run. The purpose of this test is to check whether the blocking function $B_{Mm|Q}$ can be improved when the value of the other objective function is relaxed.

As mentioned before, the numerical calculations performed in the HMOR-S2 heuristic are quite complex and because of the simplifications assumed in the traffic model underlying the heuristic and numerical errors associated with the resolution of large systems of non-linear equations, significant inaccuracies are expected. Therefore, in numerical terms, different revenue values are considered the same if they differ less than 0.5%. So, as long as a revenue value W_Q is such that $\left| \frac{W_Q - [W_Q]_{\text{basis}}}{[W_Q]_{\text{basis}}} \right| \leq 0.5\%$ then the QoS traffic revenue is considered to be practically the same as the one obtained when the basic heuristic (i.e. the HMOR-S2 heuristic without any sensitivity analysis) was run. Likewise, for the blocking probabilities, different values are those which differ by more than 5%. Note that usually the revenue values vary less (in terms of percentage) than the blocking probability values.

Generally speaking, the results of the sensitivity tests for the HMOR-S2 heuristic were as expected, allowing us to assume that the heuristic is balanced in the treatment of the different objective functions. Nonetheless, there are a few results that are worth mentioning.

In the first set of tests, one or both of the upper level objective function values were worse when a barrier (i.e. a stricter value) was imposed on one of the lower level blocking probability functions or BE traffic revenue. That is, when the improvement of one of the lower level functions is imposed, the upper level objective function values tend to be worse (at least for one of those functions). There was however one situation where one of the first level objective functions improved and the other worsened. This result is not unexpected, as the two first level objective functions are conflicting in nature, but showed that there was one non-dominated solution that the basic heuristic was not able to detect so far.

In the second set of tests, in one of the sensitivity tests where the upper level objective function $B_{Mm|Q}$ ceased to be treated in the heuristic as an objective function and a relaxed upper bound was imposed on its value, a final solution with slightly better values for both $B_{Mm|Q}$ and W_Q was obtained. Therefore, in spite of allowing the value of $B_{Mm|Q}$ to increase beyond the value obtained when the basic heuristic was run, it actually diminished, and there was a slight improvement of the QoS traffic revenue. This result suggests that, in some rare cases, the heuristic is not capable of finding a solution that slightly dominates the current selected solution.

In order to try to obtain solutions with even better values for both the upper level objective functions in these very specific types of situations, new approaches were devised.

These new approaches consist of the introduction of meta-heuristic techniques (SA and TS) in the structure of the basic heuristic HMOR-S2.

3.2 Application of a SA Technique to the Basic Heuristic

In physics, the annealing technique is used to find minimum energy states: a system is initially heated up to a very high temperature and afterwards it is slowly cooled, which allows minimum energy states to be reached. According to [24], there is an analogy between finding the minimum energy states in a physical system and finding minimum cost configurations in optimisation problems. Therefore, the reasoning behind the SA technique is the same as the one behind the physical annealing technique, and a SA-based heuristic can be used to find feasible solutions to an optimisation problem, having in mind the convergence to an optimal solution. An example of application of a SA heuristic to the problem of call routing in circuit-switched telecommunications networks is described in [50].

The SA technique can be viewed as a variant of the heuristic technique of local neighbourhood search, where a subset of feasible solutions is explored in the neighbourhood of the current solution. In an optimisation problem, the traditional implementations of local search always try to move towards an improvement of the objective function. However, with this type of strategy, the risk of remaining in a local optimum is high. The SA technique tries to prevent this from happening, by allowing solutions with worse values of the objective function (when compared with the value of that function in the current solution) to be taken into account. These moves towards worse solutions are done in a controlled way, and with the purpose of avoiding local minima or maxima. The probability of acceptance of a solution that is actually worse than the current solution is controlled by the variation of the objective function value and a parameter, a so-called temperature T , related to the state of the system, in particular related to the number of iterations that have occurred since the beginning of the search procedure.

A generic SA algorithm for a single objective problem, where a minimisation problem is considered, with solution space S , objective function f and neighbourhood structure N , can be seen, for example, in [12].

```

I. Choose an initial solution  $s_0 \in S$ ;
II. Choose an initial temperature  $T > 0$ ;
III. Choose a temperature decay function  $\vartheta(T)$ ;
IV. Repeat
  1. Repeat
    (a) Randomly choose a new solution  $s \in N(s_0)$ ;
    (b)  $\delta = f(s) - f(s_0)$ ;
    (c) If  $\delta < 0$ ,
        then  $s_0 \leftarrow s$ 
    else
      i. generate a random variable (r.v.)  $x$  with uniform distribution in  $]0; 1[$ ;
      ii. if  $x < \exp\left(\frac{-\delta}{T}\right)$ ,
          then  $s_0 \leftarrow s$ 
  until iteration number =  $nrep$ .
  2.  $T \leftarrow \vartheta(T)$ ;
until the stopping condition is satisfied.
The final value of  $s_0$  is an approximation to the optimal solution.

```

The SA technique has been successfully used to solve many different optimisation

problems. This technique is easy to implement, it can be applied to a great diversity of combinatorial optimisation problems and usually it allows for the calculation of adequate solutions [12]. However, in order to get good solutions, many parameters have to be carefully tuned: the cooling function $\vartheta(T)$, the neighbourhood area (based on the specific features of the problem to be solved), the probability function of acceptance of the new solution, the number of iterations $nrep$ and the stopping condition. Another disadvantage, apart from the need to carefully tune the system parameters, is the execution time of the SA algorithms that tends to be very long. Experiences from many authors actually show that for a specific and well-defined problem, an algorithm specifically tailored to that problem tends to provide better results than a SA algorithm [12]. Nevertheless, many authors have applied SA techniques to telecommunication network optimisation problems, such as network design and routing problems. See for instance [37, 38, 25, 44, 35, 39, 45, 40, 11, 10, 51].

3.2.1 Introduction of a SA technique in the HMOR-S2 heuristic

A variant of the basic heuristic HMOR-S2 using a SA-like technique was devised and implemented. Many issues had to be addressed to formulate this variant HMOR-S2_{SA}. Firstly the basic technique of SA had to be adapted to a hierarchical multiobjective problem. A choice was made to work only with the upper level objective functions and two different SA processes were considered simultaneously. The lower level objective functions are used as in the basic heuristic, that is, their value for the specific service under scrutiny has to improve so that the new solution may be taken into account in further steps.

Firstly, the initial temperature has to be specified. It should be high in order to guarantee that the final solution of the problem does not depend heavily on the initial solution. A high initial temperature also assures a certain diversity of solutions, which is advantageous on the initial stages of the resolution approach. Remember that the temperature decays throughout the heuristic procedure, which causes the probability of accepting new solutions that are actually worse than the current solution to diminish. This provides an intensification strategy, which should be correct for the final stages of the HMOR-S2_{SA}. Note that diversification-like and intensification-like strategies are already being used in the basic dedicated heuristic, HMOR-S2, as the parameter $nPaths$ (that represents the number of paths that can change from the current solution to the new one) starts with a high value (that is, the new solution can be quite diverse from the current one) and decays throughout the algorithm, which means that the paths remain the same for an increasing number of origin-destination pairs. As two SA sub-algorithms are considered simultaneously, two different initial temperatures have to be defined, in particular, one associated with the QoS services revenue, $T_W^0 = W_Q^0 = W_Q^{initial}$, and the other associated with the blocking probability function $B_{Mm|Q}$, $T_B^0 = B_{Mm|Q}^0 = B_{Mm|Q}^{initial}$.

The features of the neighbourhood area of the current solution have to be defined. In this implementation the features of the neighbourhood change throughout the procedure. Note that this is already being made in the basic heuristic, as the portion of the state space where new feasible solutions are sought, is defined according to the flows for which the paths may change in the current iteration. Therefore, not only the neighbourhood, where new solutions are sought, diminishes throughout the algorithm (because of the value of $nPaths$) but also it adapts to the current conditions of the resolution procedure and it is chosen in order to search for improvements in the objective function values.

The number of iterations for each temperature value also has to be determined. For

higher temperatures (initial stages of the resolution procedure), $nrep$ is small; for lower temperatures (final stages of the resolution procedure), $nrep$ is high, so as to seek a guarantee that the neighbourhood area is thoroughly searched and no maxima (or minima) for each main objective function remain undiscovered. The value that was considered is $nrep = \left\lceil \frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} \right\rceil$, where $|\overline{\mathcal{F}}| = \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} |\mathcal{F}_s|$ is the average number of traffic flows per service. Note that $nrep$ should not be excessively high because this would cause the algorithm run to be too long.

The cooling mechanism has to be devised so that the temperatures do not decay too slowly or too fast. Several experiences were conducted and the cooling functions that provided the best results were $T_W^j = [T_W^0 (1 - \frac{j}{J})]^a$ and $T_B^j = [T_B^0 (1 - \frac{j}{J})]^a$ in iteration j , with $J = 1000; 5000$ and $a = 0.1; 0.01$, for the 2 simultaneous SA procedures.

The probability of accepting a new solution that is actually worse than the most adequate solution up to the current stage of the algorithm (iteration j) is $p_W^j = \exp\left(\frac{W_Q^j - \max\{W_Q\}}{T_W^j}\right)$ and $p_B^j = \exp\left(\frac{\min\{B_{Mm|Q}\} - B_{Mm|Q}^j}{T_B^j}\right)$ for the 2 simultaneous SA procedures, where $\max\{W_Q\}$ and $\min\{B_{Mm|Q}\}$ are the upper level objective function values in the most adequate solution found so far.

The stopping criterion is the same as the one used in the basic heuristic, that is, the algorithm stops when $nPaths = 0$.

The adaptation of the SA technique to the basic heuristic HMOR-S2 can be described as follows:

- I. Let the initial temperatures be $T_W^0 = W_Q^0 = W_Q^{initial}$ and $T_B^0 = B_{Mm|Q}^0 = B_{Mm|Q}^{initial}$.
 - II. $j = 1$
 - III. Define J and a
 - IV. In the iteration $j \geq 1$,
 1. Let the current temperatures be $T_W^j = [T_W^0 (1 - \frac{j}{J})]^a$ and $T_B^j = [T_B^0 (1 - \frac{j}{J})]^a$.
 2. Cycle to be performed $nrep$ times
 - (a) Calculation of a new solution, using the MMRA-S2 bi-objective algorithm.
 - (b) For the new solution, let W_Q be the expected QoS service revenue and $B_{Mm|Q}$ the maximal average blocking probability for all QoS services.
 - (c) Let X_W and X_B be two r.v. following a uniform distribution in $]0.0;1.0[$.
 - (d) If $s \in \mathcal{S}_Q$, check whether $(B_{ms} \leq \min\{B_{ms}\} \text{ and } B_{Ms} \leq \min\{B_{Ms}\})$. If $s \in \mathcal{S}_B$, check whether $(W_B \geq \max\{W_B\})$.
 - i. If so,
 - A. Check whether $(W_Q > \max\{W_Q\} \text{ and } B_{Mm|Q} < \min\{B_{Mm|Q}\})$,
 - The solution is accepted.
 - B. Otherwise, check whether $X_W < \exp\left(\frac{W_Q - \max\{W_Q\}}{T_W^j}\right)$ and whether $X_B < \exp\left(\frac{\min\{B_{Mm|Q}\} - B_{Mm|Q}}{T_B^j}\right)$
 - The solution is accepted.
 - C. Otherwise, the solution is not accepted.
 - Else, the solution is not accepted.
 - End of the $nrep$ cycle
 3. $j \leftarrow j + 1$
- The cycle ends when all the cycles “For (s)”, “For (ape)” and “For ($nPaths$)” have been executed.

The formalisation of the meta-heuristic version of HMOR-S2 using SA, HMOR-S2_{SA}, is in Appendix B.1.

Note that one of the features of the SA technique is the random choice of the new solution (to be taken into account at each step of the algorithm) among all the feasible solutions in the neighbourhood of the best solution found so far. However, in the adaptation of a SA-like technique to HMOR-S2, the choice of the feasible solution to be compared with the most adequate solution found so far, is done with the help of the MMRA-S2 algorithm, as in the basic heuristic. Note that the solution provided by this auxiliary algorithm is likely to produce better results than a randomly chosen solution, taking into account the foundations of the resolution procedure, given in section 2.

Concerning the numerical complexity of this heuristic, it can be said that the instructions in the inner cycle of the procedure are executed $C_i^{\text{HMOR-S2SA}} = 4|\mathcal{S}||\overline{\mathcal{F}}|^2 + 2|\mathcal{S}||\overline{\mathcal{F}}|$ times. The numerical complexity of the heuristic in terms of the number of solutions that are analysed is $C_s^{\text{HMOR-S2SA}} = \frac{|\mathcal{S}||\overline{\mathcal{F}}|}{6} (2|\overline{\mathcal{F}}|^2 + 9|\overline{\mathcal{F}}| + 10)$. For comparison, the corresponding numerical complexities of the HMOR-S2 heuristic approach (see [17]) are $C_i^{\text{HMOR-S2}} = 4|\mathcal{S}||\overline{\mathcal{F}}|$ and $C_s^{\text{HMOR-S2}} = 2|\mathcal{S}||\overline{\mathcal{F}}|(|\overline{\mathcal{F}}| + 1)$. This means that the HMOR-S2 heuristic involves a significantly lower number of calculations than HMOR-S2_{SA}. For further details on these calculations, see Appendix C.

These complexity measures are an indication of the heuristic numerical complexity just at the level of the ‘optimisation’ procedures. One should note that each calculation of the objective functions and marginal costs (which are used as coefficients in the auxiliary bi-objective shortest path model) involves the numerical resolution of a large system of non-linear equations in $\{B_{ks}\}$ and $\{c_{ks}^{Q(B)}\}$ and such calculations have to be repeated whenever a candidate pair of paths ($r^1(f_s), r^2(f_s)$) is recalculated and analysed, in terms of its impact on the objective function values.

3.3 Application of a TS Technique to the Basic Heuristic

The TS technique is a local neighbourhood search technique applied to a dynamic neighbourhood defined in terms of the current solution and the history of the states encountered during the search up to the current instant. For example, in [18, 19] this technique is described in detail and some examples of application to different optimisation problems are provided. The TS can be defined as a technique where restrictions are imposed so as to guide a search process into areas that otherwise would not be explored in the search for new solutions [19]. The restrictions are usually the exclusion of some solutions that are classified as tabu, i.e. forbidden.

The reasoning behind the TS is that the resolution of problems should include an adaptive memory and an intelligent exploration of the solution space (i.e. a guided and systematic exploration rather than a random one) [18]. An adaptive memory allows for the implementation of procedures that manage to explore the solution space in an economic and efficient way. The memory can be a short-time one and its information is used to prevent the search from remaining in a local “optimum”, or it can be a long-time one and it allows for the use of intensification and diversification strategies. Two types of memory can be considered, an explicit memory where the information on specific solutions can be kept (for instance, elite solutions that should be looked into more thoroughly), and an attributive memory where the information on the attributes (tabu or not) of some moves are kept. A move is a change that is imposed on a solution in order to find another different solution.

A generic TS algorithm for a single objective problem, where a minimisation problem is considered, with solution space S , objective function f and neighbourhood structure N , can be seen, for example, in [21].

I. Choose an initial solution $s_0 \in S$;
 II. $s^* \leftarrow s_0$; $k \leftarrow 0$;
 III. Repeat

1. Obtain a subset V of solutions in $N(s_k)$, so as to guarantee that none of the tabu conditions resulting from applying a move m to s_k is violated or that one of the aspiration criteria resulting from the application of m to s_k is met;
2. Choose the solution s resulting from applying m to s_k in V , with better value of $f(s)$;
3. $k \leftarrow k + 1$;
4. $s_k \leftarrow s$;
5. If $f(s_k) < f(s^*)$, then $s^* \leftarrow s_k$;
6. Update the tabu and the aspiration conditions;

until the stopping condition is satisfied.
 The final value of s^* is an approximation to the optimal solution.

For a successful use of the TS technique in solving many different optimisation problems, many implementation choices have to be carefully made concerning key aspects: the diversification and intensification strategies, the information to be kept in memory, the neighbourhood area, the criteria to attribute a tabu status to a move, the tabu tenure (i.e. the time during which a move remains tabu), the aspirational criteria and the stopping condition.

In [19], this technique is compared with the SA technique. The emphasis of TS is on the search of adequate solutions in a neighbourhood that was chosen according to the history of the search process. In SA, the solutions are searched randomly and the criterion that is applied for the acceptance of solutions does not take into account the quality of other options. Unlike what happens in the SA technique, in the TS technique the adequate solutions are sought having in mind not only the objective function value, but also other influential factors, such as the diversification of solutions, the intensification of solutions, the aspirational criteria, the frequency of solutions and the tabu tenures.

Many authors have applied TS techniques to telecommunication network optimisation problems, such as network design and routing problems. See for instance [48, 49, 26, 15, 43, 36, 42].

3.3.1 Introduction of a TS technique in the HMOR-S2 heuristic

A variant of the dedicated heuristic HMOR-S2 obtained by introducing a TS technique was devised and implemented. Note that some aspects of TS-like techniques are already used in the basic heuristic. For instance, some paths are not allowed to change in certain steps (i.e. their change is tabu or forbidden). In each iteration the number of paths that can possibly change is $nPaths$ and the choice of the $nPaths$ flows for which the paths are liable to change is made according to the value of an auxiliary function $\xi(f_s)$ (see section 2.2). However, note that in HMOR-S2 the evaluation and choice of the flows which can not have their paths changed is made dynamically for each iteration, whereas in the usual implementation of TS techniques, a definition of the tabu tenure would have to be made.

Many issues had to be addressed to formulate this variant HMOR-S2_{TS}. Firstly the basic technique of TS had to be adapted to a hierarchical multiobjective problem. A choice was made to focus this technique on the QoS services revenue, having in mind its

central role in the system of preferences implicit in the model. In fact, given two non-dominated solutions it is usually more acceptable, from a network design point of view, to select the solution with higher QoS service revenue, at the cost of some degradation of $B_{Mm|Q}$.

The neighbourhood area where a new solution will be searched for also has to be defined. Considering a specific solution, the neighbourhood of that solution is the set of solutions that differ in the pair of routes $(r^1(f_s), r^2(f_s))$ for one flow. Therefore a move from one solution to another solution in the neighbourhood is done by choosing a new set of paths for one of the flows. The new set of paths for a flow is chosen by solving the auxiliary bi-objective shortest path problem with the MMRA-S2 algorithm. If this new set of paths for a particular flow allows for a better solution to the routing problem, then the previous set of paths for that flow becomes tabu and a move that would lead to using that previous set of paths again, is forbidden.

The tabu list is a list of moves which are tabu, so in this adaptation we consider the tabu list as a list of pairs of paths which are tabu. The maximal size of the tabu list is given by $nPaths$, which means it changes throughout the algorithm: at the beginning of the algorithm, $nPaths$ is high, which means that many moves can become tabu; towards the end of the algorithm, $nPaths$ decreases. New moves can be added to the tabu list and once it is full, the oldest move (at the top of the list) is withdrawn and the new move is added at the end of the list. Therefore, this list is a queue with FIFO discipline. The size of the tabu list also has an impact on the tabu tenure. Note that a tabu list is used for a specific service $s \in \mathcal{S}$ and when the algorithm proceeds to the analysis of a new service in the ‘services cycle’ of the basic heuristic the tabu list is reinitialised.

An aspirational criterion may be defined: if the values for the upper level objective functions and for the lower level objective functions (for the service under scrutiny) of a new solution are better than the corresponding values in the most adequate solution found so far, then this new solution should always be considered as the new most adequate solution, even if it is obtained by performing a tabu move.

The information on the tabu list is kept in the memory of the resolution procedure, along with information on a variable $\text{freq}(f_s)$, that gives the number of times a specific flow f_s has seen its set of paths changed throughout the algorithm. This information is associated with a long-term memory. As for the solutions that are found and explored, the only information that is kept is the one concerning the most adequate solution found up to the current stage of the algorithm.

In the inner cycle of the heuristic, if new sets of paths for all the $nPaths$ flows have been considered and a solution better than the current one has not been found yet, then the solution that will be used in the next stage of the algorithm will be the one originating from a non-tabu move with the highest value of $(W_Q(f_s) - W_Q^a) - a \cdot \text{freq}(f_s)$, where $W_Q(f_s)$ is the QoS services revenue value when the set of paths for flow f_s is changed, W_Q^a is the QoS services revenue value for the current solution, and a is an empirical parameter for which a value has to be chosen. The value of $(W_Q(f_s) - W_Q^a) - a \cdot \text{freq}(f_s)$ increases with the difference $(W_Q(f_s) - W_Q^a)$ (i.e. preference is given to the solutions with higher value of the QoS services revenue) and/or with lower $\text{freq}(f_s)$ (i.e. preference is given to the solutions obtained with the change of paths for a flow f_s which has not seen its paths change very often in the past stages of the algorithm). The reasoning behind this is based on a proposal in [26].

Note that this choice of solutions (with which the algorithm continues the search) tries to avoid local extremes. Instead of always proceeding with the best solution found so far, it becomes more advantageous to proceed with a solution with good value of QoS traffic

revenue.

The algorithm stops after a pre-defined number of iterations.

The adaptation of the TS technique to the basic heuristic HMOR-S2 can be described as follows:

- I. Initialisation of the frequency values $\text{freq}(f_s), \forall f_s, s \in \mathcal{S}$.
- II. Cycle of services,
 1. Initialisation of the tabu list, with length given by $nPaths$.
 2. Cycle in $nCycles$,
 - (a) Calculation and ordering of the values of the function $\xi(f_s)$.
 - (b) Use of MMRA-S2 to find pairs of paths for the flows f_s which may have their paths changed.
 - (c) Initialisation of $W_Q(f_s)$ and $(W_Q(f_s) - W_Q^a) - a \cdot \text{freq}(f_s)$ for those flows.
 - (d) Cycle in $numIterations$,
 - i. (Search up to a maximum of $numIterations$ new solutions in the neighbourhood of the current solution.)
 - ii. Go through the ordered flows f_s according to increasing values of $\xi(f_s)$,
 - A. Check whether the pair of paths proposed for the flow f_s is tabu.
 - B. Keep a copy of the current pair of paths for this flow and load the new pair of paths in the solution.
 - C. If the new solution is “better” than the current one (i.e. has better values for the upper level functions and for the lower level functions for the service under scrutiny),
 - If the move is tabu,
 - * If the aspirational criterion is met,
 - The current solution is the most adequate up to this stage of the algorithm.
 - Increment the value of $\text{freq}(f_s)$.
 - Otherwise, go back to the previous solution.
 - Otherwise,
 - * Increment the value of $\text{freq}(f_s)$.
 - * Check whether the new solution is better than the most adequate solution up to now and if it is so, the new solution becomes the most adequate solution.
 - * Add the move to the tabu list.
 - Leave the cycle of ‘going through the flows’.
 - Otherwise,
 - If the move is not tabu, keep the information on the QoS services revenue that results from using the new pair of paths for this flow f_s , and the value of $(W_Q(f_s) - W_Q^a) - a \cdot \text{freq}(f_s)$.
 - Go back to the previous solution.
 - (End of the cycle of ‘going through the flows’)
 - iii. If no new solution that improves the current solution was found,
 - A. Choose the solution obtained with a non-tabu move, which has the highest value of $(W_Q(f_s) - W_Q^a) - a \cdot \text{freq}(f_s)$.
 - (End of the cycle in $numIterations$.)
 - (End of the cycle in $nCycles$.)
- (End of the cycle of services.)

The formalisation of the TS meta-heuristic version of HMOR-S2, HMOR-S2_{TS}, is in Appendix B.2.

As for the numerical complexity of this heuristic, the instructions in the inner cycle of the procedure are executed $C_i^{\text{HMOR-S2}_{\text{TS}}} = 4|\mathcal{S}||\overline{\mathcal{F}}|$ times and the number of solutions that are analysed is $C_s^{\text{HMOR-S2}_{\text{TS}}} = 2|\mathcal{S}||\overline{\mathcal{F}}|(|\overline{\mathcal{F}}| + 1)$. Therefore, the numerical complexity

represented by any of these measures is the same as for the HMOR-S2 heuristic (see [17]). For further details on these calculations, see Appendix C.

4 Experimental Results

In this section, the computational results obtained with the HMOR-S2_{SA} and the HMOR-S2_{TS} heuristics in a network case study analogous to the one in [34] are presented. This experimental study can provide further insight into the quality of these variants. To obtain these computational results, both analytical and simulation experiments (for the cases where more promising analytical results were obtained) using a dedicated discrete-event stochastic simulator, were conducted.

4.1 Application Model

In [34] a model for traffic routing optimisation and admission control in multiservice networks supporting traffic with different QoS requirements, was proposed. This model will be used as a benchmarking study for the present work concerning upper bounds for the optimal value of the QoS traffic revenue. The objective functions to be maximised in the problem formulated in [34] are the QoS and BE flows revenues, W_Q and W_B . A bi-criteria lexicographic optimisation problem was formulated, so that the improvements in W_B are to be sought under the constraint that W_Q remains with the optimal value. A two-stage heuristic procedure based on a MCF (Multicommodity Flow) formulation was developed to solve this problem. An admission control mechanism was applied in the first stage of the heuristic. Initially only QoS traffic in the original network \mathcal{N} is taken into account and the aim is to find the optimal value of W_Q . Once this has been achieved, the BE traffic is offered to a residual network \mathcal{N}' , composed of arcs with the remaining capacities. In the first stage deterministic models are used in the calculation of paths, in particular mathematical programming models based on MCFs. At the end of this first stage, a routing plan for each service is obtained. As these models are only a rough approximation in this context and they tend to under-evaluate the blocking probabilities, Mitra et al. [34] propose an adaptation of the original model to obtain more ‘correct’ models, that is models which constitute a better approximation in a stochastic traffic environment. This adaptation of the MCF model to a stochastic environment consists of a compensation of the required bandwidth values of the flows in the MCF model with a parameter $\alpha \geq 0.0$, so as to represent the effect of the random fluctuations of the traffic that are typical of stochastic traffic flows. The parameter α should have a high value if the need for compensation is high, due to a high variability in the point processes. In this adaptation, the flow rates (input data for the flow-based problem) are compensated to make up for the variabilities in traffic intensities in a stochastic environment. The MCF-based result is mapped into the adapted model, keeping the relations between traffic intensities invariant.

Furthermore, traffic splitting was used in this traffic routing model, which means that the required bandwidth of each flow may be divided by multiple paths from source to destination, allowing for a more balanced traffic distribution in the network, hence lower blocking probabilities.

The fact that the values of W_Q obtained by this reference model provide upper bounds for the optimal value of W_Q (for the same input traffic matrix) in our model, results from the lexicographic optimisation as well as the simplifications in the traffic model, the

Table 4.1: Service features on the test network M1

Service	Class	d'_s [kbps]	d_s [channels]	w_s	h_s [s]	D_s [arcs]	m_s
1 - video	QoS	640	40	40	600	3	0.1
2 - Premium data	QoS	384	24	24	300	4	0.25
3 - voice	QoS	16	1	1	60	3	0.4
4 - data	BE	384	24	24	300	7	0.25

admission control and the traffic splitting mechanisms, adopted in [34].

4.2 Application of the Model to a Network Case Study

The routing model in [34] was applied to the test network depicted in figure 4.1. It has

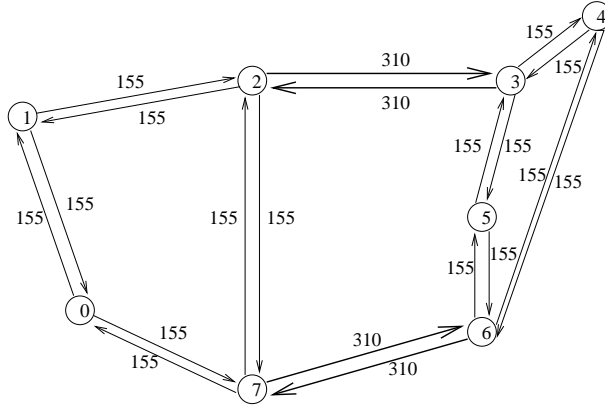


Figure 4.1: Test network M1 [34], with the indication of the bandwidth of each arc C'_k , in Mbps

$N = 8$ nodes, with 10 pairs of nodes linked by a direct arc and a total of $|\mathcal{L}| = 20$ unidirectional arcs. The bandwidth of each arc C'_k [Mbps] is shown in figure 4.1. The number of channels C_k is $C_k = \left\lceil \frac{C'_k}{u_0} \right\rceil$, with basic unit capacity $u_0 = 16$ kbps. There are $|\mathcal{S}| = 4$ service types with the features displayed in table 4.1. The values of the required effective bandwidths $d_s = \frac{d'_s}{u_0}$ [channels] $\forall s \in \mathcal{S}$ are also in the table (where d'_s is the required bandwidth in kbps). The expected revenue for a call of type s is assumed to be $w_s = d_s, \forall s \in \mathcal{S}$. The average duration of a type s call is h_s and D_s represents the maximum number of arcs for a type s call.

A base matrix $T = [T_{ij}]$ with offered total bandwidth values from node i to node j [Mbps] is provided in [34]. As mentioned above, the adaptation of the MCF model to a stochastic model was based on a compensation mechanism that models the effect of random fluctuations of traffic that are typical of a stochastic traffic model. After the introduction of the compensation factor, a relation can be established between the bandwidth demand of each flow f_s for a traffic mix $T(f_s) = m_s T_{ij}$ with $m_s \in [0.0; 1.0]$ and $\sum_{s \in \mathcal{S}} m_s = 1.0$, in the MCF model and the parameters $A(f_s)$ (the mean traffic offered associated with f_s , in Erlang) and $d'_s = d_s u_0$ of the stochastic model. From [34, eq.(5.2)], $A(f_s) \approx \frac{T(f_s)}{d'_s} - \alpha \sqrt{\frac{T(f_s)}{d'_s}} = \frac{m_s T_{ij}}{d_s u_0} - \alpha \sqrt{\frac{m_s T_{ij}}{d_s u_0}}$ [Erl] if $\frac{T(f_s)}{d'_s} = \frac{m_s T_{ij}}{d_s u_0} > \alpha^2$ and both $T(f_s)$

and $A(f_s)$ are high. Otherwise, $A(f_s) \approx \frac{T(f_s)}{d'_s} = \frac{m_s T_{ij}}{d_{su0}}$ [Erl]. From these data all the parameters needed by our traffic model can be obtained as shown in [6].

In this application example, results for the QoS flows revenue W_Q are presented for three values of α : $\alpha = 0.0$ corresponds to the deterministic situation; $\alpha = 0.5$ is the compensation parameter when calls arrive according to a Poisson process, service times follow an exponential distribution and the network is critically loaded; and $\alpha = 1.0$ is used for traffic flows with higher ‘variability’.

For further details on the application of this traffic model to the network case study under analysis, see [6].

4.3 Analytical Results

In the analytical study, the meta-heuristic versions were run only once. For the routing plan obtained at the end of this single run, values for all the objective functions are computed and if the first level objective function values dominate the corresponding values for the initial solution, then this routing plan will be the final solution.

Two different sets of tests were conducted: the (i) tests where the initial solution is the same as the one used in the basic heuristic HMOR-S2 runs, a solution which is typical of Internet routing conventional algorithms; the (f) tests where the initial solution of each meta-heuristic version is the routing plan obtained at the end of the basic heuristic runs for each specific α .

For the (i) tests, a solution with only one path for each flow, i.e. without an alternative path, is considered leaving it up to the heuristic to find an adequate solution with second choice paths. The initial solution is the same for all the services $s \in \mathcal{S}$ and the paths are symmetrical. The path for every flow f_s is the shortest one (that is, the one with minimum number of arcs); if there is more than one shortest path, the one with maximal bottleneck bandwidth (i.e. the minimal capacity of its arcs) is chosen; if there is more than one shortest path with equal bottleneck bandwidth, the choice is arbitrary.

As for the (f) tests, the aim is to check whether the meta-heuristic variants can improve the quality of the final solutions obtained with HMOR-S2 as an alternative to the direct use of the meta-heuristics (as in the case of the (i) tests).

The analytical results concerning W_Q were compared with results obtained with the previous heuristic HMOR-S2 [16] and with the model proposed in [34] for MPLS networks with two service classes that uses a lexicographic optimisation formulation based on a deterministic MCF model, which provides an upper bound to the objective function W_Q optimal value in P-M2-S2.

The experiences with the HMOR-S2_{SA} were conducted with different temperature cooling functions and the ones that provided best results for the upper level objective functions were $T_W^j = [T_W^0 (1 - \frac{j}{J})]^a$ and $T_B^j = [T_B^0 (1 - \frac{j}{J})]^a$ in iteration j , with $J = 1000; 5000$ and $a = 0.1; 0.01$. The final results were quite similar regardless of the chosen value. An example of the results is displayed in table 4.2. These results were obtained with $J = 1000$ and $a = 0.1$ in 11m30s on average in a Linux environment on a Pentium 4 processor with 3GHz CPU and 1GB of RAM.

The experiences with the HMOR-S2_{TS} were conducted with different values for *numIterations* and a , and the ones that provided best results for the upper level objective functions were *numIterations* = 10 and $a = 20$. These results are displayed in table 4.2 and they were obtained in 11m08s on average in the same computer mentioned earlier.

In the table 4.2, two different comparative analysis can be performed. For HMOR-S2_{SA}(i) and HMOR-S2_{TS}(i), the initial solution is the same as the one used in the corre-

sponding basic heuristic so the table allows for a comparison of the final results obtained with HMOR-S2 and HMOR-S2_{SA} or with HMOR-S2 and HMOR-S2_{TS}. As for the variants HMOR-S2_{SA}(f) and HMOR-S2_{TS}(f), the initial solution has the objective function values displayed in the table under HMOR-S2 (Basis) so that a comparison of the initial and the final results with HMOR-S2_{SA} and with HMOR-S2_{TS} can be performed. If an objective function value obtained with one of the variants is the same or better than the corresponding objective function value obtained with the basic heuristic, this is indicated in bold. The table also shows the obtained results for W_Q as a percentage of the upper bound optimal values given in [34].

With the (i) version of the heuristic HMOR-S2_{SA}, the final results for the upper level objective functions are worse when $\alpha = 0.0$ and are the same for the other values of α . As for the (i) version of the heuristic HMOR-S2_{TS}, the final results for the upper level objective functions improve for $\alpha = 1.0$ but are worse for the other values of α . As these variants take longer to run than the basic heuristic and generally do not provide better results for W_Q and $B_{Mm|Q}$, when the initial solution is the same, they can not be considered a better approach for solving the routing problem. However, their use on a second stage of the resolution of the routing problem (after the basic heuristic has been used on a first stage) seems to provide interesting results. In fact, for $\alpha = 0.0; 0.5$, the upper level objective function results are better with the (f) test of the heuristic HMOR-S2_{SA}. In particular, with the (f) application version of the heuristic HMOR-S2_{TS}, the upper level objective function results improve for all the values of α .

The results with $\alpha = 1.0$ for both variants are worth mentioning. After HMOR-S2_{SA}(f) is run, the final solution is actually the same as the initial solution. Note that the heuristics always give the initial solution as a final result if the algorithm has not succeeded in finding a better solution in terms of the objective functions W_Q and $B_{Mm|Q}$. As for HMOR-S2_{TS}, the values for W_Q and $B_{Mm|Q}$ in the final solution obtained with the (i) test are actually better than the ones obtained with the (f) test, although the latter are still slightly better than for the basic heuristic. This shows the dependency of the final results on the initial solution, and also shows that starting with a better solution does not necessarily lead to a better final solution.

Taking these results into account, we may conclude that a run of the basic heuristic HMOR-S2 followed by a run of the HMOR-S2_{SA} variant or a run of the HMOR-S2_{TS} variant may provide improved results for the routing problem under analysis. Finally note that the best results obtained with the meta-heuristic variants are more than 99% of the optimal value W_Q . This shows that a significant improvement on $B_{Mm|Q}$ can be obtained just with a very slight worsening on the average revenue W_Q , which gives an idea of the potential advantages of this type of multiobjective routing formulations as previously noted in [5, 6].

4.4 Simulation Results

After the analytical experiences were performed, simulation experiences, with static routing methods using the heuristics, were also carried out for the cases where more promising results were obtained. We considered that simulations with a dynamic version of the routing methods would not provide any important additional information on the quality of the variants of the heuristic. In the simulation study we used a discrete-event simulation platform developed for this type of networks, which enabled the validation of the routing model results and the evaluation of the errors intrinsic to the analytical model which provides the estimates for the objective functions.

Table 4.2: Objective function values for the final solution for HMOR-S2, HMOR-S2_{SA}(i) and HMOR-S2_{SA}(f), HMOR-S2_{TS}(i) and HMOR-S2_{TS}(f), for different traffic matrices

Objective Functions	$\alpha = 0.0$				
	HMOR-S2 (Basis)	HMOR-S2 _{SA}		HMOR-S2 _{TS}	
		(i)	(f)	(i)	(f)
W_Q	64731.51*	64517.97	64795.66 ◇	64619.61	64915.35 ★
$B_{Mm Q}$	0.0898	0.107	0.0843	0.116	0.0731
$B_{m1 Q}$	0.0898	0.107	0.0843	0.116	0.0731
$B_{m2 Q}$	0.0199	0.0218	0.0194	0.0105	0.0189
$B_{m3 Q}$	0.00216	0.00283	0.00206	0.00480	0.00179
$B_{M1 Q}$	0.691	0.673	0.700	0.854	0.721
$B_{M2 Q}$	0.0723	0.115	0.0811	0.0434	0.0953
$B_{M3 Q}$	0.0287	0.0274	0.0295	0.0467	0.0312
W_B	17007.15	17662.81	17121.51	17489.36	17163.01
Objective Functions	$\alpha = 0.5$				
	HMOR-S2 (Basis)	HMOR-S2 _{SA}		HMOR-S2 _{TS}	
		(i)	(f)	(i)	(f)
W_Q	60569.09†	60569.09	60724.32 ●	60162.90	60751.77 ◎
$B_{Mm Q}$	0.0424	0.0424	0.0289	0.0805	0.0258
$B_{m1 Q}$	0.0424	0.0424	0.0289	0.0805	0.0258
$B_{m2 Q}$	0.00534	0.00534	0.00270	0.0104	0.00259
$B_{m3 Q}$	0.00119	0.00119	0.000854	0.00254	0.000744
$B_{M1 Q}$	0.628	0.628	0.619	0.742	0.634
$B_{M2 Q}$	0.0432	0.0432	0.0108	0.0385	0.00769
$B_{M3 Q}$	0.0243	0.0243	0.0237	0.0330	0.0246
W_B	16904.99	16904.99	16738.50	17664.88	16905.73
Objective Functions	$\alpha = 1.0$				
	HMOR-S2 (Basis)	HMOR-S2 _{SA}		HMOR-S2 _{TS}	
		(i)	(f)	(i)	(f)
W_Q	56100.60‡	56100.60	56100.60 □	56191.34	56109.97 ⊗
$B_{Mm Q}$	0.0263	0.0263	0.0263	0.0179	0.0252
$B_{m1 Q}$	0.0263	0.0263	0.0263	0.0179	0.0252
$B_{m2 Q}$	0.00515	0.00515	0.00515	0.00266	0.00494
$B_{m3 Q}$	0.000560	0.000560	0.000560	0.000430	0.000555
$B_{M1 Q}$	0.544	0.544	0.544	0.489	0.556
$B_{M2 Q}$	0.0185	0.0185	0.0185	0.00955	0.0177
$B_{M3 Q}$	0.0193	0.0193	0.0193	0.0165	0.0200
W_B	16479.60	16479.60	16479.60	16288.89	16464.83

HMOR-S2: *)99.35%; †)99.57%; ‡)99.58% of the optimal revenue W_Q^{\max} in [34];
HMOR-S2_{SA}(f): ◇)99.45%; ●)99.83%; □)99.58% of the optimal revenue W_Q^{\max} in [34];
HMOR-S2_{TS}(f): ★)99.63%; ◎)99.87%; ⊗)99.59% of the optimal revenue W_Q^{\max} in [34]

The discrete-event stochastic simulation was applied to a static routing model, where the routing plan is the final solution obtained after the (f) test for each of the variants was run. This routing plan never changes throughout the simulation regardless of the random variations of traffic offered to the network. After an initialisation phase that lasts for a time $t_{warm-up}$, information on the number of offered calls and effectively carried calls in the network for each flow $f_s, s \in \mathcal{S}$, is gathered, until the end of the simulation. With this information, $B(f_s), \forall s \in \mathcal{S}$ and subsequently, the values of the upper and lower level objective functions related to blocking probabilities can be estimated. As for the expected revenues, knowing the effectively carried calls in the network allows for the calculation of the carried traffic estimates and average revenues.

The total simulated time considered in these experiments was $t_{total} = 48\text{h}$. Different values were tested for the warm-up time, which should be long enough to guarantee that the system state at the end of the initialisation phase is representative of the steady state behaviour of the system. The results displayed in table 4.3 were obtained with the total time $t_{total} = 48\text{h}$ and the warm-up time $t_{warm-up} = 8\text{h}$. It took almost 2h to get these results in the same computer mentioned earlier.

No simulations were performed for the static routing model with a routing plan equal to the final solution obtained after the (i) version of the variants was run, because in general those solutions were either not very good or the same as the solution obtained after the basic heuristic was run. As the results for the (i) version in table 4.2 show, only the final solution for the TS-like variant and $\alpha = 1.0$ is better (in terms of the upper level objective function values) than the corresponding final solution for HMOR-S2.

In the table 4.3, the analytical values of each objective function are displayed, together with the simulation results (average value \pm half length of a 95% confidence interval, computed by the independent replications method [27]) for these functions. If the statistical estimate of an objective function value obtained with one of the variants is the same or better than the corresponding value obtained with the basic heuristic, this is indicated in bold. Furthermore, if some simulation result is better than the corresponding analytical value, this is indicated in italic. The revenue values have 2 decimal places and the blocking probability values have 3 significant figures. In order to know the uncertainty level associated with the objective function values, the relative half-lengths of the 95% confidence intervals (in percentage) are also displayed in the table.

In most cases, the analytical results are outside the 95% confidence interval of the static routing model simulation results, but they are of similar magnitude. The analytical results tend to be better than the corresponding static routing model simulation results, especially in situations of lower traffic loads (which correspond to higher values of α in this routing problem application example). In fact, only for the HMOR-S2_{SA}(f) heuristic with $\alpha = 0.0$ did we get a result where an upper level objective function analytical value was in the corresponding confidence interval and had a value worse than the corresponding static routing model simulation result. These differences between the simulation and analytic results are mainly due to the inaccuracies intrinsic to the analytic/numerical resolution, particularly those associated with the simplifications of the traffic model, and the associated error propagation. As the overflow traffic is treated as Poisson traffic, the analytical model is actually a simplification which tends to underestimate the blocking probabilities in the network (and to overestimate the revenues). The errors that result from this simplification propagate throughout the complex and lengthy numerical calculations associated with the resolution, for a great number of times, of the large systems of implicit non-linear equations, (2.5) and (2.6). Further simplifications were assumed in the stochastic model for the traffic in the links: a superposition of independent Poisson flows

Table 4.3: Average objective function values with 95% confidence intervals, for simulations with the routing plan obtained with the HMOR-S2_{SA}(f) and the HMOR-S2_{TS}(f)

Objective Functions	HMOR-S2		HMOR-S2 _{SA} (f)		HMOR-S2 _{TS} (f)	
	Analytical	Static routing model	Analytical	Static routing model	Analytical	Static routing model
Results for $\alpha = 0.0$						
W_Q	64731.51	64642.53±64.17(0.10%)	64795.66	64704.03±72.85(0.11%)	64915.35	64781.55±67.82(0.10%)
$B_{Mm Q}$	0.0898	<i>0.0887±0.00336(3.79%)</i>	0.0843	0.0830±0.00389(4.68%)	0.0731	0.0749±0.00316(4.22%)
$B_{m1 Q}$	0.0898	<i>0.0887±0.00336(3.79%)</i>	0.0843	0.0830±0.00389(4.68%)	0.0731	0.0749±0.00316(4.22%)
$B_{m2 Q}$	0.0199	0.0246±0.000647(2.63%)	0.0194	0.0242±0.000551(2.27%)	0.0189	0.0243±0.000609(2.51%)
$B_{m3 Q}$	0.00216	0.00226±0.0000663(2.93%)	0.00206	0.00216±0.0000624(2.89%)	0.00179	0.00196±0.0000485(2.47%)
$B_{M1 Q}$	0.691	<i>0.684±0.00802(1.17%)</i>	0.700	<i>0.687±0.0119(1.74%)</i>	0.721	<i>0.714±0.0180(2.52%)</i>
$B_{M2 Q}$	0.0723	0.0843±0.00242(2.87%)	0.0811	0.0923±0.00377(4.09%)	0.0953	0.106±0.0107(10.05%)
$B_{M3 Q}$	0.0287	0.0291±0.000206(0.71%)	0.0295	0.0298±0.000171(0.57%)	0.0312	0.0315±0.000236(0.75%)
W_B	17007.15	16982.33±37.02(0.22%)	17121.51	17102.41±40.75(0.24%)	17163.01	17137.81±49.80(0.29%)
Results for $\alpha = 0.5$						
W_Q	60569.09	60491.22±50.79(0.08%)	60724.32	60655.12±60.57(0.10%)	60751.77	60655.33±57.72(0.10%)
$B_{Mm Q}$	0.0424	0.0460±0.00163(3.54%)	0.0289	0.0320±0.00162(5.08%)	0.0258	0.0308±0.00104(3.39%)
$B_{m1 Q}$	0.0424	0.0460±0.00163(3.54%)	0.0289	0.0320±0.00162(5.08%)	0.0258	0.0308±0.00104(3.39%)
$B_{m2 Q}$	0.00534	0.00809±0.000328(4.06%)	0.00270	0.00521±0.000329(6.32%)	0.00259	0.00577±0.000269(4.66%)
$B_{m3 Q}$	0.00119	0.00126±0.0000403(3.20%)	0.000854	0.000927±0.0000182(1.96%)	0.000744	0.000838±0.0000167(2.00%)
$B_{M1 Q}$	0.628	0.631±0.0151(2.40%)	0.619	0.615±0.0210(3.41%)	0.634	0.637±0.0157(2.46%)
$B_{M2 Q}$	0.0432	0.0503±0.00266(5.29%)	0.0108	0.0179±0.00201(11.27%)	0.00769	0.0139±0.000742(5.35%)
$B_{M3 Q}$	0.0243	0.0245±0.000196(0.80%)	0.0237	0.0239±0.000117(0.49%)	0.0246	0.0248±0.000278(1.12%)
W_B	16904.99	16899.02±38.69(0.23%)	16738.50	<i>16752.53±39.75(0.24%)</i>	16905.73	16905.09±39.59(0.23%)
Results for $\alpha = 1.0$						
W_Q	56100.60	56027.72±46.92(0.08%)	56100.60	56027.72±46.92(0.08%)	56109.97	56038.54±47.33(0.08%)
$B_{Mm Q}$	0.0263	0.0281±0.00126(4.48%)	0.0263	0.0281±0.00126(4.48%)	0.0252	0.0269±0.00126(4.70%)
$B_{m1 Q}$	0.0263	0.0281±0.00126(4.48%)	0.0263	0.0281±0.00126(4.48%)	0.0252	0.0269±0.00126(4.70%)
$B_{m2 Q}$	0.00515	0.00832±0.000685(8.23%)	0.00515	0.00832±0.000685(8.23%)	0.00494	0.00806±0.000648(8.04%)
$B_{m3 Q}$	0.000560	0.000637±0.0000154(2.42%)	0.000560	0.000637±0.0000154(2.42%)	0.000555	0.000633±0.0000168(2.65%)
$B_{M1 Q}$	0.544	0.547±0.0281(5.13%)	0.544	0.547±0.0281(5.13%)	0.556	0.558±0.0192(3.45%)
$B_{M2 Q}$	0.0185	0.0325±0.00353(10.88%)	0.0185	0.0325±0.00353(10.88%)	0.0177	0.0312±0.00331(10.60%)
$B_{M3 Q}$	0.0193	0.0195±0.000167(0.86%)	0.0193	0.0195±0.000167(0.86%)	0.0200	0.0202±0.000307(1.52%)
W_B	16479.60	16453.09±17.05(0.10%)	16479.60	16453.09±17.05(0.10%)	16464.83	16438.45±18.54(0.11%)

and independent occupations of the links. A more accurate and realistic representation of the traffic flows would allow for better estimates of the blocking probabilities (see for example the numerical algorithms proposed in [7] where the representation of the traffic flows is based on their means and variance values). Nonetheless, the approximations in our model can be considered appropriate in this context for practical reasons. In fact, if more complex models were used to represent the traffic and to calculate the blockings in overflow conditions, the computational burden would be too heavy since the analytical model has to be numerically solved many times during the execution of the heuristic and the routing method would be intractable. It is important to note that, concerning accuracy, the focus is on the relative value of the results of the traffic model rather than on the absolute accuracy of such values, since the aim of the routing optimisation procedure is just the comparison of routing solutions, in terms of the values of the objective functions.

The results displayed in the table for the upper level objective functions obtained with the two variants are close, but for the TS-like variant are slightly better than with the SA-like variant. Therefore, the HMOR-S2_{TS} heuristic may be considered more adequate to the resolution of the very complex routing problem P-M2-S2. A comparison of the results obtained with both variants shows that the analytical and simulation results are coherent, in the sense that whenever the analytical value of an objective function is better for the TS-like variant than for the SA-like variant, the same happens with the average values obtained with the static routing model simulation.

5 Conclusions and Further Work

In this work we began by reviewing a hierarchical bi-level multiobjective routing model in MPLS networks with alternative routing, with two classes of services (with different priorities in the optimisation model) and different types of traffic flows in each class. A specialised heuristic strategy, HMOR-S2, for finding “good” compromise solutions to this very complex routing optimisation problem, was also reviewed.

Sensitivity tests performed on HMOR-S2 showed that in particular cases there were “better” solutions to the routing problem that the basic heuristic was unable to find. This realisation motivated the need to devise new variants that could possibly find solutions “better” than the ones obtained with the HMOR-S2 basic heuristic. Two different variants of this heuristic HMOR-S2 were put forward by introducing meta-heuristic techniques. These variants, HMOR-S2_{SA} and HMOR-S2_{TS} were based on meta-heuristic procedures, namely SA and TS procedures (respectively).

These variants maintain the resolution framework of the HMOR-S2 heuristic. Therefore, considering HMOR-S2 as a resolution basis, procedures that are typical of those meta-heuristics were applied to that basis and allowed for the formulation of SA-like and TS-like resolutions of the addressed complex routing problem.

These variants were applied to a test network used in a benchmarking case study [34] that uses a lexicographic optimisation routing approach, including admission control for BE traffic, based on a deterministic traffic representation, with the expected revenues associated with QoS and BE traffic as objective functions. The analytical results obtained with the variants were compared with the optimal values for the QoS service expected revenue in the benchmarking study and with the values obtained with the basic heuristic HMOR-S2. The results show that the introduction of meta-heuristic techniques, in particular SA and TS, in the specialised basic heuristic, does not necessarily lead to better results. However, the introduction of these techniques is advantageous in the search for

improvements of the final solution obtained with the basic heuristic. In fact, a run of the basic heuristic HMOR-S2 followed by a run of either the variants tends to provide improved results for the routing problem. Both variants, and in particular the TS-like variant, may produce final solutions with better values for the first level objective functions, when the initial solution is the one obtained after a run of the basic heuristic.

A discrete-event simulation platform was used for a more exact evaluation of the results of the heuristic in a stochastic environment closer to real network working conditions. In most cases, the analytical results obtained with the HMOR-S2 are not inside the 95% confidence interval of the static routing model simulation results, although they are of similar magnitude, due to the inaccuracies intrinsic to the analytic/numerical resolution, namely those associated with the simplifications of the traffic model, and the associated error propagation.

Finally note that these variants have added a greater complexity to the basic heuristic. The computational burden of the resolution has also increased. These remain the major limitations of this type of routing method and restrain its potential practical application, at present, to networks with a limited number of nodes, such as the core and intermediate (metro-core) level networks of low dimension.

Further work on this model will focus on the search for possible simplifications and improvements in the heuristic resolution approaches. Also the extension of the model to broader routing principles such as probabilistic load sharing or traffic splitting might be studied and tested.

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A Formalisation of the Heuristic HMOR-S2

- I. $\bar{R}_a \leftarrow \bar{R}_o$
- II. Compute \bar{B} and $W_Q, B_{Mm|Q}$ for \bar{R}_a
- III. $W_Q^o \leftarrow W_Q, B_{Mm|Q}^o \leftarrow B_{Mm|Q}$
- IV. $\bar{R}_* \leftarrow \bar{R}_a$
- V. Compute \bar{B} for \bar{R}_a
 Compute $W_Q, B_{Mm|Q}, B_{ms|Q}, B_{Ms|Q} (\forall s \in \mathcal{S}_Q), W_B$ for \bar{R}_a
- VI. $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
 $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q} (\forall s \in \mathcal{S}_Q)$ and $\max\{W_B\} \leftarrow W_B$
- VII. For $nPaths = |\mathcal{F}|$ to $nPaths = 1$
 1. For $ape = 0$ to $ape = 1$
 - (a) If $ape = 0$, $z_{APR} \leftarrow 1.0$
 Else, $z_{APR} \leftarrow 0.01 \cdot nPaths$
 - (b) For $s = 1$ to $s = |\mathcal{S}|$
 - i. For $nCycles = 1$ to $nCycles = 0$
 - A. Compute \bar{B} and $\bar{c}^Q, s \in \mathcal{S}_Q$ or $\bar{c}^B, s \in \mathcal{S}_B$ for \bar{R}_a
 - B. Compute and order the values of the function $\xi(f_s)$, with $\xi(f_s) = F_L(f_s)$ if $nCycles = 1$ and $\xi(f_s) = F_C^{Q(B)}(f_s)$ if $nCycles = 0$
 - C. Find the $nPaths$ flows with lower value of $\xi(f_s)$
 - D. Compute with MMRA-S2 new candidate paths for the corresponding O-D pairs and define a new set of first and second choice paths for the service $s, \bar{R}_a(s)$, according to the rules established for each service
 - E. Compute \bar{B} for \bar{R}_a
 Compute $B_{ms|Q}, B_{Ms|Q}$ if $s \in \mathcal{S}_Q$ or W_B if $s \in \mathcal{S}_B$ for \bar{R}_a
 - F. If $s \in \mathcal{S}_Q$ then
 - If $(B_{ms|Q} < \min\{B_{ms|Q}\})$ and $B_{Ms|Q} < \min\{B_{Ms|Q}\})$ then
 - Compute $W_Q, B_{Mm|Q}$

- If $(W_Q > \max\{W_Q\} \text{ and } B_{Mm|Q} < \min\{B_{Mm|Q}\})$ then
 - * $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}$
 - * $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
 - * $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$

G. Else ($s \in \mathcal{S}_B$)

- If $(W_B > \max\{W_B\})$ then
 - Compute $W_Q, B_{Mm|Q}$
 - If $(W_Q > \max\{W_Q\} \text{ and } B_{Mm|Q} < \min\{B_{Mm|Q}\})$ then
 - * $\max\{W_B\} \leftarrow W_B$
 - * $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
 - * $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$

H. $\overline{R}_a(s) \leftarrow \overline{R}_*(s)$

End of the cycle For ($nCycles$)

End of the cycle For (s)

End of the cycle For (ape)

End of the cycle For ($nPaths$)

VIII. If $W_Q^o > \max\{W_Q\}$ or $B_{Mm|Q}^o < \min\{B_{Mm|Q}\}$ then

1. The best solution is \overline{R}_o

IX. Otherwise, the best solution is \overline{R}_*

X. Compute the objective function values for the best solution

B Formalisation of the Meta-Heuristics

B.1 HMOR-S2_{SA}

I. $\overline{R}_a \leftarrow \overline{R}_o$

II. Compute \overline{B} and $W_Q, B_{Mm|Q}$ for \overline{R}_a

III. $W_Q^o \leftarrow W_Q, B_{Mm|Q}^o \leftarrow B_{Mm|Q}$

IV. $\overline{R}_* \leftarrow \overline{R}_a$

V. Compute \overline{B} for \overline{R}_a

Compute $W_Q, B_{Mm|Q}, B_{ms|Q}, B_{Ms|Q} (\forall s \in \mathcal{S}_Q), W_B$ for \overline{R}_a

VI. $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$

$\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q} (\forall s \in \mathcal{S}_Q)$ and $\max\{W_B\} \leftarrow W_B$

VII. $T_W^0 \leftarrow W_Q^0 = W_Q^{initial}; T_B^0 \leftarrow B_{Mm|Q}^0 = B_{Mm|Q}^{initial}$

VIII. Initialisation of J ; Initialisation of a ;

IX. $j \leftarrow 0$

X. For $nPaths = |\overline{\mathcal{F}}|$ to $nPaths = 1$

1. For $ape = 0$ to $ape = 1$

(a) If $ape = 0, z_{APR} \leftarrow 1.0$

Else, $z_{APR} \leftarrow 0.01 \cdot nPaths$

(b) For $s = 1$ to $s = |\mathcal{S}|$

i. For $nCycles = 1$ to $nCycles = 0$

A. $j \leftarrow j + 1$

B. $T_W^j \leftarrow [T_W^0 (1 - \frac{j}{J})]^a$; $T_B^j \leftarrow [T_B^0 (1 - \frac{j}{J})]^a$

C. $nrep \leftarrow \left\lceil \frac{|\mathcal{F}|+1-nPaths}{2} \right\rceil$

D. Cycle in $nrep$

- Compute \bar{B} and $\bar{c}^Q, s \in \mathcal{S}_Q$ or $\bar{c}^B, s \in \mathcal{S}_B$ for \bar{R}_a
- Compute and order the values of the function $\xi(f_s)$, with $\xi(f_s) = F_L(f_s)$ if $nCycles = 1$ and $\xi(f_s) = F_C^{Q(B)}(f_s)$ if $nCycles = 0$
- Find the $nPaths$ flows with lower value of $\xi(f_s)$
- Compute with MMRA-S2 new candidate paths for the corresponding O-D pairs and define a new set of first and second choice paths for the service s , $\bar{R}_a(s)$, according to the rules established for each service
- Compute \bar{B} for \bar{R}_a
Compute $B_{ms|Q}, B_{Ms|Q}$ if $s \in \mathcal{S}_Q$ or W_B if $s \in \mathcal{S}_B$ for \bar{R}_a
- Generate $X_W \sim \mathcal{U}[0.0; 1.0]$ and $X_B \sim \mathcal{U}[0.0; 1.0]$
- If $s \in \mathcal{S}_Q$ then
 - If $(B_{ms|Q} \leq \min\{B_{ms|Q}\})$ and $B_{Ms|Q} \leq \min\{B_{Ms|Q}\})$ then
 - * Compute $W_Q, B_{Mm|Q}$
 - * If $(W_Q \geq \max\{W_Q\})$ and $B_{Mm|Q} \leq \min\{B_{Mm|Q}\})$ or $(X_W < \left(\frac{W_Q - \max\{W_Q\}}{T_W^j}\right))$ and $X_B < \left(\frac{\min\{B_{Mm|Q}\} - B_{Mm|Q}}{T_B^j}\right)$ then
 - * \diamond $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}$
 - * \diamond $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
 - * \diamond $\bar{R}_*(s) \leftarrow \bar{R}_a(s)$
 - Else ($s \in \mathcal{S}_B$)
 - If $(W_B \geq \max\{W_B\})$ then
 - * Compute $W_Q, B_{Mm|Q}$
 - * If $(W_Q \geq \max\{W_Q\})$ and $B_{Mm|Q} \leq \min\{B_{Mm|Q}\})$ or $(X_W < \left(\frac{W_Q - \max\{W_Q\}}{T_W^j}\right))$ and $X_B < \left(\frac{\min\{B_{Mm|Q}\} - B_{Mm|Q}}{T_B^j}\right)$ then
 - * \diamond $\max\{W_B\} \leftarrow W_B$
 - * \diamond $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
 - * \diamond $\bar{R}_*(s) \leftarrow \bar{R}_a(s)$
- $\bar{R}_a(s) \leftarrow \bar{R}_*(s)$

End of the Cycle in $nrep$

End of the cycle For ($nCycles$)

End of the cycle For (s)

End of the cycle For (ape)

End of the cycle For ($nPaths$)

XI. If $W_Q^o > \max\{W_Q\}$ or $B_{Mm|Q}^o < \min\{B_{Mm|Q}\}$ then

1. The best solution is \bar{R}_o

XII. Otherwise, the best solution is \bar{R}_*

XIII. Compute the objective function values for the best solution

B.2 HMOR-S2_{TS}

- I. $\bar{R}_a \leftarrow \bar{R}_o$
- II. Compute \bar{B} and $W_Q, B_{Mm|Q}$ for \bar{R}_a
- III. $W_Q^o \leftarrow W_Q, B_{Mm|Q}^o \leftarrow B_{Mm|Q}$
- IV. $\bar{R}_* \leftarrow \bar{R}_a$
- V. Compute \bar{B} for \bar{R}_a
 Compute $W_Q, B_{Mm|Q}, B_{ms|Q}, B_{Ms|Q}(\forall s \in \mathcal{S}_Q), W_B$ for \bar{R}_a
- VI. $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$
 $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}(\forall s \in \mathcal{S}_Q)$ and $\max\{W_B\} \leftarrow W_B$
- VII. $\text{freq}(f_s) \leftarrow 0, \forall f_s, s \in \mathcal{S}$;
- VIII. Initialisation of a ; Initialisation of numIterations ;
- IX. For $nPaths = |\mathcal{F}|$ to $nPaths = 1$
 1. For $ape = 0$ to $ape = 1$
 - (a) If $ape = 0, z_{APR} \leftarrow 1.0$
 Else, $z_{APR} \leftarrow 0.01 \cdot nPaths$
 - (b) For $s = 1$ to $s = |\mathcal{S}|$
 - i. Create a tabu list with length $nPaths$
 - ii. For $nCycles = 1$ to $nCycles = 0$
 - A. Compute \bar{B} and $\bar{c}^Q, s \in \mathcal{S}_Q$ or $\bar{c}^B, s \in \mathcal{S}_B$ for \bar{R}_a
 - B. Compute and order the values of the function $\xi(f_s)$, with $\xi(f_s) = F_L(f_s)$ if $nCycles = 1$ and $\xi(f_s) = F_C^{Q(B)}(f_s)$ if $nCycles = 0$
 - C. Find the $nPaths$ with lower value of $\xi(f_s)$
 - D. Compute with MMRA-S2 new candidate paths for the corresponding O-D pairs and define a new set of first and second choice paths $(r_{\text{new}}^1(f_s); r_{\text{new}}^2(f_s))$ for the service s , according to the rules established for each service
 - E. For the $nPaths$ flows with lower value of $\xi(f_s)$, initialise $(W_Q(f_s) - W_Q) - a \cdot \text{freq}(f_s)$
 - F. Cycle in numIterations
 - Go through the flows f_s ordered according to $\xi(f_s)$
 - * Check whether the pair of paths for this flow is tabu.
 - * Compute \bar{B} for \bar{R}_a
 - * Compute $W_Q^{\text{old}}, B_{Mm|Q}^{\text{old}}$ and $B_{ms|Q}^{\text{old}}, B_{Ms|Q}^{\text{old}}$ if $s \in \mathcal{S}_Q$ or W_B^{old} if $s \in \mathcal{S}_B$ for \bar{R}_a
 - * $(r_{\text{old}}^1(f_s); r_{\text{old}}^2(f_s)) \leftarrow (r_a^1(f_s); r_a^2(f_s))$
 - * $(r_a^1(f_s); r_a^2(f_s)) \leftarrow (r_{\text{new}}^1(f_s); r_{\text{new}}^2(f_s))$
 - * Compute \bar{B} for \bar{R}_a
 - * Compute $W_Q, B_{Mm|Q}$ and $B_{ms|Q}, B_{Ms|Q}$ if $s \in \mathcal{S}_Q$ or W_B if $s \in \mathcal{S}_B$ for \bar{R}_a
 - * If $(W_Q \geq W_Q^{\text{old}}$ and $B_{Mm|Q} \leq B_{Mm|Q}^{\text{old}})$ and $(B_{ms|Q} \leq B_{ms|Q}^{\text{old}}$ and $B_{Ms|Q} \leq B_{Ms|Q}^{\text{old}})$ for $s \in \mathcal{S}_Q$ or $(W_B \geq W_B^{\text{old}})$ for $s \in \mathcal{S}_B$
 - *◇ If the move is tabu,

* \diamond If $(W_Q \geq \max\{W_Q\} \text{ and } B_{Mm|Q} \leq \min\{B_{Mm|Q}\})$ and $(B_{ms|Q} \leq \min\{B_{ms|Q}\} \text{ and } B_{Ms|Q} \leq \min\{B_{Ms|Q}\})$ for $s \in \mathcal{S}_Q$ or $(W_B \geq \max\{W_B\})$ for $s \in \mathcal{S}_B$ then

* \diamond \square $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}$

* \diamond \square $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$

* \diamond \square $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$

* \diamond \square $\text{freq}(f_s) \leftarrow \text{freq}(f_s) + 1$

Update the tabu list.

* \diamond Else, $(r_a^1(f_s); r_a^2(f_s)) \leftarrow (r_{\text{old}}^1(f_s); r_{\text{old}}^2(f_s))$

* \diamond Else

* \diamond \square $\text{freq}(f_s) \leftarrow \text{freq}(f_s) + 1$

* \diamond If $(W_Q \geq \max\{W_Q\} \text{ and } B_{Mm|Q} \leq \min\{B_{Mm|Q}\})$ and $(B_{ms|Q} \leq \min\{B_{ms|Q}\} \text{ and } B_{Ms|Q} \leq \min\{B_{Ms|Q}\})$ for $s \in \mathcal{S}_Q$ or $(W_B \geq \max\{W_B\})$ for $s \in \mathcal{S}_B$ then

* \diamond \square $\min\{B_{ms|Q}\} \leftarrow B_{ms|Q}, \min\{B_{Ms|Q}\} \leftarrow B_{Ms|Q}$

* \diamond \square $\max\{W_Q\} \leftarrow W_Q, \min\{B_{Mm|Q}\} \leftarrow B_{Mm|Q}$

* \diamond \square $\overline{R}_*(s) \leftarrow \overline{R}_a(s)$

* \diamond Add move to the tabu list.

* \diamond Leave the cycle of going through the flows

* Else,

* \diamond If the move is not tabu, then compute $(W_Q(f_s) - W_Q) - a \cdot \text{freq}(f_s)$

* \diamond $(r_a^1(f_s); r_a^2(f_s)) \leftarrow (r_{\text{old}}^1(f_s); r_{\text{old}}^2(f_s))$

End of the cycle of going through the flows

- If no new solution that improves the current solution was found,
 - * Find the solution $\overline{R}_{\text{best}}(s)$ obtained with a non-tabu move with best value of $(W_Q(f_s) - W_Q) - a \cdot \text{freq}(f_s)$
 - * $\overline{R}_a(s) \leftarrow \overline{R}_{\text{best}}(s)$

End of the Cycle in $numIterations$

End of the cycle For $(nCycles)$

End of the cycle For (s)

End of the cycle For (ape)

End of the cycle For $(nPaths)$

X. If $W_Q^o > \max\{W_Q\}$ or $B_{Mm|Q}^o < \min\{B_{Mm|Q}\}$ then

1. The best solution is \overline{R}_o

XI. Otherwise, the best solution is \overline{R}_*

XII. Compute the objective function values for the best solution

C Calculation of the Numerical Complexity of the Meta-Heuristics

C.1 HMOR-S2_{SA}

The instructions in the inner cycle of the procedure HMOR-S2_{SA} are executed $C_i^{\text{HMOR-S2}_{\text{SA}}}$ times, with $C_i^{\text{HMOR-S2}_{\text{SA}}}$ given by

$$\begin{aligned}
C_i^{\text{HMOR-S2}_{\text{SA}}} &= (\#ape)(\#nCycles) \sum_{u=1}^{|\mathcal{S}|} \sum_{nPaths=1}^{|\mathcal{F}_u|} \left\lceil \frac{|\mathcal{F}_u| + 1 - nPaths}{2} \right\rceil \\
&\downarrow \#ape = 2; \#nCycles = 2; |\mathcal{F}_u| = |\overline{\mathcal{F}}| \forall u \in \mathcal{S} \\
&= 4|\mathcal{S}| \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} \left\lceil \frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} \right\rceil \\
&\downarrow |\overline{\mathcal{F}}| \text{ is an even number} \\
&= 4|\mathcal{S}| \left(\sum_{\substack{nPaths=1 \\ nPaths \text{ is odd}}}^{|\overline{\mathcal{F}}|} \frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} + \sum_{\substack{nPaths=1 \\ nPaths \text{ is even}}}^{|\overline{\mathcal{F}}|} \left(\frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} + \frac{1}{2} \right) \right) \\
&= 2|\mathcal{S}| \left(\frac{|\overline{\mathcal{F}}|}{2} (2|\overline{\mathcal{F}}| + 3) - \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} nPaths \right) \\
&= 2|\mathcal{S}| \left(\frac{|\overline{\mathcal{F}}|}{2} (2|\overline{\mathcal{F}}| + 3) - \frac{|\overline{\mathcal{F}}| (|\overline{\mathcal{F}}| + 1)}{2} \right) \\
&= |\mathcal{S}| |\overline{\mathcal{F}}| (|\overline{\mathcal{F}}| + 2)
\end{aligned}$$

As for the number of solutions that are analysed,

$$\begin{aligned}
C_s^{\text{HMOR-S2SA}} &= (\#ape)(\#nCycles) \sum_{u=1}^{|\mathcal{S}|} \sum_{nPaths=1}^{|\mathcal{F}_u|} \left\lceil \frac{|\mathcal{F}_u| + 1 - nPaths}{2} \right\rceil nPaths \\
&\downarrow \#ape = 2; \#nCycles = 2; |\mathcal{F}_u| = |\overline{\mathcal{F}}| \forall u \in \mathcal{S} \\
&= 4|\mathcal{S}| \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} \left\lceil \frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} \right\rceil nPaths \\
&\downarrow |\overline{\mathcal{F}}| \text{ is an even number} \\
&= 4|\mathcal{S}| \left(\sum_{\substack{nPaths=1 \\ nPaths \text{ is odd}}}^{|\overline{\mathcal{F}}|} \frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} nPaths + \right. \\
&\quad \left. + \sum_{\substack{nPaths=1 \\ nPaths \text{ is even}}}^{|\overline{\mathcal{F}}|} \left(\frac{|\overline{\mathcal{F}}| + 1 - nPaths}{2} + \frac{1}{2} \right) nPaths \right) \\
&= 2|\mathcal{S}| \left((|\overline{\mathcal{F}}| + 1) \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} nPaths - \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} nPaths^2 + \sum_{\substack{nPaths=1 \\ nPaths \text{ is even}}}^{|\overline{\mathcal{F}}|} nPaths \right) \\
&= 2|\mathcal{S}| \left((|\overline{\mathcal{F}}| + 1) \frac{|\overline{\mathcal{F}}|(|\overline{\mathcal{F}}| + 1)}{2} - \frac{|\overline{\mathcal{F}}|(|\overline{\mathcal{F}}| + 1)(2|\overline{\mathcal{F}}| + 1)}{6} + \frac{|\overline{\mathcal{F}}|^2 + 2|\overline{\mathcal{F}}|}{4} \right) \\
&= \frac{1}{6} |\mathcal{S}| |\overline{\mathcal{F}}| (2|\overline{\mathcal{F}}|^2 + 9|\overline{\mathcal{F}}| + 10)
\end{aligned}$$

C.2 HMOR-S2_{TS}

The instructions in the inner cycle of the procedure HMOR-S2_{TS} are executed $C_i^{\text{HMOR-S2TS}}$ times, with $C_i^{\text{HMOR-S2TS}}$ given by

$$\begin{aligned}
C_i^{\text{HMOR-S2TS}} &= (\#ape)(\#nCycles) \sum_{u=1}^{|\mathcal{S}|} \sum_{nPaths=1}^{|\mathcal{F}_u|} 1 \\
&\downarrow \#ape = 2; \#nCycles = 2; |\mathcal{F}_u| = |\overline{\mathcal{F}}| \forall u \in \mathcal{S} \\
&= 4|\mathcal{S}| \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} 1 \\
&= 4|\mathcal{S}| |\overline{\mathcal{F}}|
\end{aligned}$$

As for the number of solutions that are analysed,

$$\begin{aligned}
C_s^{\text{HMOR-S2TS}} &= (\#ape)(\#nCycles) \sum_{u=1}^{|\mathcal{S}|} \sum_{nPaths=1}^{|\mathcal{F}_u|} nPaths \\
&\downarrow \#ape = 2; \#nCycles = 2; |\mathcal{F}_u| = |\overline{\mathcal{F}}| \forall u \in \mathcal{S} \\
&= 4|\mathcal{S}| \sum_{nPaths=1}^{|\overline{\mathcal{F}}|} nPaths \\
&= 2|\mathcal{S}| |\overline{\mathcal{F}}| (|\overline{\mathcal{F}}| + 1)
\end{aligned}$$