RUBIS : a bipolar-valued outranking method for the choice method

Raymond Bisdorff

Computer Science and Communication, University of Luxembourg

3rd DECISION DECK Workshop, Coimbra, June, 2008

Content Introductory Example The Ruby choice method Solving the introductory example

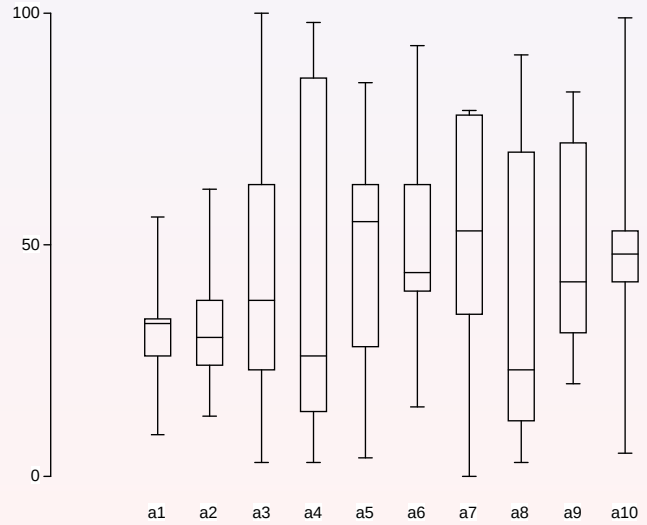
Introductory example

Decision:problem: *Choose the best from a set of ten alternatives evaluated on 7 criteria as shown below.*

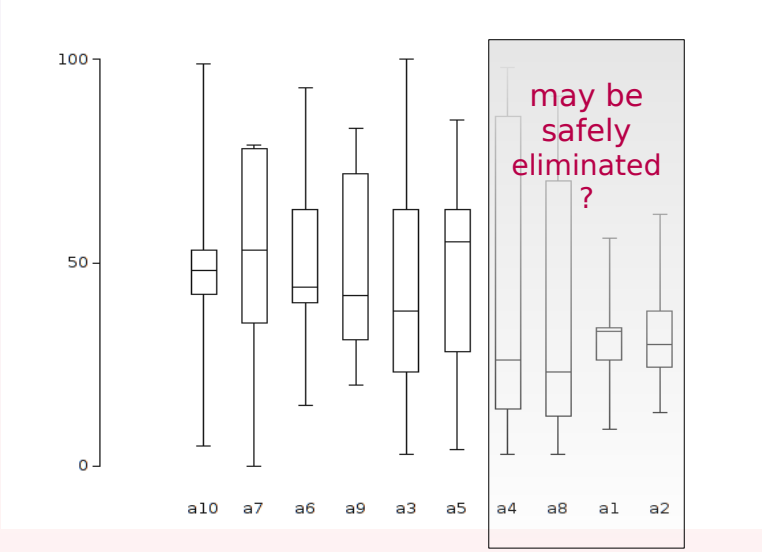
criterion	weight	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
g_1	7	33	13	3	14	48	44	18	47	31	98
g_2	7	9	30	23	86	63	40	79	3	83	48
g_3	5	34	38	63	16	85	53	78	91	47	42
g_4	5	53	24	38	3	28	93	35	12	72	5
g_5	5	26	44	60	98	62	15	53	23	37	44
g_6	4	26	29	100	36	4	63	54	70	24	53
g_7	1	56	62	33	36	21	49	0	13	20	99

- The performance scale on each criteria is $0 - 100$ pts, with a weak preference threshold of 10 points, a preference threshold of 20 pts, and a veto threshold of 80 pts.
- We assume that the criteria are not commensurable.

Introductory example: Boxplots of the performances



Introductory example: Boxplots of the performances



Introductory example: Ranking the performances?

Criterion	a10	a7	a6	a9	a3	a5	a4	a8	a1	a2
"g1"	98	18	44	31	3	48	14	47	33	13
"g2"	48	79	40	83	23	63	86	3	9	30
"g3"	42	78	53	47	63	85	16	91	34	38
"g4"	5	35	93	72	38	28	3	12	53	24
"g5"	44	53	15	37	60	62	98	23	26	44
"g6"	53	54	63	24	100	4	36	70	26	29
"g7"	99	0	49	20	33	21	36	13	56	62

Introductory example: Pairwise comparisons

Is a_{10} globally at least as good as a_7?

Outranking thresholds: weak preference ($\geqslant 10$), preference ($\geqslant 20$), veto ($\leqslant -80$).

criterion	w_i	a_{10}	a_7	$\Delta_i(10,7)$	balance	veto ?
g_1	7	98	18	80	+7	no
g_2	7	48	79	-31	-7	no
g_3	5	42	78	-36	-5	no
g_4	5	5	35	-30	-5	no
g_5	5	44	53	-9	+5	no
g_6	4	53	54	-1	+4	no
g_7	1	99	0	99	+1	no
				total balance	0	

We observe a balanced situation.
No conclusion can be drawn.

Introductory example: Pairwise comparisions (continued)

Is a_7 globally at least as good as a_{10}?

Outranking thresholds: weak preference ($\geqslant 10$), preference ($\geqslant 20$), veto ($\leqslant -80$).

criterion	w_i	a_7	a_{10}	$\Delta_i(10,7)$	balance	veto ?
g_1	7	18	98	-80	-7	yes
g_2	7	79	48	+31	+7	no
g_3	5	78	42	+36	+5	no
g_4	5	35	5	+30	+5	no
g_5	5	53	44	+9	+5	no
g_6	4	54	53	+1	+4	no
g_7	1	0	99	-99	-1	yes
				total balance	+18-34	

We observe a veto situation on criteria g_1 and g_7.
a_7 is clearly not globally at least as good as a_{10}? !

Introductory example: Pairwise comparisions (continued)

Is a_{10} (resp. a_6) globally at least as good as a_6 (resp. a_{10}) ?

g_i	w_i	a_{10}	a_6	$\Delta_i(10,6)$	balance	veto?	$\Delta_i(6,10)$	balance	veto?
g_1	7	98	44	54	+7	no	-54	-7	no
g_2	7	48	40	8	+7	no	-8	+7	no
g_3	5	42	53	-11	-5	no	11	+5	no
g_4	5	5	93	-88	-5	yes	88	+5	no
g_5	5	44	15	29	+5	no	-29	-5	no
g_6	4	53	63	-10	0	no	10	+4	no
g_7	1	99	49	50	+1	no	-50	-1	no
				total balance	+10 -34		total balance	+8	

- a_{10} is clearly not globally at least as good as a_6 (veto (-88) on criterion g_4)!
- Note the weak preference situation on criterion g_6 !
- a_6 is globally at least as good as a_{10} (balance of +8 in favour).

Introductory example: Pairwise comparisions (continued)

Is a_6 globally at least as good as a_7 ?

criteria	weight	a_7	a_6	$\Delta_i(7,6)$	balance	veto ?
g_1	7	44	18	26	+7	no
g_2	7	40	79	-39	-7	no
g_3	5	53	78	-25	-5	no
g_4	5	93	35	58	+5	no
g_5	5	15	53	-38	-5	no
g_6	4	63	54	9	+4	no
g_7	1	49	0	49	+1	no
				total balance	0	

We observe again a balanced situation.
No conclusion can be drawn.

Introductory example: Global outranking relation

$\tilde{S}$	a_{10}	a_7	a_6	a_9	a_3	a_5	a_4	a_8	a_1	a_2
a_{10}	-	0	-34	10	1	2	10	20	24	29
a_7	-34	-	8	15	24	18	22	10	20	32
a_6	8	0	-	10	11	0	-34	24	29	23
a_9	10	11	7	-	10	7	19	9	32	32
a_3	-34	8	2	-4	-	-4	3	10	13	25
a_5	10	19	14	2	-34	-	1	26	14	24
a_4	-34	10	-34	7	6	0	-	2	10	12
a_8	-34	0	-34	-34	-10	5	-34	-	22	3
a_1	-9	-8	-10	5	-1	-7	6	9	-	15
a_2	-34	-3	-10	3	6	-9	10	2	10	-

Backbone of RUBIS : $\widetilde{S}$

- Let X be a finite set of p alternatives.
- Let N be a finite set of $n > 1$ criteria.
- Let m be the total significance of the criteria.
- Let x and y be two alternatives from X.
- Let x_i be the value taken by x on criterion g_i

Definition (The outranking situation)

- x outranks y ($x\,S\,y$) if there is a significant majority of criteria which support an at least as good statement and there is no criterion which raises a veto against it.
- The bipolar valued relation $\widetilde{S} \in [-m, m]$ expresses the credibility of the validation or the non-validation of the outranking relation S.

Backbone of RUBIS : $\widetilde{\mathrm{S}}$

Definition (The bipolar valued outranking situation)

$$\widetilde{\mathrm{S}}(x,y) = \min\Big\{ \big(\sum_{i\in N} w_i \cdot C_i(x,y) \big), \min_{i\in N} \big(-V_i(x,y) \big) \cdot m \Big\}$$

$$C_i(x,y) = \begin{cases} 1 & \text{if} \quad x_i + q_i > y_i; \\ -1 & \text{if} \quad x_i + p_i \leqslant y_i \\ 0 & \text{otherwise} \end{cases}$$

$$-V_i(x,y) = \begin{cases} 1 & \text{if} \quad x_i + wv_i > y_i; \\ -1 & \text{if} \quad x_i + v_i \leqslant y_i \\ 0 & \text{otherwise} \end{cases}$$

where q_i, p_i represent the weak preference, resp. the preference, and wv_i, v_i, the weak veto, resp. the veto, threshold on criterion g_i.

Backbone of RUBIS : $\widetilde{\mathrm{S}}$

$\widetilde{\mathrm{S}}$ is defined on a bipolar-valued credibility scale $\mathcal{L} = [-m, m]$ supporting the following demantics denotation:

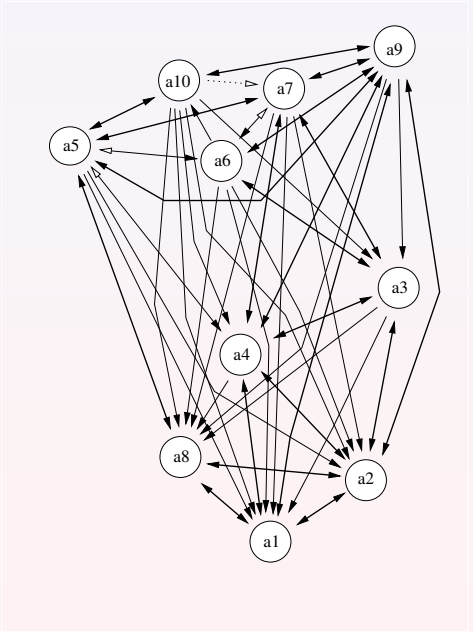
- $\widetilde{\mathrm{S}}(x,y) = +m$ means that assertion $x\,\mathrm{S}\,y$ is clearly validated.
- $\widetilde{\mathrm{S}}(x,y) = -m$ means that assertion $x\,\mathrm{S}\,y$ is clearly non-validated.
- $\widetilde{\mathrm{S}}(x,y) > 0$ means that assertion $x\,\mathrm{S}\,y$ is more validated than non-validated.
- $\widetilde{\mathrm{S}}(x,y) < 0$ means that assertion $x\,\mathrm{S}\,y$ is more non-validated than validated.
- $\widetilde{\mathrm{S}}(x,y) = 0$ means that assertion $x\,\mathrm{S}\,y$ is undetermined.

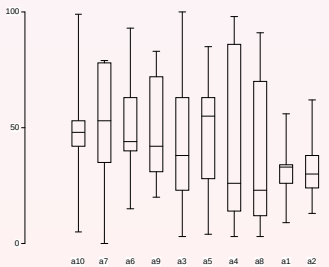
Backbone of RUBIS : $\widetilde{G}(X, \widetilde{\mathrm{S}})$

Definition (The bipolar valued outranking digraph)

- We denote $\widetilde{G}(X, \widetilde{\mathrm{S}})$ the bipolar-valued outranking digraph modelled via $\widetilde{\mathrm{S}}$ on $X \times X$.
- The associated crisp outranking relation S may be recovered from $\widetilde{\mathrm{S}}$ as the set of pairs (x,y) such that $\widetilde{\mathrm{S}} > 0$.
- $G(X, \mathrm{S})$ is called the crisp outranking digraph associated with $\widetilde{G}(X, \widetilde{\mathrm{S}})$.

Introductory example: The crisp outranking digraph





RUBIS decision aiding approach

- A choice problem traditionally consists in the search for a single best alternative.
- We adopt a progressive decision analysis process which allows to uncover the best single choice via possible intermediate recommendations.
- These intermediate choice recommendations, the case given, have to be refined at some further stages of the decision analysis.

Pragmatic choice recommendation (CR) principles

$\mathcal{P}_1$: Non-retainement for well motivated reasons.
all eliminated alternative must be considered worse as at least one recommended alternative.

$\mathcal{P}_2$: Minimal size.
the CR should be as limited as possible.

$\mathcal{P}_3$: Efficient and informative.
each CR must deliver a stable recommendation.

$\mathcal{P}_4$: Effectively better.
the CR should not correspond simultaneously to a choice and an elimination recommendation.

$\mathcal{P}_5$: Maximally credible.
the CR must be as credible as possible wrt the preferential knowedge modelled via $\widetilde{\mathrm{S}}$.

Useful choice qualifications in $\widetilde{G}(X, \widetilde{\mathrm{S}})$

Let Y be a non-empty subset of X, called a choice in $\widetilde{G}$.

- Y is said to be outranking (resp. outranked) iff $x \notin X \Rightarrow \exists y \in Y : \widetilde{\mathrm{S}}(x, y) > 0)$.
- Y is said to be independent iff for all $x \neq y$ in Y we have X $\widetilde{\mathrm{S}}(x, y) \leqslant 0)$.
- Y is called an outranking kernel (resp. outranked kernel) iff it is an outranking (resp. outranked) and indendent choice.
- Y is called an outranking hyperkernel (resp. outranked hyperkernel) iff it is an outranking (resp. outranked) choice which consists of independent chordless circuits of odd order $p \geqslant 1$.

Tranlating CR principles into choice qualifications

$\mathcal{P}_1$: Non-retainment for well motivated reasons.
A CR is an outranking choice.

$\mathcal{P}_{2+3}$: Minimal size & stable.
A CR is a hyperkernel.

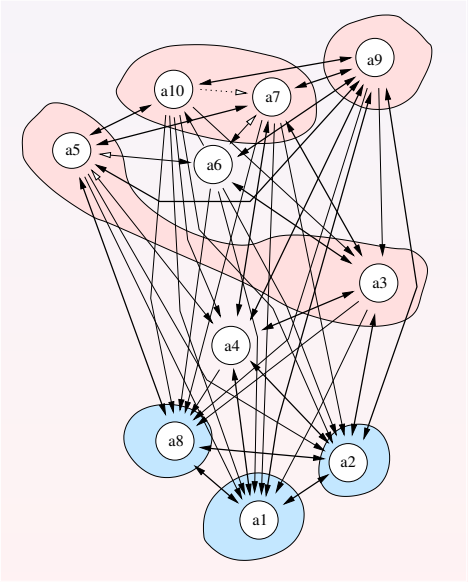
$\mathcal{P}_4$: Effectivity.
A CR is a stricly more outranking than outranked choice.

$\mathcal{P}_5$: Maximal credibility.
A CR has maximal determinateness.

Theorem

Any bipolar outranking digraph contains at least one outranking and one outranked hyperkernel.

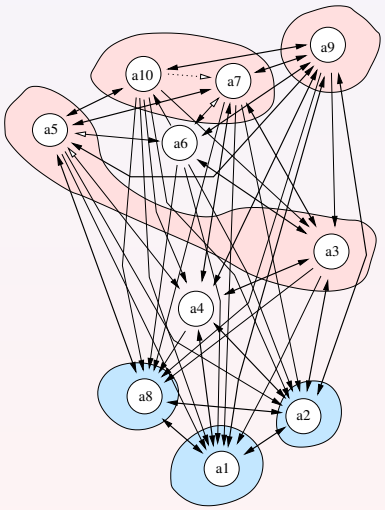
Introductory example: All outranking and outranked hyperkernels



The Rubis choice recommendation (RCR)

- A RCR verifies the five CR principles.
- A maximally determined strict outranking hyperkernel, if it exists in $\widetilde{G}$, gives a RCR.
- A RCR is a provisional subset of alternatives, most certainly containing the best alternative, if it exists !.
- A RCR must not be confused with the ultimate best choice of the decision maker.
- The Rubis choice method is only convenient in a progressive decision aiding approach.

Introductory example: all kernels and hyperkernels



outranking choices:

$\{a_9\}$

$\{a_3, a_5\}$

$\{a_{10}, a_7\}$

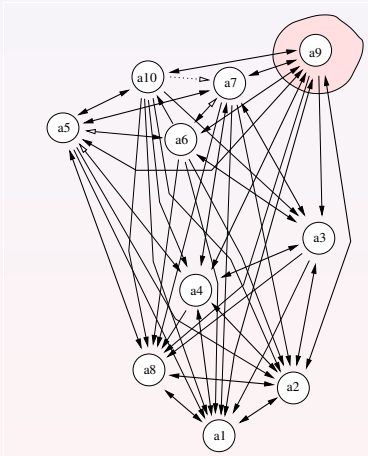
outranked choices:

$\{a_8\}$

$\{a_2\}$

$\{a_1\}$

Introductory example: The Rubis choice recommendation



choice : $\{a_9\}$

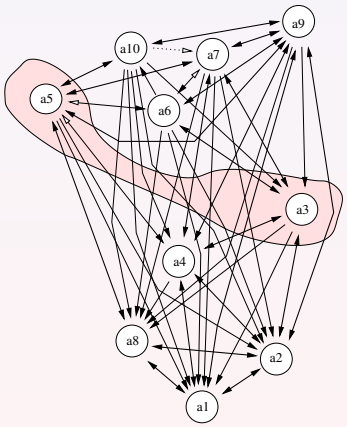
determinateness : 60%

(weighted majority of criterion)

outrankingness : 60%

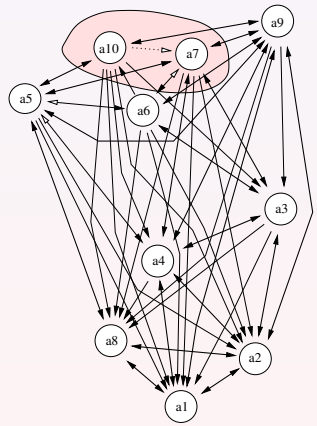
outrankedness : 0%

Introductory example: Potential choice recommendation



choice : $\{a_3, a_5\}$
determinateness : 53%
(weighted majority of criterion)
outrankingness : 53%
outrankedness : 48.5%

Introductory example: Other potential choice recommendation



choice : $\{a_{10}, a_7\}$
determinateness : 50%
(weighted majority of criterion)
outrankingness : 62%
outrankedness : 38%

Concluding remarks

Properties of the RUBIS choice recommendation:

- Progressiveness: intermediate solutions are proposed to the decision maker;
- Existence: A RCR always exists in a non-symmetrical bipolar-valued outranking digraph;
- Multiplicity: In case multiple RCR coexist, their union gives a suitable intermediate choice recommendation;
- Missing values: They are treated as information which is not available at a given stage of the decision analysis; which might be determined later on;
- Efficient decision aiding: Strongly motivated conclusions can nevertheless be drawn.

Bibliography I

R. Bisdorff, P. Meyer, M. Roubens (2008)
RUBIS: a bipolar-valued outranking method for the choice problem.
4OR, A Quarterly Journal of Operations Research, Springer-Verlag,6:143-165
(Online) Electronic version: DOI: 10.1007/s10288-007-0045-5.

R. Bisdorff, M. Pirlot and M. Roubens (2006).
Choices and kernels from bipolar valued digraphs.
European Journal of Operational Research, 175 (2006) 155-170.

R. Bisdorff (2007)
The Python Digraph implementation for Ruby: User manual.
University of Luxembourg, 2007,
http://ernst-schroeder.uni.lu/bisdorff/Digraph

Bibliography II

- D2-Decision-Deck Consortium (2007)
  The D2-Decision-Deck project.
  http://www.decision-deck.org/
- Ernst-Schroeder: The Decision Deck RIA server
  University of Luxembourg.
  http://ernst-schroeder.uni.lu/